Physics Illinois	**Exam # 1 Review Session** Fall 2010 Instructor: Prof. Nadya Mason	PHYS 436 - Electromagnetic Fields II Date: Sept. 20th, 2010

Problem 1

An infinitely long wire with linear charge density $-\lambda$ lies along the z axis. An insulating cylindrical shell of radius R and moment of inertia I per unit length is concentric with the wire, and can rotate freely about the z axis. The surface charge density on the shell is $\sigma = \lambda/(2\pi R)$ and is uniformly distributed. The cylinder is immersed in an external magnetic field $\mathbf{B}_{ext} = B\hat{z}$, and is initially at rest. Starting at $t = 0$, the external magnetic field is gradually switched off.

a) Using conservation of angular momentum, determine the final angular velocity ω of the shell.

b) Check your answer by using Faraday's law.

Problem 2 (Griffiths 9.12)

Find all elements of the Maxwell stress tensor for a monochromatic plane wave traveling in the z direction and linearly polarized in the x direction, i.e.

$$\mathbf{E}(z,t) = E_0 \cos(kz - \omega t + \delta)\hat{x}\,, \qquad \mathbf{B}(z,t) = \frac{E_0}{c}\cos(kz - \omega t + \delta)\hat{y}\,. \tag{1}$$

Comment on the form of your answer (remember that $\overleftrightarrow{\mathbf{T}}$ represents the momentum flux density).

Problem 3

The electric and magnetic fields can be written in terms of a scalar potential V and a vector potential $\mathbf{A}$ as

$$\mathbf{B} = \nabla \times \mathbf{A}\,, \qquad \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}\,. \tag{2}$$

Maxwell's equations in vacuum then imply

$$0 = \nabla^2 V + \frac{\partial}{\partial t}(\nabla \cdot \mathbf{A}) \tag{3}$$

$$0 = \left(\nabla^2 \mathbf{A} - \mu_0\epsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2}\right) - \nabla\left(\nabla \cdot \mathbf{A} + \mu_0\epsilon_0 \frac{\partial V}{\partial t}\right). \tag{4}$$

Find a condition that would make both V and the components of $\mathbf{A}$ satisfy decoupled wave equations. Assuming a monochromatic wave solution, write down this condition in terms of the amplitudes of the corresponding waves.