Problem 1

An infinitely long wire with linear charge density $-\lambda$ lies along the $z$ axis. An insulating cylindrical shell of radius $R$ and moment of inertia $I$ per unit length is concentric with the wire, and can rotate freely about the $z$ axis. The surface charge density on the shell is $\sigma = \lambda/(2\pi R)$ and is uniformly distributed. The cylinder is immersed in an external magnetic field $B_{\text{ext}} = B\hat{z}$, and is initially at rest. Starting at $t = 0$, the external magnetic field is gradually switched off.

a) Using conservation of angular momentum, determine the final angular velocity $\omega$ of the shell.

b) Check your answer by using Faraday’s law.

Problem 2 (Griffiths 9.12)

Find all elements of the Maxwell stress tensor for a monochromatic plane wave traveling in the $z$ direction and linearly polarized in the $x$ direction, i.e.

$$E(z,t) = E_0 \cos(kz - \omega t + \delta) \hat{x}, \quad B(z,t) = \frac{E_0}{c} \cos(kz - \omega t + \delta) \hat{y}.$$  \hspace{1cm} (1)

Comment on the form of your answer (remember that $\mathbf{T}$ represents the momentum flux density).

Problem 3

The electric and magnetic fields can be written in terms of a scalar potential $V$ and a vector potential $\mathbf{A}$ as

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}. \hspace{1cm} (2)$$

Maxwell’s equations in vacuum then imply

$$0 = \nabla^2 V + \frac{\partial}{\partial t} (\nabla \cdot \mathbf{A}) \hspace{1cm} (3)$$

$$0 = \left( \nabla^2 \mathbf{A} - \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} \right) - \nabla \left( \nabla \cdot \mathbf{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t} \right). \hspace{1cm} (4)$$

Find a condition that would make both $V$ and the components of $\mathbf{A}$ satisfy decoupled wave equations. Assuming a monochromatic wave solution, write down this condition in terms of the amplitudes of the corresponding waves.