Physics 460 Homework 9 Solutions

1. From eq. (10.9)

$$\Delta F = F_N(0) - F_S(0) = B_{ac}^2/8\pi$$

For Pb, $B_{ac} = 803\ G$

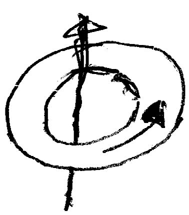
$$\Delta F = \frac{803\ G^2}{8\pi} = 2.566 \times 10^4 \frac{erg}{cm^3} = 1.602 \times 10^{16} \frac{eV}{cm^3}$$

Pb has a concentration of 13.2×10^{22} valence e^-/cm^3

$$\text{energy difference per valence electron} = \frac{1.602 \times 10^{16}}{13.2 \times 10^{22}} = 1.214 \times 10^{-7}\ eV$$

which is quite small compared to Pb's Fermi energy $\epsilon_F = 9.37\ eV$ in Table 1, chapter 6

and average kinetic energy $= \frac{3}{5}\epsilon_F = 5.622\ eV$.

2. 

A supercurrent flowing in a ring must have a magnetic field through the hole in the ring (Maxwell's Equations). The magnetic field can decrease only if the field goes through the superconductor which costs energy. If we define that energy to be ΔE, the thermal probability is $e^{-\Delta E/k_BT}$ which $\rightarrow 0$ for low T.

Therefore the probability of the current to decrease $\rightarrow 0$ for $T \rightarrow 0$, and the current can flow without resistance for arbitrarily long times — as long as the age of the universe!

3. $\psi_{1s} = (\pi a_0^3)^{-\frac{1}{2}} e^{-r/a_0}$

$$\langle r^2 \rangle = \int d^3r\, |\psi_{1s}|^2 r^2 = (\pi a_0^3)^{-1} \cdot 4\pi \int_0^\infty dr\, r^4 e^{-2r/a_0} = 3a_0^2 \quad \text{using} \int_0^\infty dx\, x^n e^{-sx} = \frac{n!}{s^n}$$

$$\chi = -\frac{Ze^2 N \langle r^2 \rangle}{6mc^2} = -\frac{Ze^2 N a_0^2}{2mc^2} \qquad N = \frac{\#}{V}$$

$\boxed{CGS}$

For a mole of hydrogen atoms, $Z = 1$, $\# = 6.02 \times 10^{23}$

$$V \cdot \chi = -\frac{e^2 a_0^2}{2mc^2} \# = -\frac{(4.803\times10^{-10}\,\text{esu})^2 \cdot (.529\times10^{-9}\,\text{cm})^2}{2\cdot(9.11\times10^{-28}\,\text{g})\cdot(3\times10^{10}\,\text{cm/s})^2} \times 6.02\times10^{23} = -2.367\times10^{-6}\ \text{cm}^3/\text{m.}$$

which is the molar diamagnetic susceptibility of atomic hydrogen atom

In SI units $[V\chi] = \mu_0 \frac{N_A \mu}{B} = -\mu_0 \frac{N_A e^2 a_0^2}{2m}$

But note one must divide by 4π to compare with CGS

CGS $B = H + 4\pi M$ SI $B = \mu_0 H + \mu_0 M$

4 From eq. (11.18)

$$M \simeq N\mu(\mu B/k_B T) \Rightarrow \chi = \frac{N\mu^2}{k_B T} \quad \text{where } \mu = \mu_B$$

For $N = 1\ /\text{cm}^3$

$$\chi = \frac{(9.27\times10^{-21}\ \text{erg}\ G^{-1})^2}{1.38\times10^{-16}\text{erg}\,K^{-1}\cdot 300\,K\cdot\text{cm}^3} = 2.076\times10^{-27}$$

Molar susceptibility $V\chi = 2.076\times10^{-27}\times 6.02\times10^{23}\ \text{cm}^3/\text{mole} = 1.25\times10^{-3}\ \text{cm}^3/\text{mole}$

which is much larger than $-2.36\times10^{-6}\ \text{cm}^3/\text{mole}$.

It seems that hydrogen atom should behave as a paramagnet. But hydrogens normally appear in the form of hydrogen molecules, with 2 hydrogen atoms binding together. The paramagnetic effect will disappear since the two electrons with opposite spins will cancel each other.

5.

Magnetization $= \frac{N}{V}\, g\, m\, \mu_B$ for electron $m = \frac{1}{2}, g = 2$

- The density of Fe is $8.5\times10^{22}\ \text{cm}^{-3}$ and assume 1 electron per unit cell that can contribute to the magnetization.

Max Magnetization $= 8.5\times10^{22}\ \text{cm}^{-3}\cdot 1\cdot 9.27\times10^{-21}\ \text{erg}\ G^{-1}$

$= 7.88\times10^{2}\ G$

Comparing to $M_s = 1740\ G$ at $0\,K$ in Table 2, chapter 15.

we see they differ by a factor about 2, so the simple estimation is not bad.