Additional Mid1 Review Problems

An inner cylinder of radius R aligned along the z-axis carries a surface current of $K_0 \sin\omega t\ \hat{z}$. An outer cylinder of radius 2R carries a canceling surface current of $(-K_0 \sin\omega t / 2)\ \hat{z}$. Work in the quasi-static limit and ignore all terms of order ω^2 or higher.

(a) Start by computing $\vec{E}\left(R < s < 2R\right)$ in terms of $K_0, \omega,$ and R, and cylindrical coordinates. $\boxed{\vec{E} = \hat{z}\mu_0 R K_0 \omega \cos\omega t \ln\left(s/R\right)}$

(b) Construct the Poynting vector and show that it satisfies $\vec{\nabla} \bullet \vec{S} + \partial_t u_B + \vec{E} \bullet \vec{J} = 0$ in the region $R < s < 2R$. We are ignoring $\partial_t u_E$ since we are in the quasi-static limit. $\boxed{\vec{S} = -\frac{\mu_o \omega R^2 K_0^2}{s} \sin\omega t \cos\omega t \ln\left(s/R\right)\hat{s}}$

(c) Verify that $\int \vec{S} \bullet d\vec{a} + \partial_t \int u_B d\tau + \int \vec{E} \bullet \vec{K}\ da = 0$ for a cylindrical shell with $s > 2R$ and length Δz. $\boxed{\int \vec{E} \bullet \vec{K}\ da = -2\pi \Delta z \mu_0 \ln\left(2\right) R^2 K_0^2 \omega \cos\omega t \sin\omega t}$