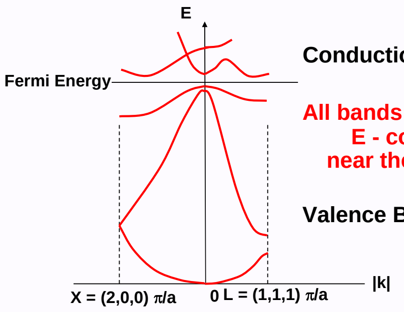
Lecture 17 - Semiconductors - continued

Lecture 17: Semiconductors - continued (Kittel Ch. 8)



Conduction Band

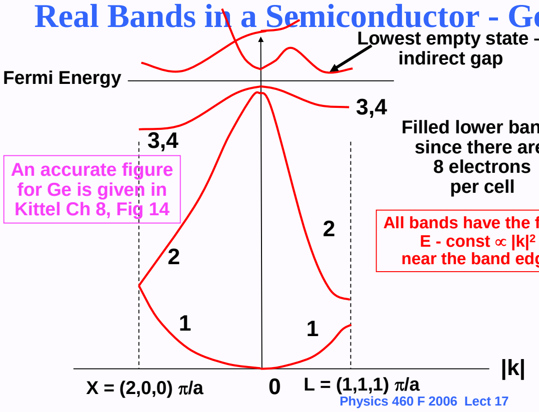
All bands have the form
E - const $\propto$ $|k|^2$
near the band edge

Valence Bands

Outline

- **Electrical carriers in Semiconductors**
  Bands near maximum of filled bands, and minimum of empty bands
- **Equations of motion in electric and magnetic fields**
  Effective mass
  Electrons and Holes
- **Intrinsic concentrations in a pure material**
  Law of mass action
- **(Read Kittel Ch 8)**

Real Bands in a Semiconductor - Ge



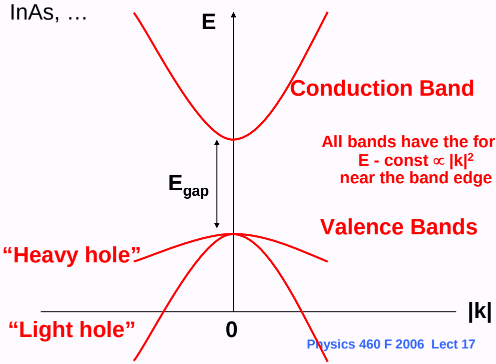
Filled lower bands since there are 8 electrons per cell

An accurate figure for Ge is given in Kittel Ch 8, Fig 14

All bands have the form
E - const $\propto$ $|k|^2$
near the band edge

Bands in semiconductor near k = 0

- Applies to "direct gap" semiconductors like GaAs, InAs, …



Conduction Band

All bands have the form
E - const $\propto$ $|k|^2$
near the band edge

Valence Bands

Motion of carrier in field

- Consider one electron in an otherwise empty band (a similar analysis applies to a missing electron in an otherwise full band)
- Group velocity: $\underline{\mathbf{v}} = \dfrac{d\omega}{d\underline{\mathbf{k}}} = \dfrac{1}{\hbar}\dfrac{dE}{d\underline{\mathbf{k}}}$
- If a force is applied the work done on the electron is the change in energy

$$dE/dt = \underline{\mathbf{F}} \cdot \underline{\mathbf{v}} = \frac{dE}{d\underline{\mathbf{k}}} \cdot d\underline{\mathbf{k}}/dt$$

- Using the above relations we find

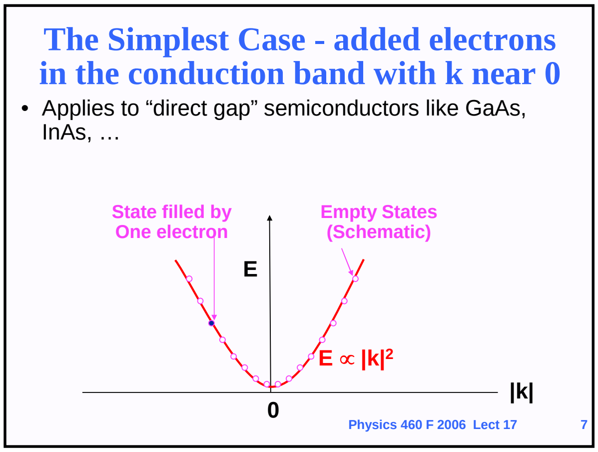
$$\underline{\mathbf{F}} = \hbar \; d\underline{\mathbf{k}}/dt$$

just as in free case! - independent of the form of the bands!

Effective Mass

- Consider the acceleration of the electron in a band in the presence of a force (e.g. $\underline{\mathbf{F}} = -e\,\underline{\mathbf{E}}$)
- Acceleration: $\dfrac{d}{dt}\underline{\mathbf{v}} = \dfrac{1}{\hbar}\dfrac{d}{dt}\dfrac{dE}{d\underline{\mathbf{k}}} = \dfrac{1}{\hbar}\dfrac{d^2E}{d^2\underline{\mathbf{k}}}\dfrac{d\underline{\mathbf{k}}}{dt} = \dfrac{1}{\hbar^2}\dfrac{d^2E}{d^2\underline{\mathbf{k}}}\underline{\mathbf{F}}$
- Thus the electron acts like it has an "effective mass" m*, where $\dfrac{1}{m^*} = \dfrac{1}{\hbar^2}\dfrac{d^2E}{d^2\underline{\mathbf{k}}}$
- This is the same as for free electrons, but with an "effective mass" m* - the motion of the electrons is changed because the electron is in a periodic potential (remember - d**k** /dt does not depend on the bands - but the relation of the velocity to k does depend on the bands!

Lecture 17 - Semiconductors - continued



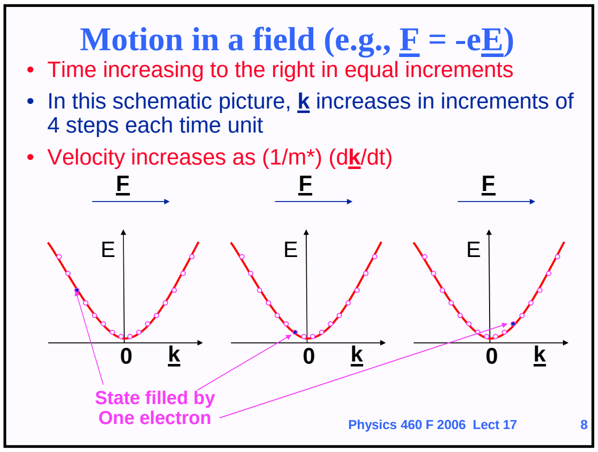
The Simplest Case - added electrons in the conduction band with k near 0

- Applies to "direct gap" semiconductors like GaAs, InAs, …

State filled by One electron

Empty States (Schematic)

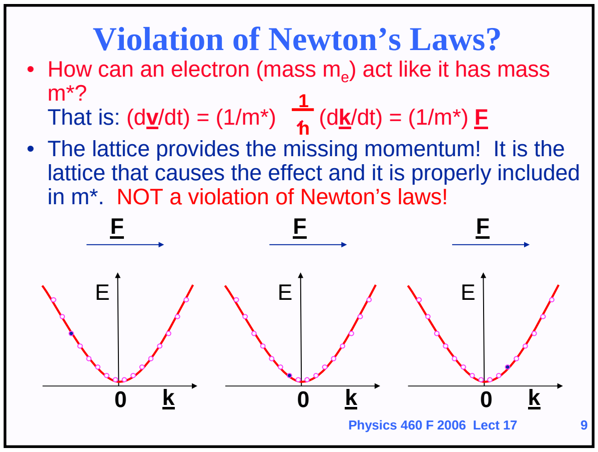
$E \propto |k|^2$



Motion in a field (e.g., F = -eE)

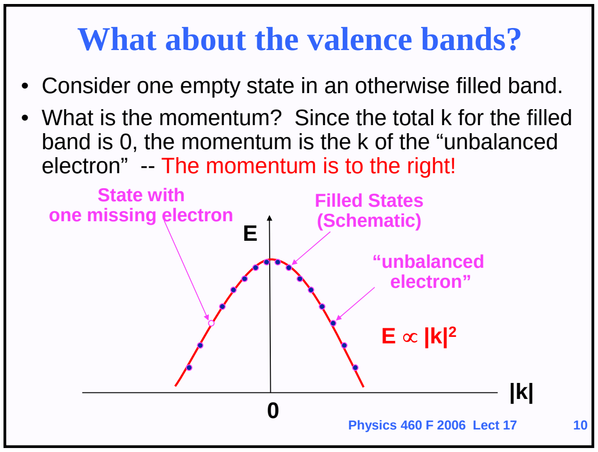
- Time increasing to the right in equal increments
- In this schematic picture, **k** increases in increments of 4 steps each time unit
- Velocity increases as (1/m*) (d**k**/dt)

State filled by One electron



Violation of Newton's Laws?

- How can an electron (mass m_e) act like it has mass m*?
  That is: (d**v**/dt) = (1/m*) $\frac{1}{\hbar}$ (d**k**/dt) = (1/m*) **F**
- The lattice provides the missing momentum! It is the lattice that causes the effect and it is properly included in m*. NOT a violation of Newton's laws!



What about the valence bands?

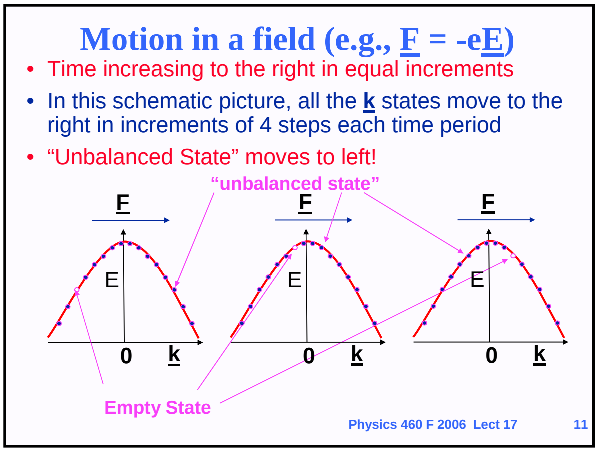
- Consider one empty state in an otherwise filled band.
- What is the momentum? Since the total k for the filled band is 0, the momentum is the k of the "unbalanced electron" -- The momentum is to the right!

State with one missing electron

Filled States (Schematic)

"unbalanced electron"

$E \propto |k|^2$

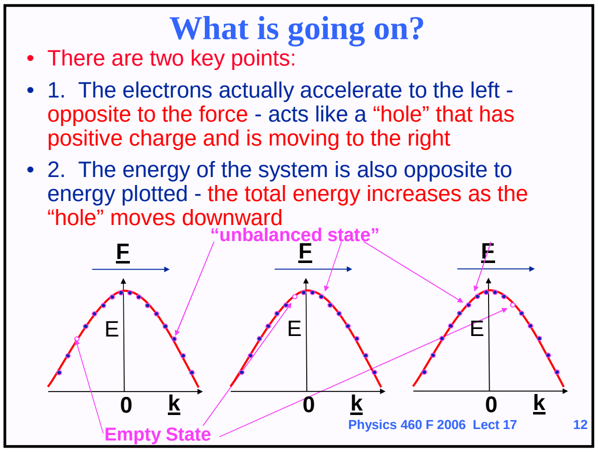


Motion in a field (e.g., F = -eE)

- Time increasing to the right in equal increments
- In this schematic picture, all the **k** states move to the right in increments of 4 steps each time period
- "Unbalanced State" moves to left!

"unbalanced state"

Empty State



What is going on?

- There are two key points:
- 1. The electrons actually accelerate to the left - opposite to the force - acts like a "hole" that has positive charge and is moving to the right
- 2. The energy of the system is also opposite to energy plotted - the total energy increases as the "hole" moves downward

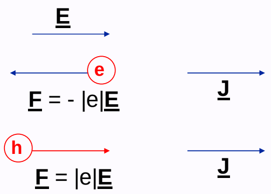
"unbalanced state"

Empty State

Lecture 17 - Semiconductors - continued

Conductivity

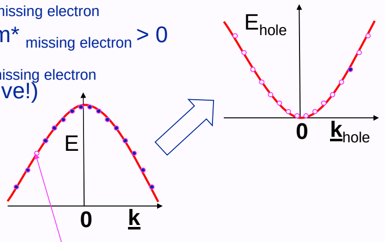
- Both electrons and holes contribute
- 1. An electron in the conduction bands has negative charge
- 2. A “hole” in the valence band has positive charge



- Ohm’s law results from scattering that limits the velocity

Holes in semiconductors

- This can all be put together (see Kittel p. 191-205) by defining:
- 1. $k_{hole} = - k_{missing\ electron}$
- 2. $E_{hole} = - E_{missing\ electron}$
- 3. $v_{hole} = + v_{missing\ electron}$
- 4. $m^*_{hole} = - m^*_{missing\ electron} > 0$
- 5. $q_{hole} = - q_{missing\ electron} = +|e|$ (positive!)



Equilibrium Concentration

- Details - See Kittel p 205-208
- Density of electrons = $n = \int_c^\infty D_c(E)\, f(E)\, dE$
  Parabolic Approx. for conduction band:
  $n = 2(m_c k_B T/ 2 \pi^2)^{3/2} \exp(-(E_c - \mu)/k_B T)$
- Density of holes = $p = \int_v^\infty D_v(E)\, (1-f(E))\, dE$
  Parabolic Approx. for valence band:
  $p = 2(m_v k_B T/ 2 \pi^2)^{3/2} \exp(-(\mu - E_v)/k_B T)$
- Product:
  $n\, p = 4\, (k_B T/ 2 \pi^2)^3\, (m_c m_v)^{3/2} \exp(-(E_c - E_v)/k_B T)$

Law of Mass Action

- Product
  $n\, p = 4\, (k_B T/ 2 \pi^2)^3\, (m_c m_v)^{3/2} \exp(-(E_c - E_v)/k_B T)$
  is independent of the Fermi energy
- Even though n and p vary by huge amounts, the product np is constant!
- Why?
  There is an equilibrium between electrons and holes! Like a chemical reaction, the reaction rate for an electron to fill a hole is proportional to the product of their densities. If one creates more electrons by some process, they will tend to fill more of the holes leaving fewer holes, etc.

Summary

- **Electrical carriers in semiconductors involve bands near maximum of filled bands, minimum of empty bands**
- **Equations of motion in electric and magnetic fields**
  Effective mass
  Acts like m*, with $1/m^* = d^2E/dk^2$
  Electrons and Holes
  A hole is the absence of electron in a filled band - Acts like positive charge, with change of sign of k and E, positive m*, with $1/m^*$ - d^2E/dk^2
- **Intrinsic concentrations in a pure material**
  Law of mass action
  n p = value that depends on material and T
- **(Read Kittel Ch 8)**

Next time

- **More on concentrations of electrons and holes in Semiconductors**
  Control of conductivity by doping (impurities)
- **Mobility**
- **Carriers in a magnetic field**
  Cyclotron resonance
  Hall effect
- **Thermoelectric effect**
- **(Read Kittel Ch 8)**