Physics 460 Homework 8 Solutions

1 Kittel 8-1

(a)

$$E_d = \frac{e^4 m_e}{2\epsilon^2 \hbar^2} = \frac{13.6}{\epsilon^2}\frac{m_e}{m}\ eV = \frac{13.6}{18^2}\cdot 0.015 = 6.29\times 10^{-4}\ eV$$

(b)

$$a_d = \frac{\epsilon \hbar^2}{m_e e^2} = \frac{0.53\epsilon}{m_e/m}\ \text{Å} = \frac{0.53\times 18}{0.015}\ \text{Å} = 636\ \text{Å}$$

(c)

Overlap when $\frac{4}{3}\pi a_d^3 \cdot N \simeq V \Rightarrow$ concentration $n \simeq \dfrac{1}{\frac{4}{3}\pi a_d^3} = 9.28\times 10^{20}\ \dfrac{1}{m^3}$

2 Kittel 8-2

Error in original solution - the final answer is the same

(a)

$$n \cong (n_0 N_d)^{1/2} e^{-E_d/2k_BT}$$

$N_d = 10^{13}$ donors $/cm^3$, $m_e = 0.01\ m$

$T = 4K$ $E_d = 1\ meV$

$$n_0 = 2\left(m_e k_B T/2\pi\hbar^2\right)^{3/2} = 2\left(0.01\cdot 9.11\times 10^{-31}\ kg\cdot 1.38\times 10^{-23}\cdot 4\ J\ /\ 2\pi(1.055\times 10^{-34}\ Js)^2\right)^{3/2}$$

$$= 3.856\times 10^{19}\ /m^3$$

$$n \cong \left(3.856\times 10^{19}\times 10^{13}\times 10^{6}\right)^{1/2} e^{-(10^{-3}/2\cdot 4)\cdot 1.16\times 10^{4}} = 4.606\times 10^{18}\ \frac{1}{m} = 4.606\times 10^{12}/cm^3$$

(b)

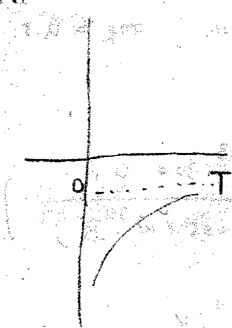
$$R_H = -\frac{1}{ne} = -\frac{1}{4.606\times 10^{12}\cdot 1.602\times 10^{-19}} = -1.355\times 10^{6}\ \frac{cm^3}{Coul}$$

3

(a)

$$R_H = -\frac{1}{ne} = -\frac{1}{(n_0 N_d)^{1/2} e}\, e^{E_d/2k_BT}$$

as $T\to 0$ $R_H \to -\infty$



(b)

$R_H = -\frac{1}{ne}$ roughly constant

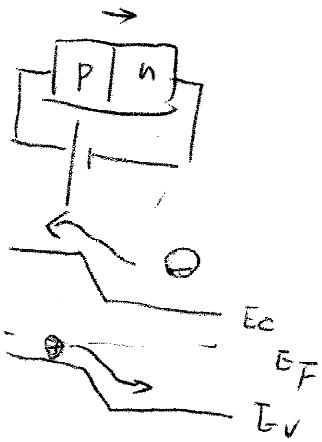
(c)

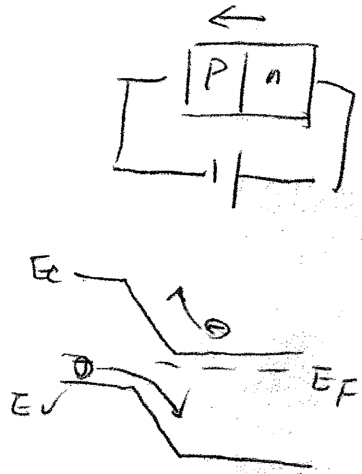
$$R_H = \frac{1}{ne} = \frac{1}{(n_0 N_d)^{1/2} e}\, e^{E_d/2k_BT}$$

changes sign relative to (a)

4

Forward bias reverse bias

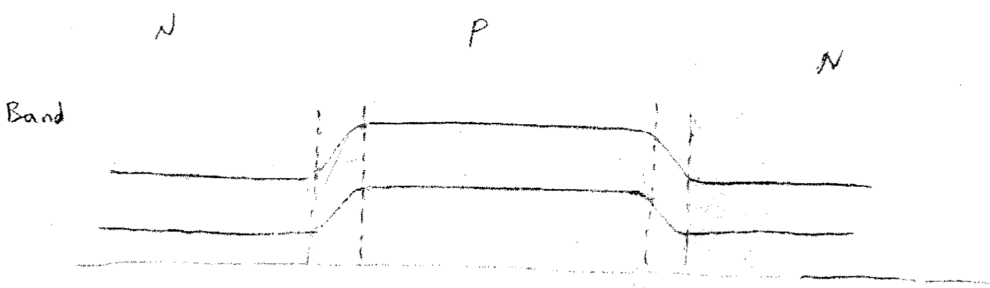


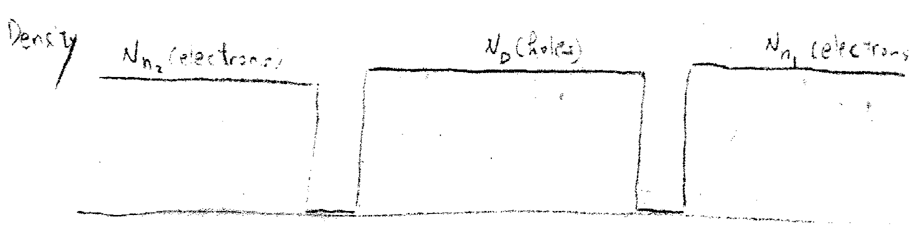


Allowed : p → ~~to~~ n

block : n → p.

5 From the result of prob. 2





N_{n_1}, N_D, N_{n_2} can be different

← result from the model in prob 2

6 In z-direction, the electron motion is quantized

$$k_z = \frac{n\pi}{L} \quad (n=1,2,3,\ldots) \quad \text{while } k_x\ k_y \text{ are not restricted}$$

Thus $$E_k = \frac{\hbar^2}{2m^*}\left(k_x^2+k_y^2+\left(\frac{n\pi}{L}\right)^2\right)$$

$\therefore$ The lowest possible energy relative to that of the Conduction band bottom

is $$E_{lowest} = \frac{\hbar^2}{2m^*}\left(\frac{\pi}{L}\right)^2$$

$m^* = 0.066\ m$ for GaAs

$$E_{lowest} = \frac{(1.055\times10^{-34}\ Js)^2}{2\cdot0.066\cdot9.11\times10^{-31}\ kg}\left(\frac{\pi}{2\times10^{-8}\ m}\right)^2 = 2.284\times10^{-21}\ J = 1.426\times10^{-2}\ eV$$

7, Because the electrons can be considered to be only in the lowest quantum state in the z-direction, they are not free to move in that direction, They are two-dimensiona

8 $k_x, k_y\ k_z$ are quantized

$$k_x = \frac{n_x\pi}{L}, \quad k_y = \frac{n_y\pi}{L} \quad k_z = \frac{n_z\pi}{L}$$

$$E_k = \frac{\hbar^2}{2m^*}\left(\frac{\pi}{L}\right)^2(n_x^2+n_y^2+n_z^2) \qquad n_x, n_y, n_z = 1,2,3,\ldots$$

$$= 1.426\times10^{-2}\ eV\ (n_x^2+n_y^2+n_z^2)$$

The lowest $n_x=n_y=n_z=1$ $\quad E_k = 4.278\times10^{-2}\ eV$

The next one $n_x=n_y=1,\ n_z=2$

$$E_k = 1.426\times10^{-2}\ eV\times6 = 8.556\times10^{-2}\ eV$$