Physics 460 Homework 7 Solutions

1. If we compare two samples with the same geometry. We also run the same currents through both samples. Then $j = n_1 e v_1 = n_2 e v_2$,

The Hall effect is proportional to the velocity of the carriers (for the Lorentz force $\vec{F} = q\vec{v}\times\vec{B}$): Hall effect $\propto v$

But $v \propto \frac{1}{n}$, hence, Hall effect $\propto \frac{1}{n}$.

b) One valence electron per atom:

From table 4, chapter 1, concentration of Cu is

$8.45 \times 10^{28}\ m^{-3}$, if we count one valence eletron per Cu atom the density of electrons in Cu is $8.45 \times 10^{28}\ m^{-3}$

In SI units

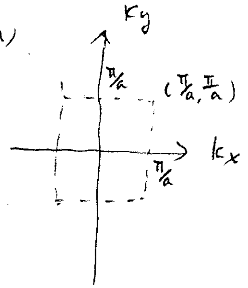
$$R_H = -\frac{1}{ne}$$

$$= -0.739 \times 10^{-10}\ \frac{m^2}{\text{coulomb}}$$

Note negative sign since carriers are electrons

2 Kittel 7-1

(a)



At corner

$$E_k = \frac{\hbar^2}{2m}\left(\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{a}\right)^2\right) = 2 \cdot \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2$$

At midpoint of a side face

$$E_k = \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2 \Rightarrow \frac{E_{k\,corner}}{E_{k\,mid}} = 2$$

(b)

For simple cubic At corner $\vec{k} = (\pm\frac{\pi}{a}, \pm\frac{\pi}{a}, \pm\frac{\pi}{a}) \Rightarrow E_k = 3 \cdot \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2$

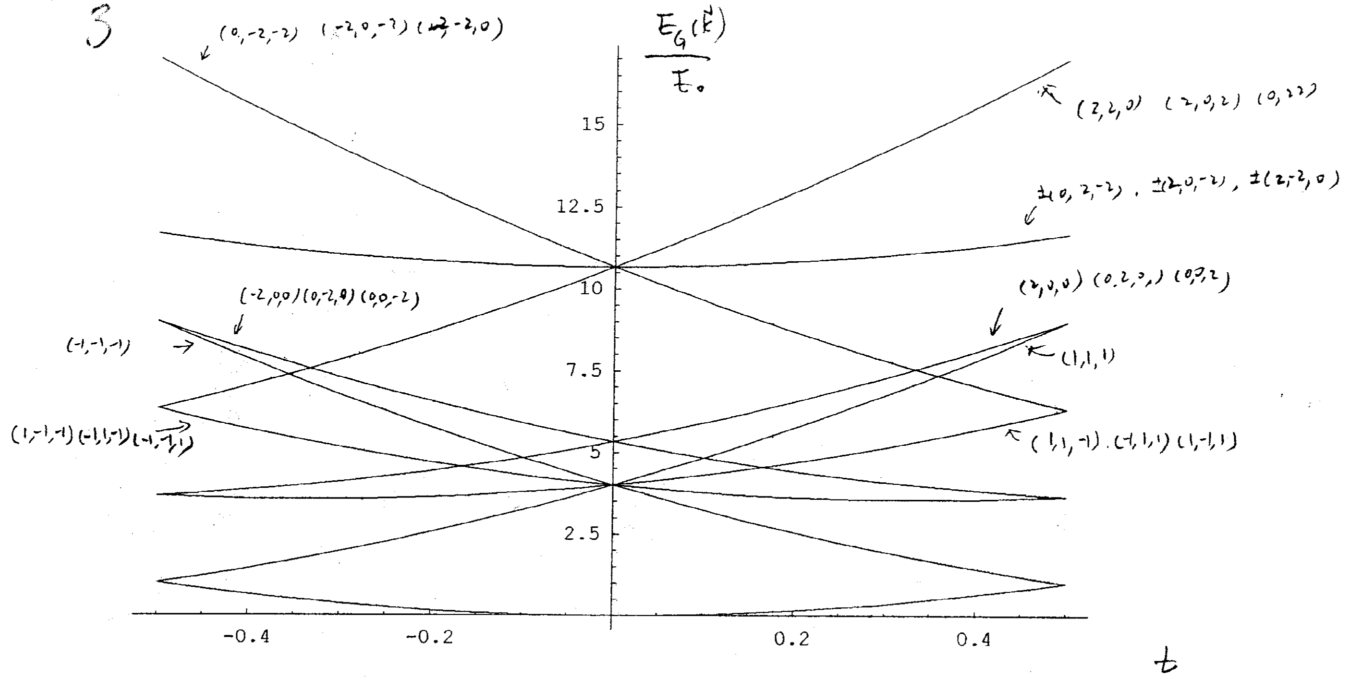
At midpoint of a side face $\vec{k} = (\pm\frac{\pi}{a}, 0, 0)$ $(0, \pm\frac{\pi}{a}, 0)$ $(0, 0, \pm\frac{\pi}{a})$

$$E_k = \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2 \Rightarrow \text{Ratio} = 3$$

(c)

If the corners of the 1st BZ had the same energy as the midpoint of a side face, the 1st BZ would have been filled up for divalent metals. Then the "metals" would become insulator. But now, the corners have high energies, that means the regions around the corners are not filled up (some holes there) and there are regions in 2nd BZ (near the midpoint of a side face) that are filled. Therefore, "hole" in 1st BZ and electrons in 2nd BZ contribute to conductivity, making "divalent metals" metallic!

3



$$E_{\vec{G}}(\vec{k}) = \frac{\hbar^2}{2m}(\vec{k}+\vec{G})^2$$

In (111) direction $\vec{k} = \frac{2\pi}{a}(t,t,t)$ $\quad t \in [-\frac{1}{2}, \frac{1}{2}]$.

$\vec{G}$'s are reciprocal lattice vectors of fcc.

$$\vec{G} = h\vec{b}_1 + k\vec{b}_2 + l\vec{b}_3$$

$$= \frac{2\pi}{a}(k+l-h,\ l+h-k,\ h+k-l) \equiv \frac{2\pi}{a}(v_1, v_2, v_3)$$

possible $\vec{G}/(\frac{2\pi}{a})$ are $(\pm1,\pm1,\pm1)$ $(\pm2,0,0)$ $(0,\pm2,0)$ $(0,0,\pm2)$
$(\pm2,\pm2,0)$ $(0,\pm2,\pm2)$ $(\pm2,0,\pm2)$ ...

$$E_{\vec{G}=0}\left(\vec{k} = \tfrac{2\pi}{a}(1,1,1)\right) = \frac{\hbar^2}{2m}\left(\frac{2\pi}{a}\right)^2 \cdot \frac{3}{4} \equiv E_0$$

$$\frac{E_{\vec{G}}(\vec{k})}{E_0} = \frac{4}{3}\left[(t+v_1)^2 + (t+v_2)^2 + (t+v_3)^2\right]$$

The band structure is plotted above.

4

a) Something like the chemical reaction balance the electrons & holes

b) $n_i = p_i = \sqrt{np}$

$\Rightarrow n = \sqrt{2.1\times10^{19}} = 4.58\times10^{9}\ a^{-3}$

$n \propto T^{\frac{3}{2}} \exp\left(-\frac{E_g}{2kT}\right)$

$\therefore \frac{n_{200k}}{n_{300k}} = \left(\frac{200}{300}\right)^{\frac{3}{2}} \exp\left[\frac{E_g}{2k_BT}\left(\frac{1}{300}-\frac{1}{200}\right)\right]$

$= 5.4\times10^{4}\ a^{-3}$

c) $n_e = 10^{18}\ a^{-3}$

$np = 2.1\times10^{19}\ a^{-6}$

$\Rightarrow p = 21\ a^{-3}.$

5.

Using

$\frac{m_e}{m} = \frac{1}{\frac{2\lambda}{u}-1} \qquad \frac{m_R}{m} = \frac{1}{\frac{2\lambda}{u}+1}$

λ is the free electron energy while $2u$ is the band gap

(a) $\frac{m_e}{m} = \frac{1}{\frac{2\cdot10}{1/2}-1} = \frac{1}{39}. \qquad \frac{m_R}{m} = \frac{1}{\frac{2\cdot10}{1/2}+1} = \frac{1}{41}$

(b) $\frac{m_e}{m} = \frac{1}{\frac{2\cdot10}{0.1/2}-1} = \frac{1}{399} \qquad \frac{m_R}{m} = \frac{1}{\frac{2\cdot10}{0.1/2}+1} = \frac{1}{401}$