Homework #4

1. In lecture we obtained the Fresnel Equations for the case of electric field vectors lying in the plane of the three k-vectors using just electric field boundary conditions. Assume that μ is the same on either side of the boundary. Use the r and t expressions we obtained for the electric fields, to show that the magnetic boundary conditions are also satisfied.

2. In lecture we considered reflection from a dielectric boundary at normal incidence. Let $\tilde{E}_1 = \left(E_I e^{ikz-i\omega t} + E_R e^{-ikz-i\omega t}\right)\hat{x}$ and $\tilde{B}_1 = \frac{1}{v_1}\left[E_I e^{ikz-i\omega t} - E_R e^{-ikz-i\omega t}\right]\hat{y}$. Use $\left\langle \vec{S}_1 \right\rangle = \frac{\text{Re}\left\{\tilde{E}^* \times \tilde{B}\right\}}{2\mu_1}$ to show $\left\langle \vec{S}_1 \right\rangle = \frac{\hat{z}\left|E_I\right|^2}{2\mu_1 v_1}\left(1-\left|r\right|^2\right)$ where $r = \frac{E_R}{E_I}$.

3. Consider an incident wave traveling in air of the form $\vec{E}_I = E_I \hat{x} e^{ikz-i\omega t}$ which is normally incident on a transparent, non-magnetic, dielectric with $\mu = \mu_0$, and $\varepsilon = \varepsilon_0 N^2$ which starts at z=0. Calculate the time average surface pressure on the dielectric using the Maxwell Stress tensor and a pill box bounding surface which extends just below and just above the z=0 plane in terms of E_I and the index of refaction, N . Is the surface force parallel or antiparallel to $\hat{z}$? Assume that the Stress tensor in the dielectric is the same form as the vacuum stress tensor with $\varepsilon_0 \to \varepsilon$ and $\mu_0 \to \mu$, **Warning: this analysis is rather controversial and may be wrong**. Although this problem was first discussed by Poynting over 100 years ago, if you google “pressure on a dielectric boundary” you will see papers published earlier recently discussing Poynting’s approach and others which contradict it.

4. Griffiths problem 9.16

5. In lecture we discussed an evanescent wave crossing the gap between two matching right angle prisms and concluded that $\vec{E}_T = \hat{y}\left[\left(ae^{-\gamma z} + be^{\gamma z}\right)e^{ikx-i\omega t}\right]$ where $k,\ \gamma$ are real and a,b are complex.

(a) Calculate the magnetic field using $i\omega\vec{B} = \vec{\nabla}\times\vec{E}$.

(b) Show that $\left\langle \vec{S}_z \right\rangle = S_0 \,\text{Im}\left\{a^* b\right\}$ and find S_0 in terms of ω, γ, a, b.

6. Recall that for electromagnetic waves traveling in perfect conductors, the magnetic field is 45^0 out of phase with the electric field. Now consider an electromagnetic wave traveling through a conductor with ε, μ and conductivity σ.

(a) Show that angular frequency ω such that the magnetic field is 30^0 out of phase can be written as $\omega = G\sigma / \varepsilon$ where G is a numeric factor. Find *G*.

(b) Show that σ/ε has the dimensions of a frequency.

7. A simple model for high frequency AC current flow through a round wire of radius r ,aligned in the z direction, is $\vec{J}\left(r - d_{skin} < s < r\right) = J_0 \hat{z}$ and $\vec{J} = 0$ otherwise. At frequencies of 10GHz, the skin depth for good conductors is $d_{skin} \approx 1\ \mu\text{m}$. Use this model to compute the ratio of the resistance for a 10 GHz current to the resistance for a 60Hz ($d_{skin} \to \infty$) current flowing through an arbitrary length of 1 cm diameter wire ($r = 5$ mm). Incidentally, the resistance ratio is the ratio of the power required to transmit the same r.m.s. current at these two frequencies.