Physics 436
Fall 2009

Homework #10

1) Consider an incident photon that collides on a stationary proton to form a particle --called the $\psi(3095)$ -- with a rest energy of 3095 MeV along with a proton in the center of momentum frame. $\gamma p \to \psi(3095)\ p$. Use 940 MeV for the rest energy of the proton.

(a) What is the minimum (threshold) lab energy of the incident photon?

(b) What is the momentum of the incident photon in the center of momentum frame at threshold? Hint—Use the dot product between the incident photon 4-momentum and the sum of the incident photon and proton 4-momenta.

2) Consider the symmetric decay of a particle into two pions $(m_\pi c^2 = 140$ MeV$)$ with lab momenta of $\vec{p}_1 = 707\hat{x} + 207\hat{y}$ MeV/c and $\vec{p}_2 = 707\hat{x} - 207\hat{y}$ MeV/c . The decay point lies 7.64 cm downstream of the point where the particle was created in the lab frame.

a) Find the mass of the decaying particle.

b) Find the lifetime of the decaying particle in its own rest-frame.

3) In this problem you will find formulae used to design a high energy neutrino beam based on the decay $\pi \to \mu\nu$ where ν (or neutrino) has very nearly zero mass and $m_\pi c^2 = 0.140$ GeV/c^2 and $m_\mu c^2 = 0.105$ GeV/c^2.

(a) Use $(\tilde{\pi} - \tilde{\nu})\bullet(\tilde{\pi} - \tilde{\nu}) = (m_\mu c)^2$ to obtain $\mathcal{E}_\nu(\mathcal{E}_\pi - c\vec{p}_\pi \bullet \hat{k}) = \frac{(m_\pi^2 - m_\mu^2)c^4}{2}$ where $\hat{k}$ is a unit vector that points along the momentum of the neutrino.

(b) Show that $\mathcal{E}_\nu = \frac{(m_\pi^2 - m_\mu^2)c^4}{2\mathcal{E}_\pi}\frac{1}{1 - \beta\cos\theta}$ where β is the v/c of the pion in the lab frame and θ is the (lab) angle between the pion velocity and the neutrino.

(c) In the limit $\theta \ll 1$ and $\mathcal{E}_\pi \gg m_\pi c^2$ show that $\mathcal{E}_\nu \to \frac{\mathcal{E}_o}{1 + (\theta/\theta_0)^2}$. Find $\mathcal{E}_o$ and θ_0 for the case of a 100 GeV incident pion beam. Hints: use the approximations $\cos\theta \approx 1 - \theta^2/2$ and $\beta \approx 1 - \frac{1}{2}\left(\frac{m_\pi c^2}{\mathcal{E}_\pi}\right)^2$.