Physics 460 Homework 6 Solutions

1.

$$U_0 = \int_0^{\epsilon_F} \epsilon D(\epsilon)\, d\epsilon \qquad \text{where} \quad D(\epsilon) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar}\right)^{3/2} \epsilon^{1/2}$$

$$= \frac{V}{2\pi^2}\left(\frac{2m}{\hbar}\right)^{3/2} \frac{2}{5}\epsilon_F^{5/2} = \frac{3}{5}N\epsilon_F \qquad \text{note} \quad \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3} = \epsilon_F$$

2.

$$k_i = \frac{2\pi}{L} n_i \quad \Rightarrow \quad d^2 n = \left(\frac{L}{2\pi}\right)^2 d^2 k$$

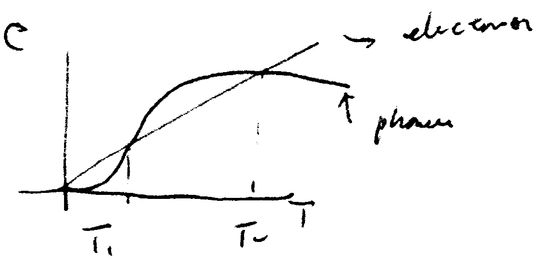
$$N = 2 \cdot \int \left(\frac{L}{2\pi}\right)^2 d^2 k = 2\frac{A}{(2\pi)^2}\pi k^2$$

$$\epsilon = \frac{\hbar^2 k^2}{2m} \quad \Rightarrow \quad N = \frac{2A}{(2\pi)^2}\pi \cdot \frac{2m\epsilon}{\hbar^2} = \frac{Am}{\pi\hbar^2}\epsilon$$

$$D(\epsilon) = \frac{dN}{d\epsilon} = \frac{mA}{\pi\hbar^2}$$

HW 6

3.



$$T_1 \Rightarrow \quad 234\, N k_B \left(\frac{T}{\theta}\right)^3 = \frac{3}{2}\pi^2 N k_B \frac{T}{T_F}$$

$$\Rightarrow T_1 \approx 6\text{ K}$$

$$T_2 \Rightarrow \quad 3 N k_B = \frac{3}{2}\pi^2 N k_B \frac{T}{T_F} \quad \Rightarrow \quad T_2 = 27300\text{ K}.$$

4. Kittel 6-2

(a)
$$P = -\frac{\partial U_0}{\partial V} = -\frac{3}{5}N\frac{\partial \epsilon_F}{\partial V}$$

Since $\epsilon_F \propto V^{-2/3}$, $\frac{\partial \epsilon_F}{\partial V} = -\frac{2}{3}\frac{\epsilon_F}{V}$

$$\therefore P = \frac{3}{5}N\frac{2}{3}\frac{\epsilon_F}{V} = \frac{2}{3}\frac{U_0}{V}$$

(b)
$$B = -V\frac{\partial P}{\partial V} = -V\frac{2}{3}\frac{\partial}{\partial V}\left(\frac{U_0}{V}\right)$$

Since $\frac{U_0}{V} = \frac{3}{5}N\frac{1}{V}\epsilon_F \propto V^{-5/3}$

$$\frac{\partial}{\partial V}\left(\frac{U_0}{V}\right) = -\frac{5}{3}\left(\frac{U_0}{V}\right)/V = -\frac{5}{3}\frac{U_0}{V^2}$$

$$\therefore B = -V\frac{\partial P}{\partial V} = \frac{2}{3}\cdot\frac{5}{3}\frac{U_0}{V} = \frac{10}{9}\frac{U_0}{V}$$

(c)
$$B = \frac{10}{9}\frac{U_0}{V} = \frac{10}{9}\frac{3}{5}\frac{N}{V}\epsilon_F = \frac{2}{3}\left(\frac{N}{V}\right)\epsilon_F$$

$$= \frac{2}{3}\cdot 1.40\times10^{22}\ \mathrm{cm}^{-3}\cdot 2.12\ \mathrm{eV}$$

$$= \frac{2}{3}\cdot 1.40\times10^{22}\times10^{6}\times2.12\times1.602\times10^{-19}\ \frac{\mathrm{joule}}{\mathrm{m}^3}$$

$$= 3.17\times10^{9}\ \frac{\mathrm{joule}}{\mathrm{m}^3}$$

5

A. The repulsive kinetic term

$$\frac{U_0}{N} = \frac{3}{5}\epsilon_F = \frac{3}{5}\frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$$

$$a_H = \frac{\hbar^2}{me^2} \qquad \frac{N}{V} = \frac{1}{\frac{4\pi r_0^3}{3}}$$

$$\frac{U_0}{N} = \frac{3}{5}\frac{\hbar^2}{2m}\left[\frac{3\pi^2}{\frac{4}{3}\pi r_s^3 a_H^3}\right]^{2/3} = \frac{3}{5}\frac{\hbar^2}{2m}\left(\frac{9\pi}{4}\right)^{2/3}\left(\frac{me^2}{\hbar^2}\right)^2\frac{1}{r_s^2}$$

$$= \frac{3}{5}\left(\frac{9\pi}{4}\right)^{2/3}\frac{me^4}{2\hbar^2}\frac{1}{r_s^2} \simeq 2.21\frac{1}{r_s^2}\times\left(\frac{me^4}{2\hbar^2}\right) \rightarrow \text{rydbergs}$$

$U(r_s)$ = repulsive kinetic term + attractive term

$$= \frac{2.21}{r_s^2} - \frac{1.8}{r_s}$$

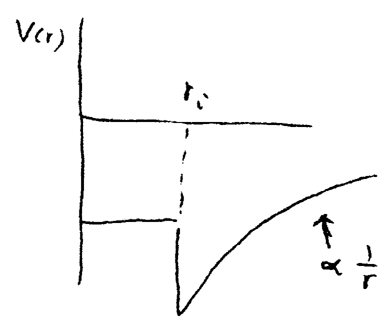
$$U'(r_s) = -\frac{2\cdot 2.21}{r_s^3} + \frac{1.8}{r_s^2} = 0$$

at $r_s = \frac{2\cdot 2.21}{1.8} \simeq 2.45$ $U(r_s)$ is minimum

B.

	Li	Na	K	Rb	Cs
r_s	3.25	3.93	4.86	5.20	5.63

The deviation is due to the finite size of nuclei (which increases from Li → Cs); the electron-ion potential has different r_i for different nuclei.



(c) At the minimum $r_s = 2.45$

$$U \simeq \frac{2.21}{(2.45)^2} - \frac{1.8}{2.45} = -0.367 \text{ Rydbergs}$$

∴ Binding energy = 0.367 Rydbergs.