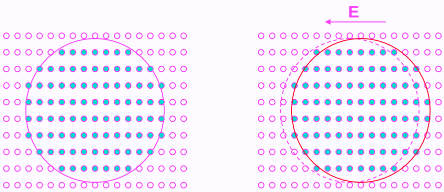
Part II - Electronic Properties of Solids
Lecture 13: The Electron Gas Continued
(Kittel Ch. 6)



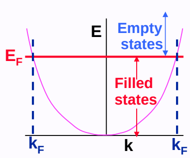
Equilibrium - no field With applied field

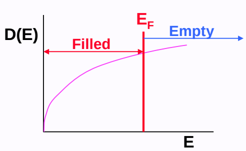
Outline

- **From last time:**
  - **Success of quantum mechanics**
  - **Pauli Exclusion Principle, Fermi Statistics**
  - **Energy levels in 1 and 3 dimensions**
  - **Density of States, Heat Capacity**
- **Today:**
  - **Fermi surface**
  - **Transport**
    - **Electrical conductivity and Ohm's law**
    - **Impurity, phonon scattering**
    - **Hall Effect**
    - **Thermal conductivity**
    - **Metallic Binding**
- **(Read Kittel Ch 6)**

Electron Gas in 3 dimensions

- Recall from last lecture:
- Energy vs k
  $E(k) = ((\hbar^2/2m)(k_x^2 + k_y^2 + k_z^2) = (\hbar^2/2m)k^2$
- Density of states
  $D(E) = (1/2\pi^2)\, E^{1/2} (\hbar^2/2m)^{-3/2} \sim E^{1/2}$

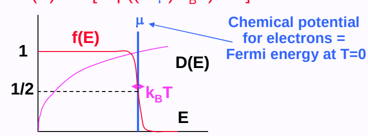




•Electrons obey exclusion Principle:
The lowest energy possible is for all states filled up to the Fermi momentum k_F and Fermi energy $E_F = (\hbar^2/2m)k_F^2$ given by
$k_F = (3\pi^2 N_{elec}/V)^{1/3}$ and $E_F = (\hbar^2/2m)(3\pi^2 N_{elec}/V)^{2/3}$

Fermi Distribution

- At finite temperature, electrons are not all in the lowest energy states. Thermal energy causes states to be partially occupied.
- Fermi Distribution (Kittel appendix)
  $f(E) = 1/[\exp((E-\mu)/k_BT) + 1]$



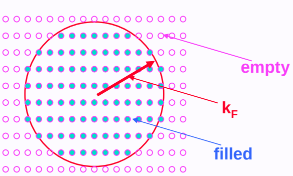
• For typical metals the Fermi energy is much greater than ordinary temperatures. Example:
For Al, E_F = 11.6 eV, i.e., $T_F = E_F/k_B$ = 13.5 x10[4] K

• At ordinary temperature, the only change in the occupation of the states is very near the chemical potential μ. States are filled for states with $E \ll \mu$, and empty for states with $E \gg \mu$.

• Heat capacity $C = dU/dT \sim N_{elec}\, k_B\, (T/T_F)$

Electrical Conductivity & Ohm's Law

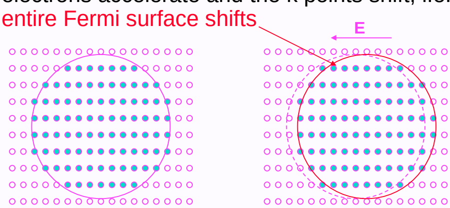
- The filling of the states is described by the Fermi surface – the surface in k-space that separates filled from empty states
- For the electron gas this is a sphere of radius k_F.



Lowest energy state filled for states with $k < k_F$, i.e., $E < E_F$

Electrical Conductivity & Ohm's Law

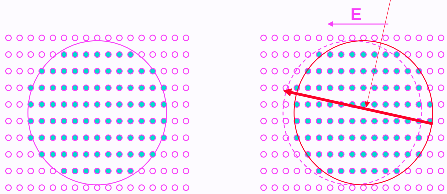
- Consider electrons in an external field E. They experience a force F = -eE
- Now $F = dp/dt = \hbar\, dk/dt$, since $p = \hbar k$
- Thus in the presence of an electric field all the electrons accelerate and the k points shift, i.e., the entire Fermi surface shifts



Equilibrium - no field With applied field

Electrical Conductivity & Ohm's Law

- What limits the acceleration of the electrons?
- Scattering increases as the electrons deviate more from equilibrium
- After field is applied a new equilibrium results as a balance of acceleration by field and scattering



Equilibrium - no field

With applied field

Electrical Conductivity and Resitivity

- The conductivity σ is defined by $j = \sigma E$, where j = current density
- How to find σ?
- From before $F = dp/dt = m\, dv/dt = \hbar\, dk/dt$
- Equilibrium is established when the rate that k increases due to E equals the rate of decrease due to scattering, then $dk/dt = 0$
- If we define a scattering time τ and scattering rate $1/\tau$

  $\hbar\,(dk/dt + k/\tau) = F = q E$ (q = charge)
- Now $j = n q v$ (where n = density) so that

  $j = n q (\hbar k/m) = (n q^2/m)\, \tau E$

  $\Rightarrow \sigma = (n q^2/m)\, \tau$

  Note: sign of charge does not matter
- Resistance: $\rho = 1/\sigma \propto m/(n q^2 \tau)$

Scattering mechanisms

- Impurities - wrong atoms, missing atoms, extra atoms, ….

  Proportional to concentration
- Lattice vibrations - atoms out of their ideal places

  Proportional to mean square displacement
- This also applies to a crystal (not just the electron gas) using the fact that there is no scattering in a perfect crystal as discussed in the next lectures

Electrical Resitivity

- Resistivity ρ is due to scattering: Scattering rate inversely proportional to scattering time τ

  $\rho \propto$ scattering rate $\propto 1/\tau$
- Matthiesson's rule - scattering rates add

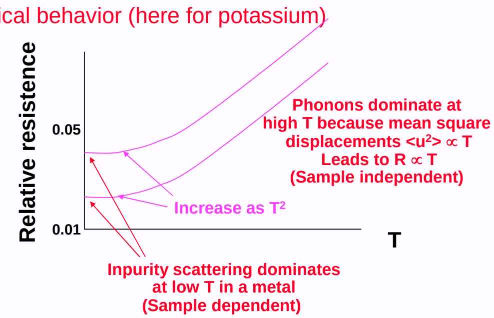
  $\rho = \rho_{vibration} + \rho_{impurity} \propto 1/\tau_{vibration} + 1/\tau_{impurity}$

  Temperature dependent $\propto \langle u^2 \rangle$

  Temperature independent - sample dependent

Electrical Resitivity

- Consider relative resistance R(T)/R(T=300K)
- Typical behavior (here for potassium)

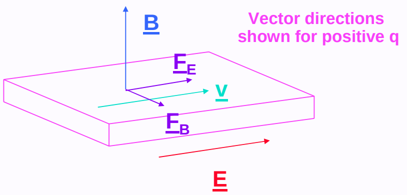


Interpretation of Ohm's law
Electrons act like a gas

- A electron is a particle - like a molecule.
- Electrons come to equilibrium by scattering like molecules (electron scattering is due to defects, phonons, and electron-electron scattering).
- Electrical conductivity occurs because the electrons are charged, and it shows the electrons move and equilibrate
- What is different from usual molecules? Electrons obey the exclusion principle. This limits the allowed scattering which means that electrons act like a weakly interacting gas.

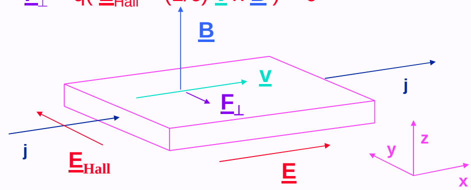
Hall Effect I

- Electrons moving in an electric and a perpendicular magnetic field
- Now we must carefully specify the vector force
  $\underline{F} = q(\, \underline{E} + (1/c)\, \underline{v} \times \underline{B}\,)$ (note: $c \rightarrow 1$ for SI units)
  ($q = -e$ for electrons)



Hall Effect II

- Relevant situation: current $j = \sigma E = nqv$ flowing along a long sample due to the field $\underline{E}$
- But NO current flowing in the perpendicular direction
- This means there must be a Hall field $\underline{E}_{Hall}$ in the perpendicular direction so the net force $\underline{F}_{\perp} = 0$
  $\underline{F}_{\perp} = q(\, \underline{E}_{Hall} + (1/c)\, \underline{v} \times \underline{B}\,) = 0$

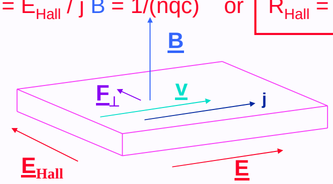


Hall Effect III

- Since
  $\underline{F}_{\perp} = q(\, \underline{E}_{Hall} + (1/c)\, \underline{v} \times \underline{B}\,) = 0$ and $v = j/nq$

then defining $v = (\underline{v})_x$, $E_{Hall} = (\underline{E}_{Hall})_y$, $B = (\underline{B})_z$,
$E_{Hall} = -\,(1/c)\,(j/nq)\,(-B)$ Sign from cross product

and the Hall coefficient is
$R_{Hall} = E_{Hall} / j B = 1/(nqc)$ or $R_{Hall} = 1/(nq)$ in SI



Hall Effect IV

- Finally, define the Hall resistance as

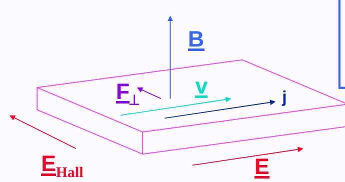
$\rho_{Hall} = R_{Hall}\, B = E_{Hall} / j$ Each of these quantities can be measured directly

which has the same units as ordinary resistivity

- $R_{Hall} = E_{Hall} / j B = 1/(nq)$

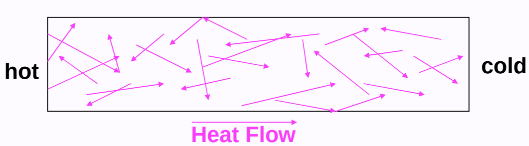
Note: R_{Hall} determines sign of charge q

Since magnitude of charge is known R_{Hall} determines density n



Heat Transport due to Electrons

- A electron is a particle that carries energy - just like a molecule.
- Electrical conductivity shows the electrons move, scatter, and equilibrate
- What is different from usual molecules?
  Electrons obey the exclusion principle. This limits scattering and helps them act like weakly interacting gas.



Heat Transport due to Electrons

- Definition (just as for phonons):
  $j_{thermal}$ = heat flow (energy per unit area per unit time)
  $= -\,K\, dT/dx$
- If an electron moves from a region with local temperature T to one with local temperature $T - \Delta T$, it supplies excess energy $c\,\Delta T$, where c = heat capacity per electron. (Note ΔT can be positive or negative).
- On the average for a thermal :
  $\Delta T = (dT/dx)\, v_x\, \tau$, where τ = mean time between collisions
- Then $j = -\,n\ \ v_x\, c\, v_x\, \tau\, dT/dx\ = -\,n\ \ c\, v_x^2\, \tau\, dT/dx$

Density Flux

Electron Heat Transport - continued

- Just as for phonons:
 Averaging over directions gives $(v_x^2)_{average} = (1/3) v^2$
 and
 $$j = - (1/3)\, n\, c\, v^2\, \tau\, dT/dx$$
- Finally we can define the mean free path $L = v\,\tau$
 and $C = nc$ = total heat capacity,
 Then
 $$j = - (1/3)\, C\, v\, L\, dT/dx$$
 and
 $K = (1/3) C v L = (1/3) C v^2 \tau$ = thermal conductivity

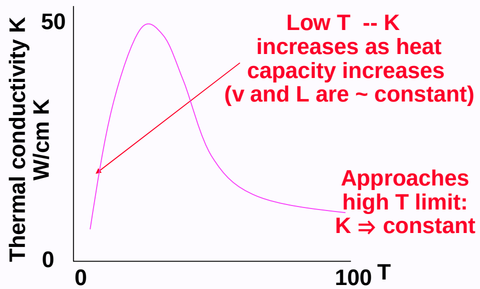
 (just like an ordinary gas!)

Electron Heat Transport - continued

- What is the appropriate v?
- The velocity at the Fermi surface = v_F
- What is the appropriate τ ?
- Same as for conductivity (almost).
- Results using our previous expressions for C:
 $$K = (\pi^2/3)\ (n/m)\ \tau\ k_B^2\ T$$
- Relation of K and σ -- From our expressions:
 $$K / \sigma = (\pi^2/3)\ (k_B/e)^2\ T$$
- This justifies the Weidemann-Franz Law that
 $K / \sigma \propto T$

Electron Heat Transport - continued

- $K \propto \sigma T$
- Recall $\sigma \to$ constant as $T \to 0$, $\sigma \to 1/T$ as $T \to$ large



Electron Heat Transport - continued

- Comparison to Phonons

 Electrons dominate in good metal crystals

 Comparable in poor metals like alloys

 Phonons dominate in non-metals

Metallic Binding

- (Treated only in problems in Kittel)
- Electron gas kinetic energy is positive, i.e., replusive.
 See homework for E, pressure, bulk modulus
 Key point: $E_{kinetic} \propto (1/V)^{2/3}$
- What is the attraction that holds metals together?
 Coulomb attraction for the nuclei
 NOT included in gas so far - must be added
- Energy of point nuclei in uniform electron gas:
 Key point: $E_{Coulomb} \propto - (1/V)^{1/3}$
 Approximate expressions in Kittel problem 8
 Energy per electron:
 $E_{Coulomb} \propto - 1.80/r_s$ Ryd, where $(4\pi/3) r_s^3 = V$
- Net effect is metallic binding

Where can the electron gas be found?

- In semiconductors!
 More later - in doped semiconductors, the extra electrons (or missing electrons) can act like an electron gas in a background
- Where can 1d or 2d gas be found?
 In semiconductor structures!
 Layers of GaAs and AlAS can make nearly Ideal 2d gasses
 1d “wires” can also be made
- More later

Summary

- **Electrical Conductivity - Ohm's Law**
  $\sigma = (n\, q^2/m)\, \tau$ $\rho = 1/\sigma$
- **Hall Effect**
  $\rho_{Hall} = R_{Hall}\, B = E_{Hall} / j$
  ρ and ρ_{Hall} determine n and the charge of the carriers
- **Thermal Conductivity**
  $K = (\pi^2/3)\ (n/m)\ \tau\ k_B^2\ T$
  Weidemann-Franz Law:
  $K / \sigma = (\pi^2/3)\ (k_B/e)^2\ T$
- **Metallic Binding**
  Kinetic repulsion
  Coulomb attraction to nuclei
  (not included in gas model - must be added)

Next time

- EXAM Wednesday, October 11
- **Next week: Electrons in crystals**
  - **Energy Bands**
  - **We will use many ideas from the understanding of crystals and lattice vibrations to describe electron waves in a periodic crystal!**
  - **(Read Kittel Ch 7)**

Comments on Exam

- **Three types of problems:**
- **Short answer questions**
- **Order of Magnitudes**
- **Essay question**
- **Quantitative problems**