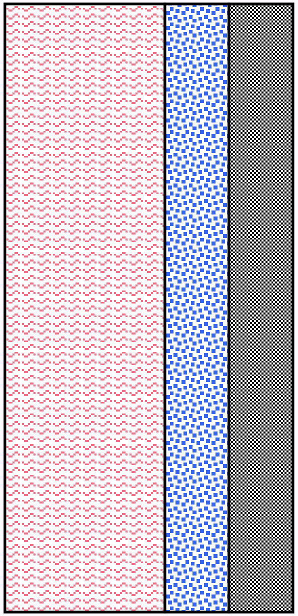
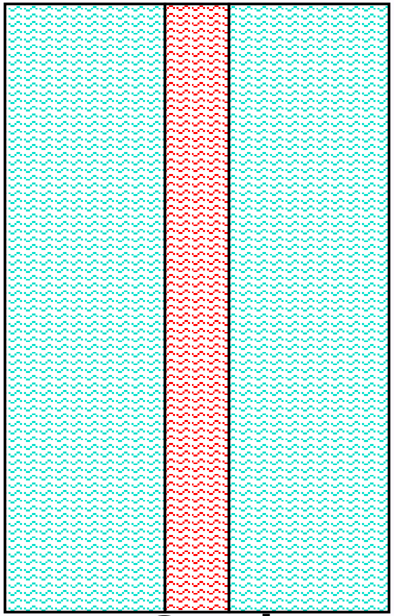
Lecture 20: Semiconductor Structures
Kittel Ch 17, p 494-503, 507- 511
+ extra material in the class notes

MOS Structure



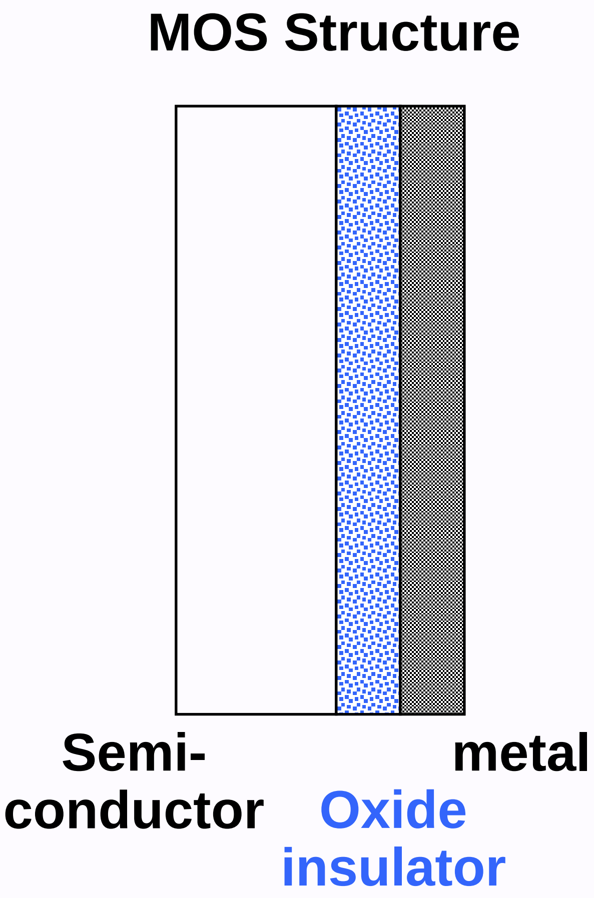
Layer Structure

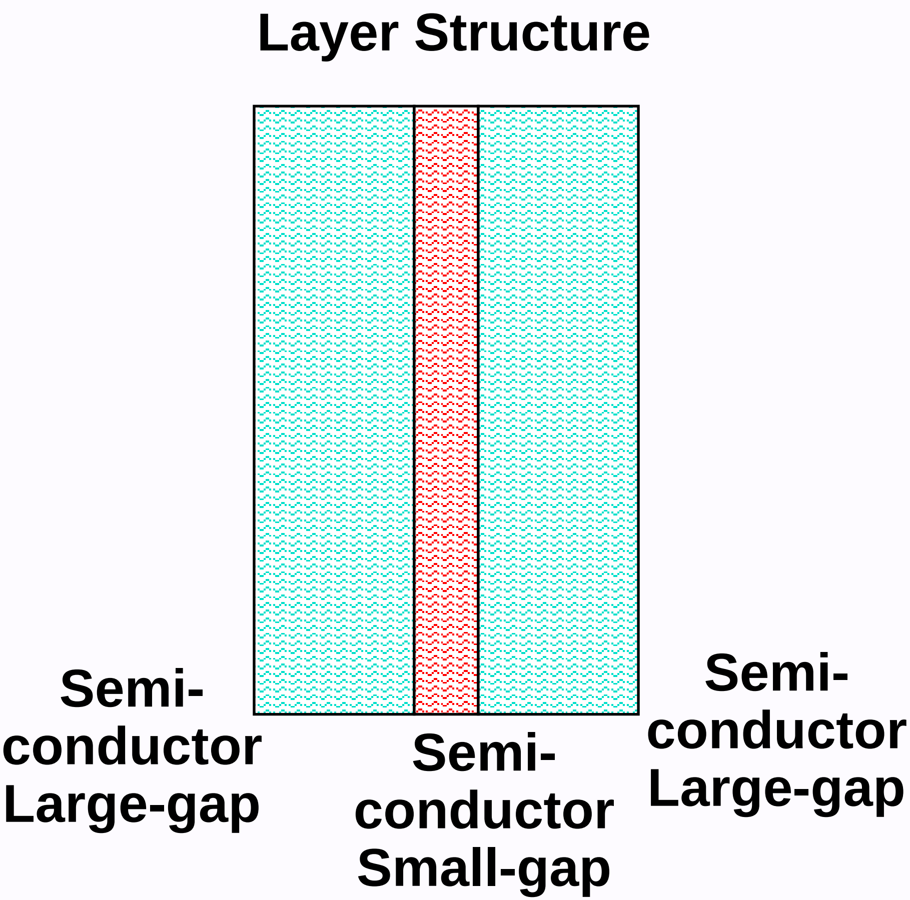


Outline

- **What is a semiconductor Structure?**
- **Created by Applied Voltages**
  - **Conducting channels near surfaces**
  - **Controlled by gate voltages**
  - **MOSFET**
- **Created by material growth**
  - **Layered semiconductors**
  - **Grown with atomic layer control by "MBE"**
  - **Confinement of carriers**
    - **High mobility devices**
    - **2-d electron gas**
    - **Quantized Hall Effect**
  - **Lasers**
- **Covered briefly in Kittel Ch 17, p 494-503, 507- 511**
  - added material in the lecture notes

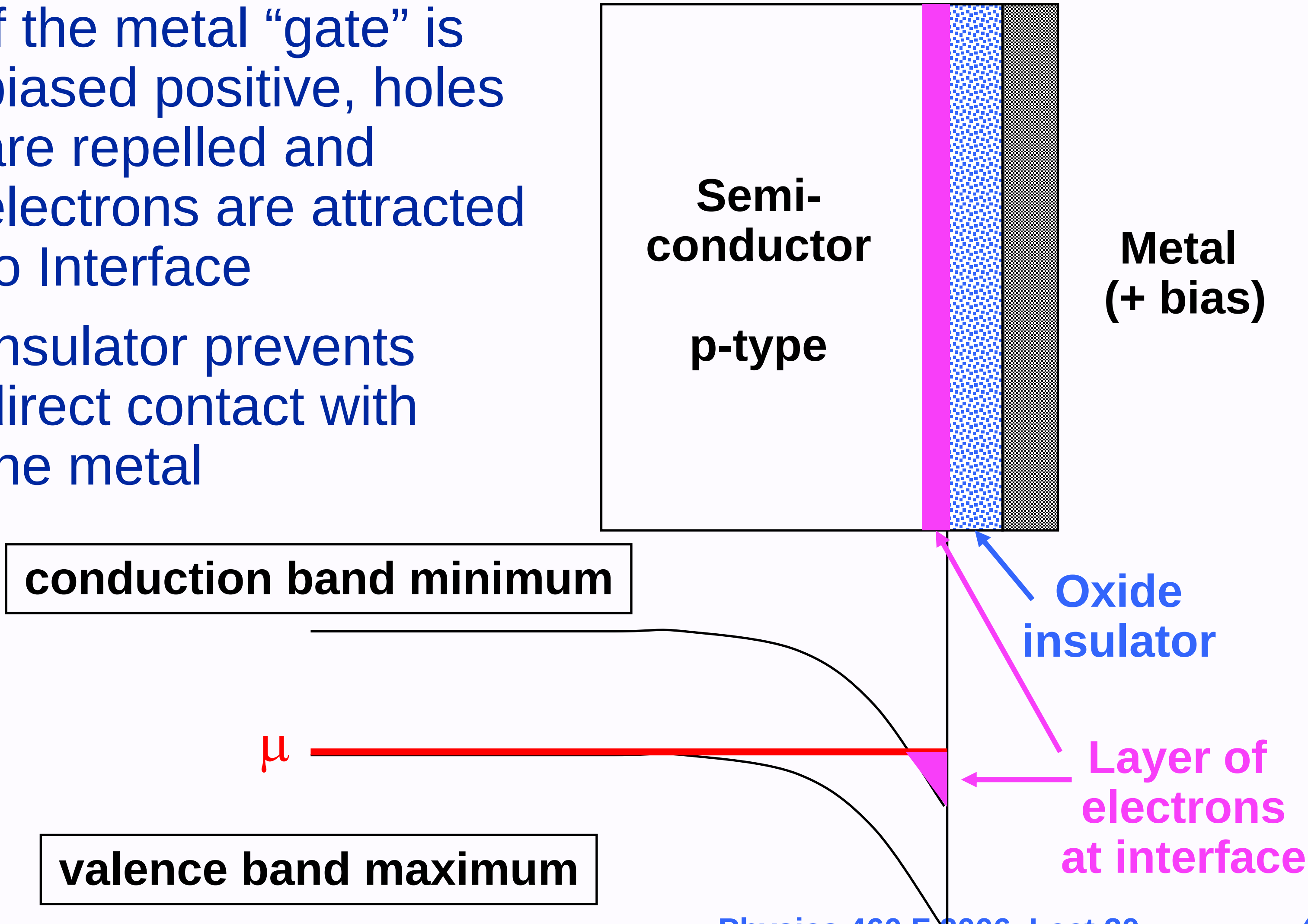
Semiconductor Structure





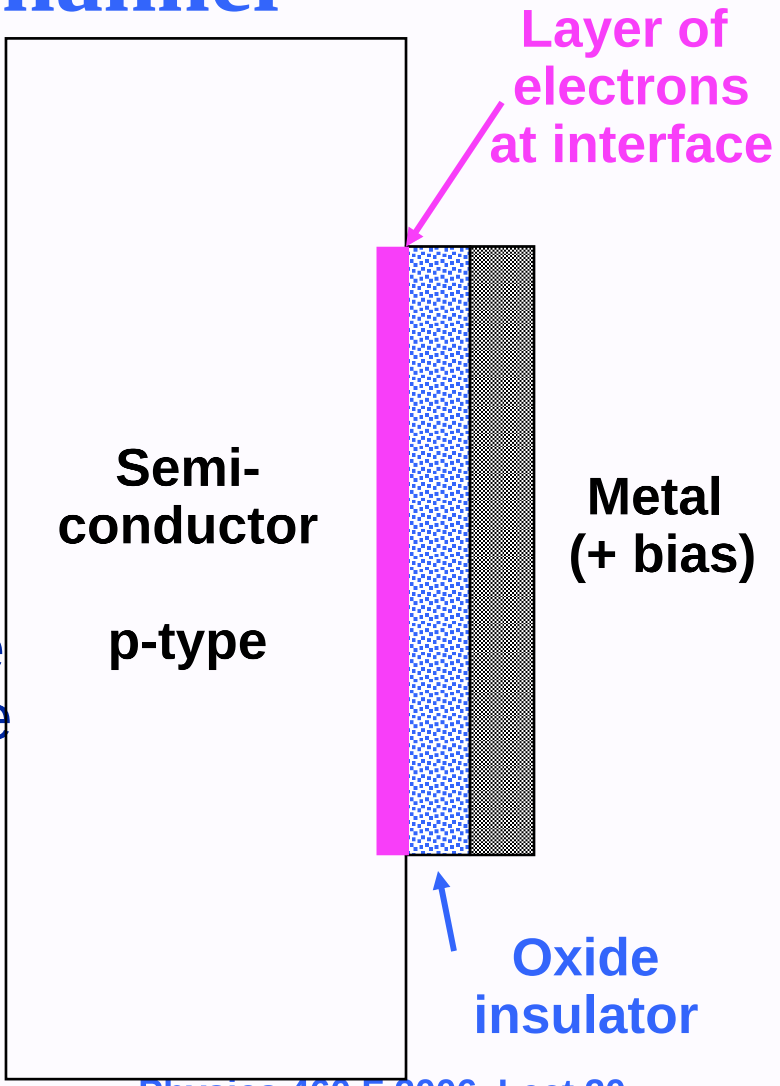
MOS Structure

- If the metal "gate" is biased positive, holes are repelled and electrons are attracted to Interface
- Insulator prevents direct contact with the metal



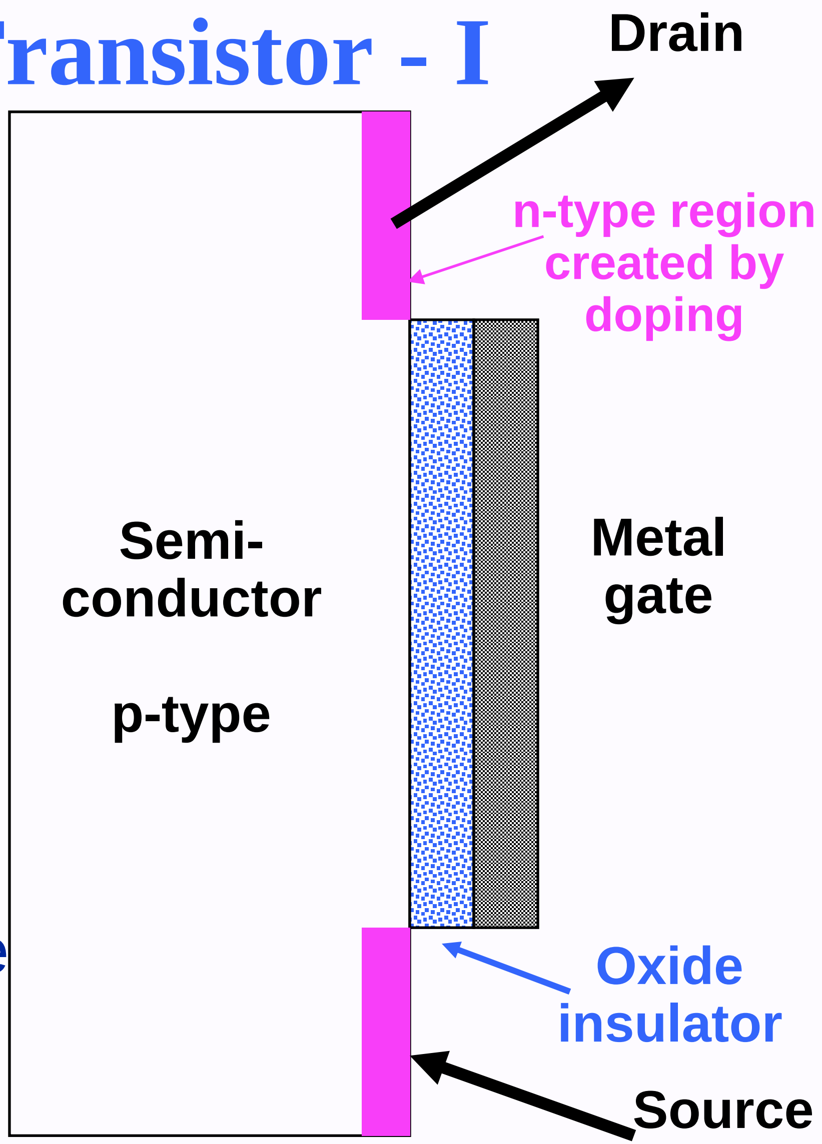
MOS channel

- If the metal is biased positive, there can be an electron layer formed at the interface
- Insulator prevents direct contact with the metal
- Electrons are bound to interface but free to move along interface across the full extent of the metal region
- How can this be used?



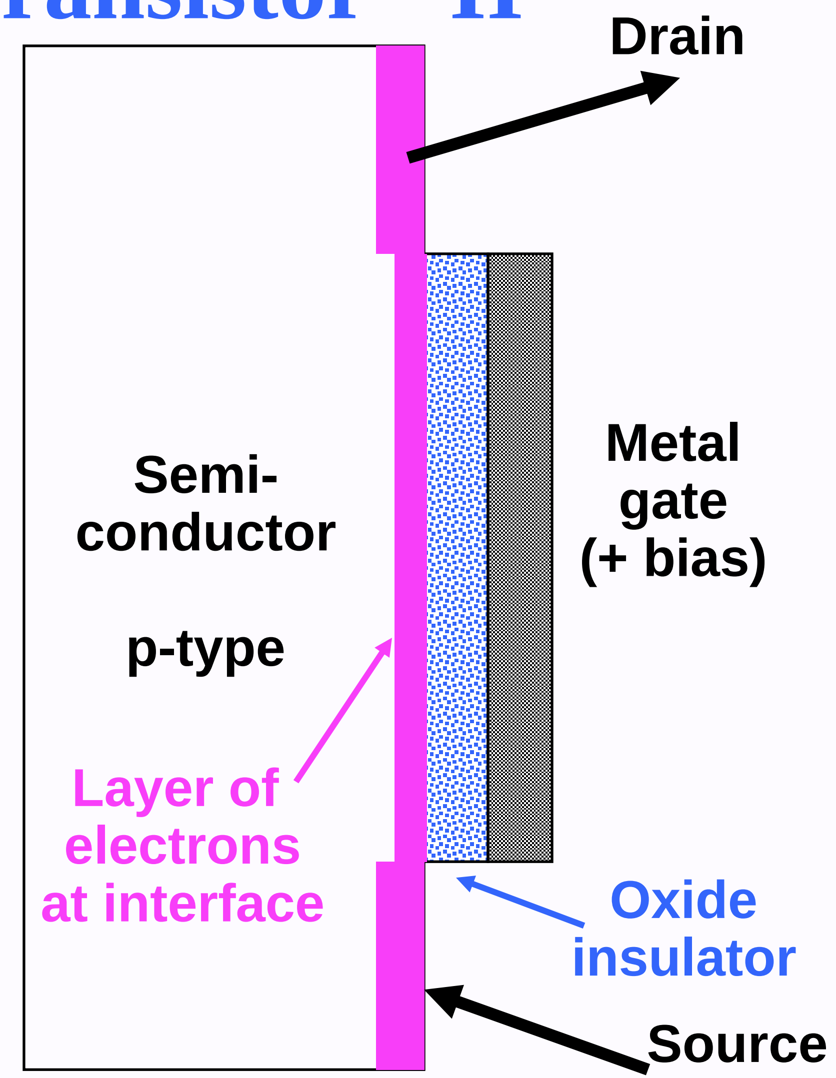
MOSFET Transistor - I

- The structure at the right has n regions at the surface of the p-type semiconductor
- Contacts to the n regions are source and drain
- If there is no bias on the metal gate, no current can flow from source to drain
- Why? There are two p-n junctions – one is reverse biased whenever a voltage is applied
- "Off state"



MOSFET Transistor - II

- A positive bias on the metal gate creates an n-type channel
- Current can flow from source to drain through the n region
- “On state”
- MOSFET transistor!

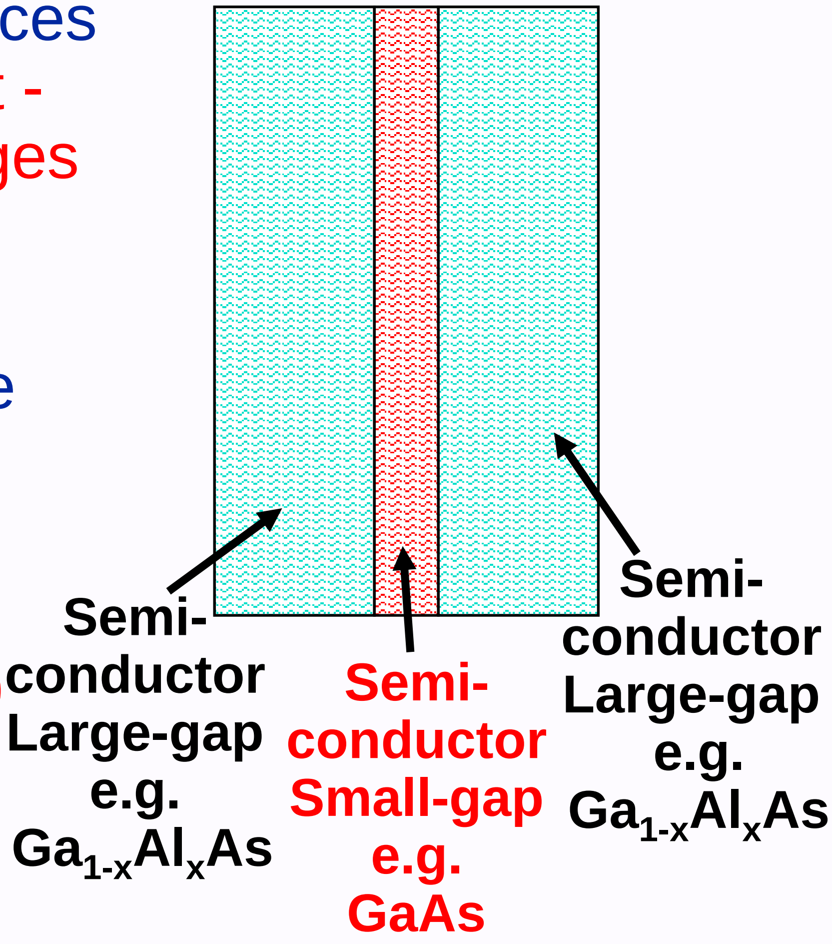


Semiconductor Layered Structures

- Electrons and holes can be confined and controlled by making structures with different materials
- Different materials with different band gaps ⇒ electrons and/or holes can be confined
- Structures can be grown with interfaces that are atomically perfect - sharp interface between the different materials with essentially no defects

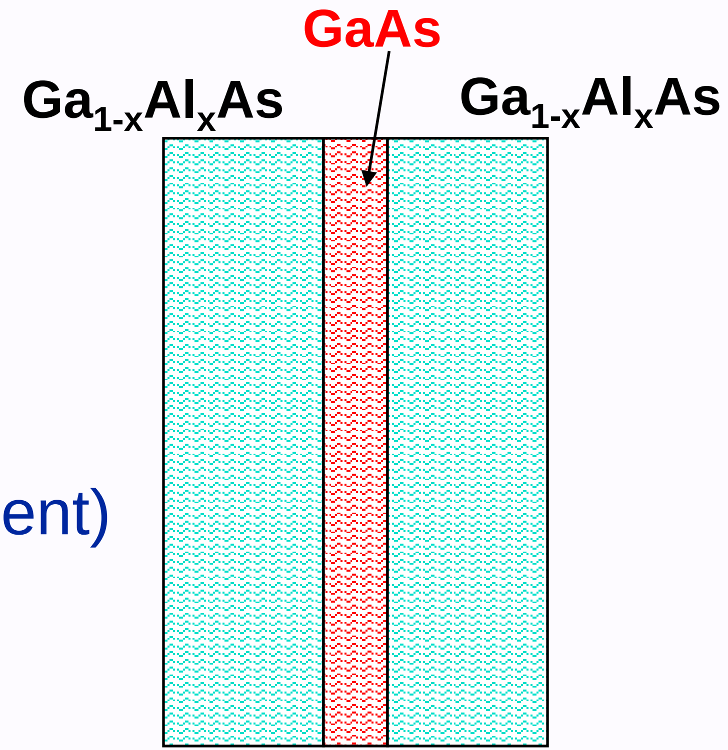
Semiconductor Layered Structures

- Can be grown with interfaces that are atomically perfect - a single crystal that changes from one layer to another
- Example: GaAs/AlAs Single crystal (zinc blende structure) with layers of GaAs and AlAs
- Grown using "MBE" (Molecular Beam Epitaxy)
- Can "tune" properties by making an **alloy** of GaAs and AlAs, called **$Ga_{1-x}Al_xAs$**

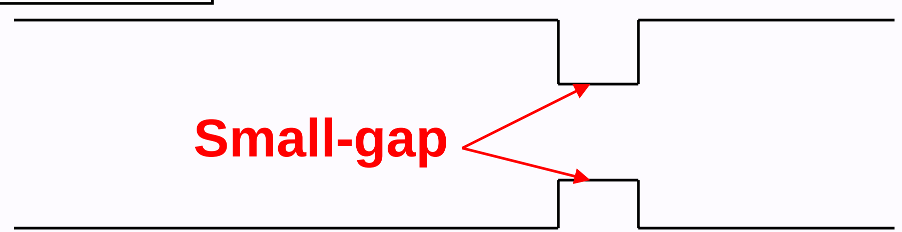


Semiconductor Layered Structures

- Example of bands in GaAs-AlAs structures
- Both electrons and holes are confined
- (Other systems can be different)



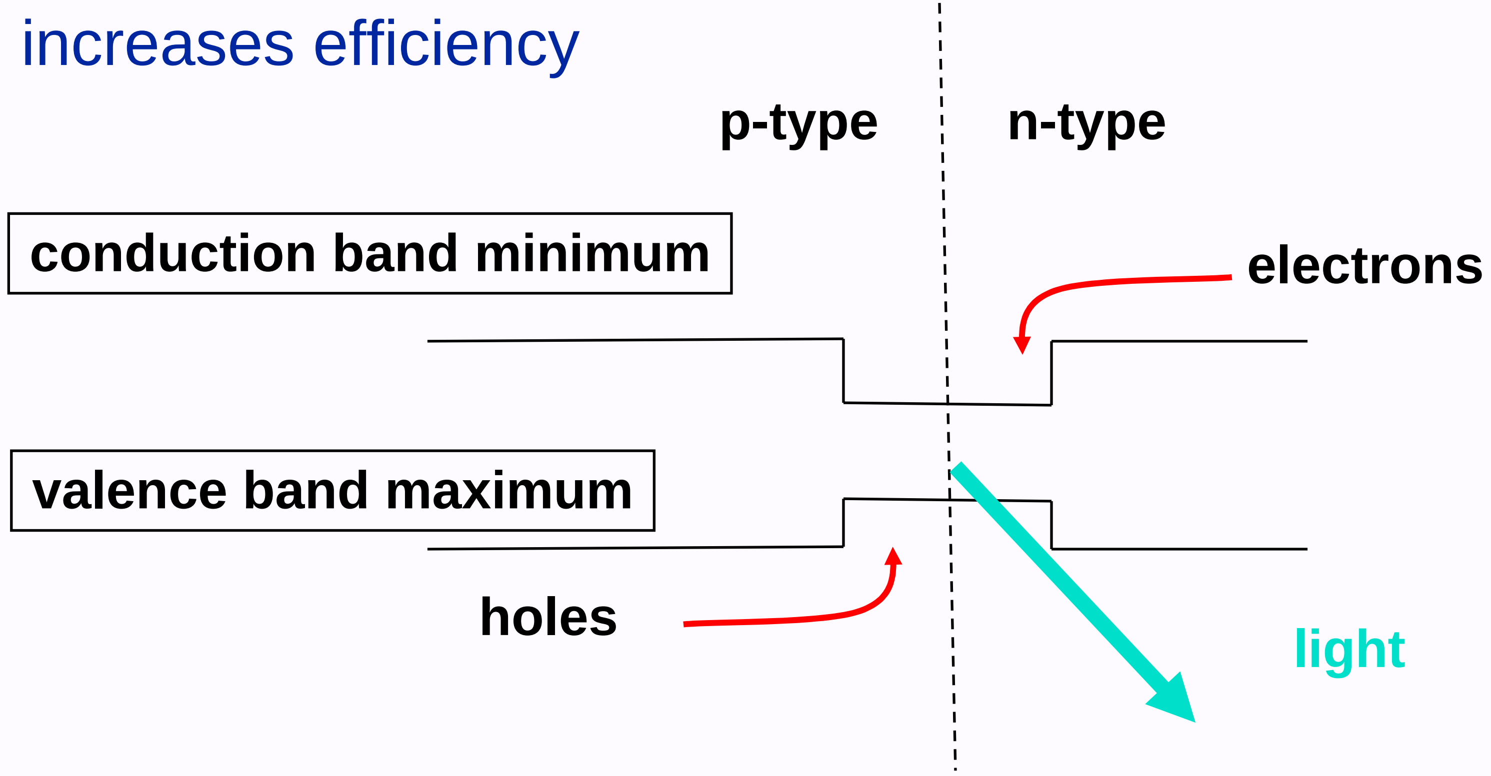
conduction band minimum



valence band maximum

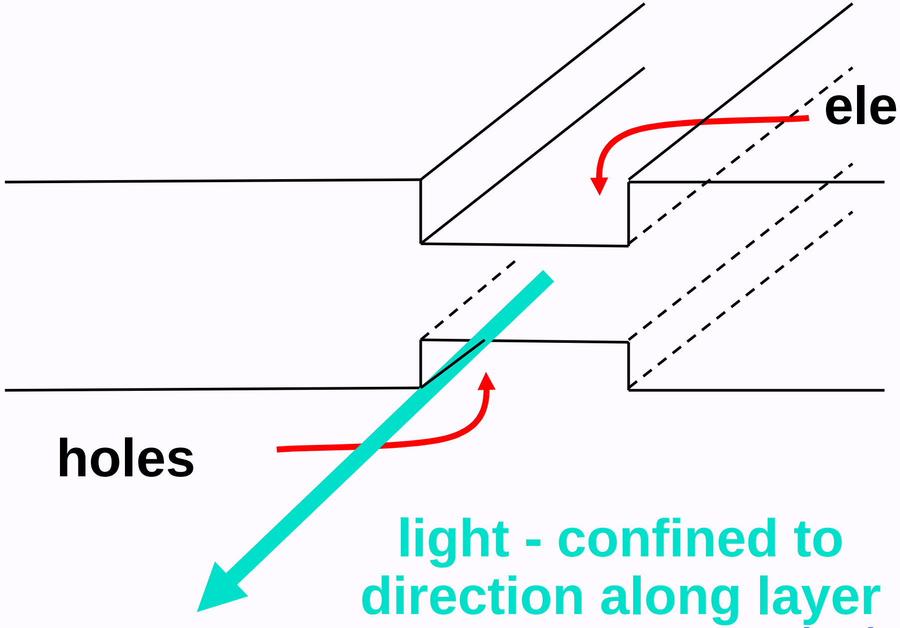
Uses of Layered Structures

- Confinement of carriers can be very useful
- Example - light emitting diodes
- Confinement of both electrons and holes increases efficiency



Uses of Layered Structures

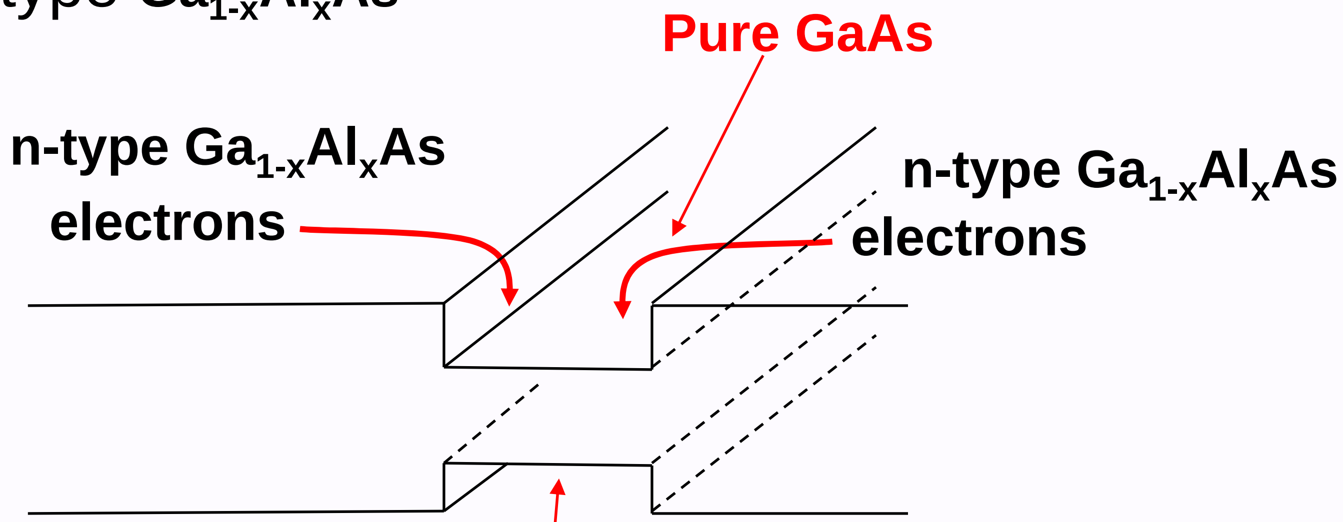
- Confinement of light can be very useful
- Example - light emitting diodes, lasers
- Confinement of light is due to larger dielectric constant of the low band gap material - total internal reflection



Used to make the lasers in your CD player!

Uses of Layered Structures

- The highest mobility for electrons (or holes) in semiconductors are made with layer structures
- Example – pure GaAs layer between layers of doped n-type $Ga_{1-x}Al_xAs$



High mobility for the electrons in GaAs because the impurity dopant atoms are in the $Ga_{1-x}Al_xAs$

Quantum Layered Structures

- If the size of the regions is very small quantum effects become important.
- How small?
- Quantum effects are important when the energy difference between the quantized values of the energies of the electrons is large compared to the temperature and other classical effects
- In a semiconductor the quantum effects can be large!

Electron in a box

- Here we consider the same problem that we treated for metals – the "electron in a box" – see lecture 12 and Kittel, ch. 6
- There are two differences here:
  1. The electrons have an effective mass m*
  2. The box can be small! This leads to large quantum effects
- We will treat the simplest case – a "box" in which each electron is free to move except that it is confined to the box
- To describe a thin layer, we consider a box with length L in one direction (call this the z direction and define $L = L_z$) and very large in the other two directions (L_x, L_y very large)

Schrodinger Equation

- Basic equation of Quantum Mechanics

$$[\, -(\hbar/2m)\,\nabla^2 + V(\underline{r})\,]\,\Psi(\underline{r}) = E\,\Psi(\underline{r})$$

where

m = mass of particle

$V(\underline{r})$ = potential energy at point $\underline{r}$

$\nabla^2 = (d^2/dx^2 + d^2/dy^2 + d^2/dz^2)$

E = eigenvalue = energy of quantum state

$\Psi(\underline{r})$ = wavefunction

$n(\underline{r}) = |\Psi(\underline{r})|^2$ = probability density

From Lecture 12
See Kittle, Ch 6

Schrodinger Equation - 1d line

- Suppose particles can move freely on a line with position x, $0 < x < L$

0 **L**

- Schrodinger Eq. In 1d with V = 0

  $$-\left(\hbar^2/2m\right) d^2/dx^2\, \Psi(x) = E\, \Psi(x)$$

- Solution with $\Psi(x) = 0$ at $x = 0, L$ ← **Boundary Condition**

  $$\Psi(x) = 2^{1/2} L^{-1/2} \sin(kx)\ ,\quad k = m\,\pi/L,\ m = 1,2, \ldots$$

  (Note similarity to vibration waves)

  Factor chosen so $\int_0^L dx\, |\Psi(x)|^2 = 1$

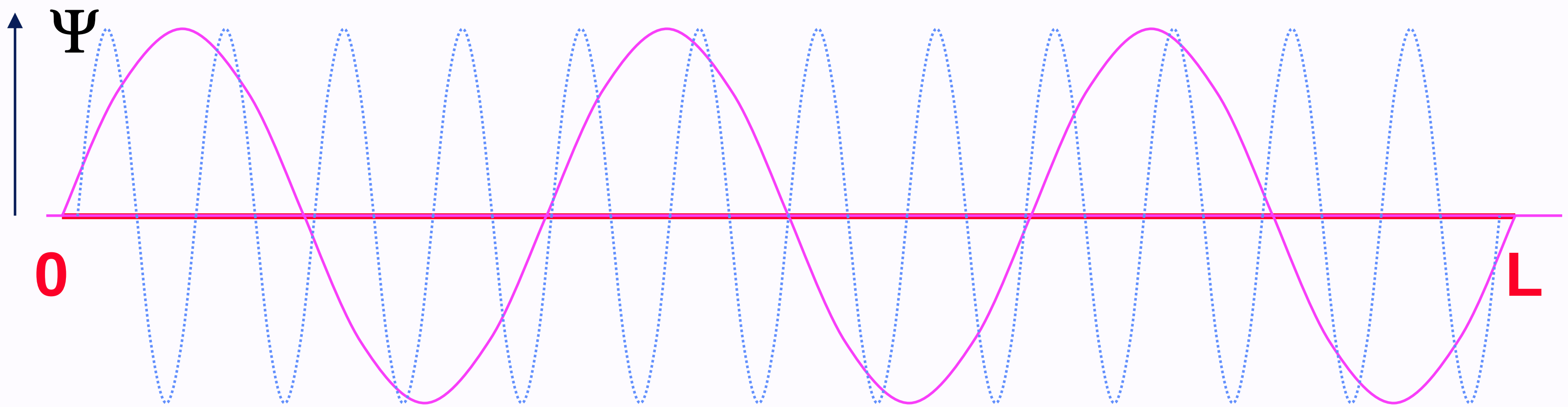
- $E(k) = \left(\hbar^2/2m\right) k^2$

From Lecture 12
See Kittle, Ch 6

Electrons on a line

- Solution with $\Psi(x) = 0$ at $x = 0, L$

  Examples of waves - same picture as for lattice vibrations except that here $\Psi(x)$ is a continuous wave instead of representing atom displacements

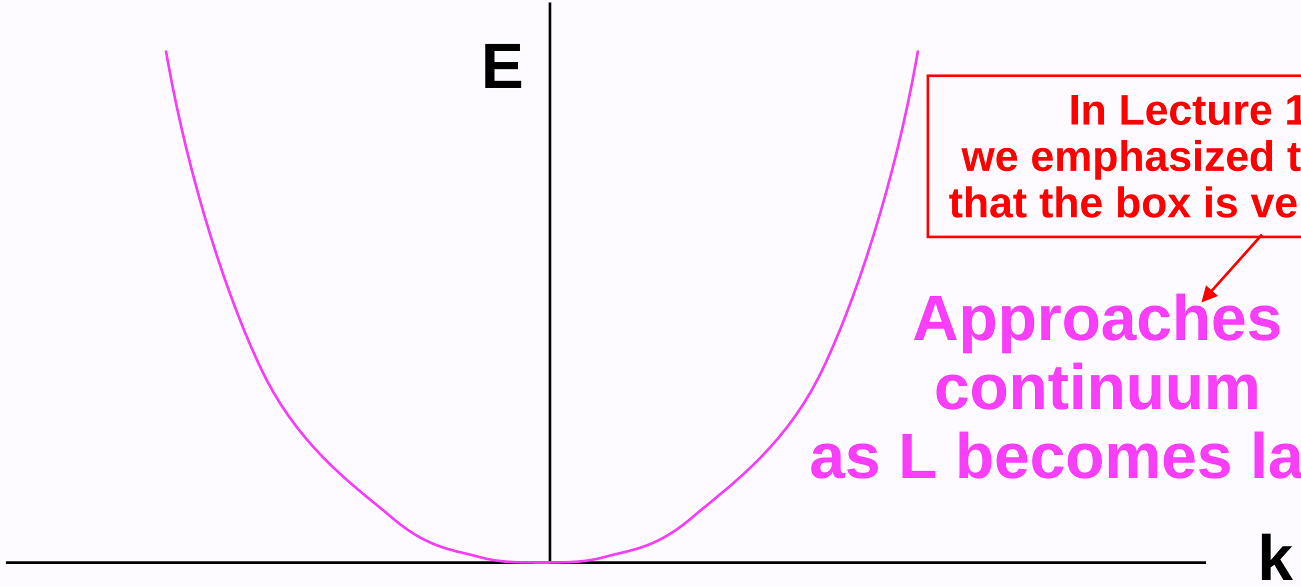


From Lecture 12
See Kittle, Ch 6

Electrons on a line

- For electrons in a box, the energy is just the kinetic energy which is quantized because the waves must fit into the box

$E(k) = (\hbar^2/2m)\, k^2$, $k = m\,\pi/L$, $m = 1,2, \ldots$



In Lecture 12 we emphasized the limit that the box is very large

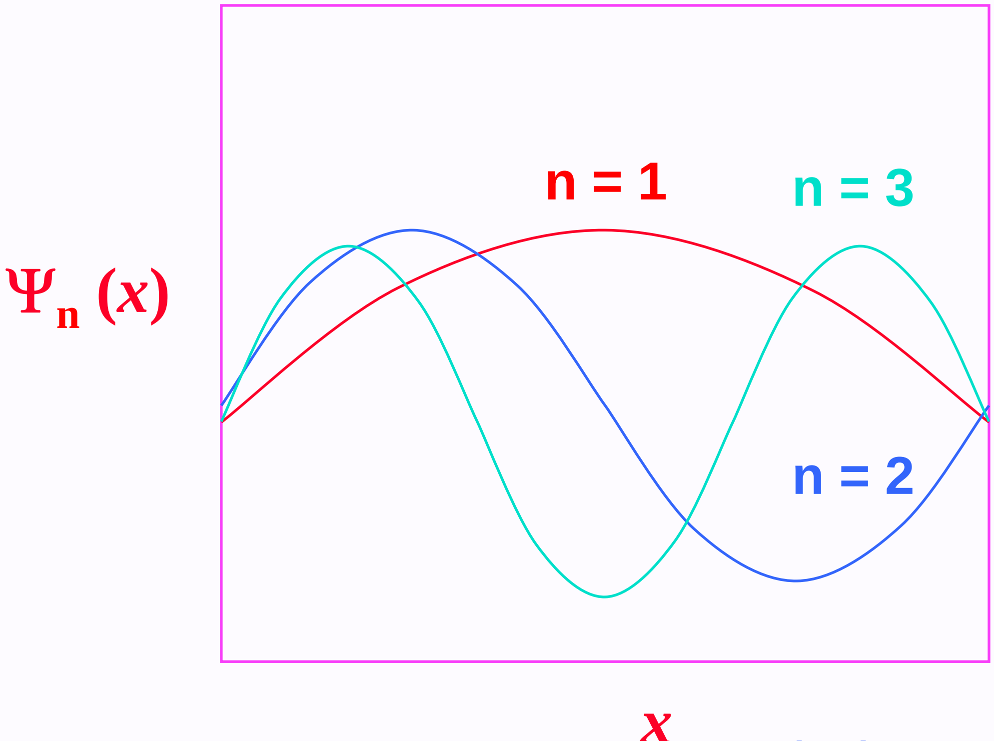
Approaches continuum as L becomes large

From Lecture 12
See Kittle, Ch 6

Quantization for motion in z direction

- $E_n = (h^2/2m) k_z^{\,2}$, $k_z = n \pi/L$, n = 1,2, ...

- Lowest energy solutions with $\Psi_n (x) = 0$ at x = 0,L



Here we emphasize the case where the box is very small

Total energies of Electrons

- Including the motion in the x,y directions gives the total energy for the electrons:

$$E(\underline{\mathbf{k}}) = (\hbar^2/2m^*)(k_x^2 + k_y^2 + k_z^2)$$

$$= E_n + (\hbar^2/2m^*)(k_x^2 + k_y^2)$$

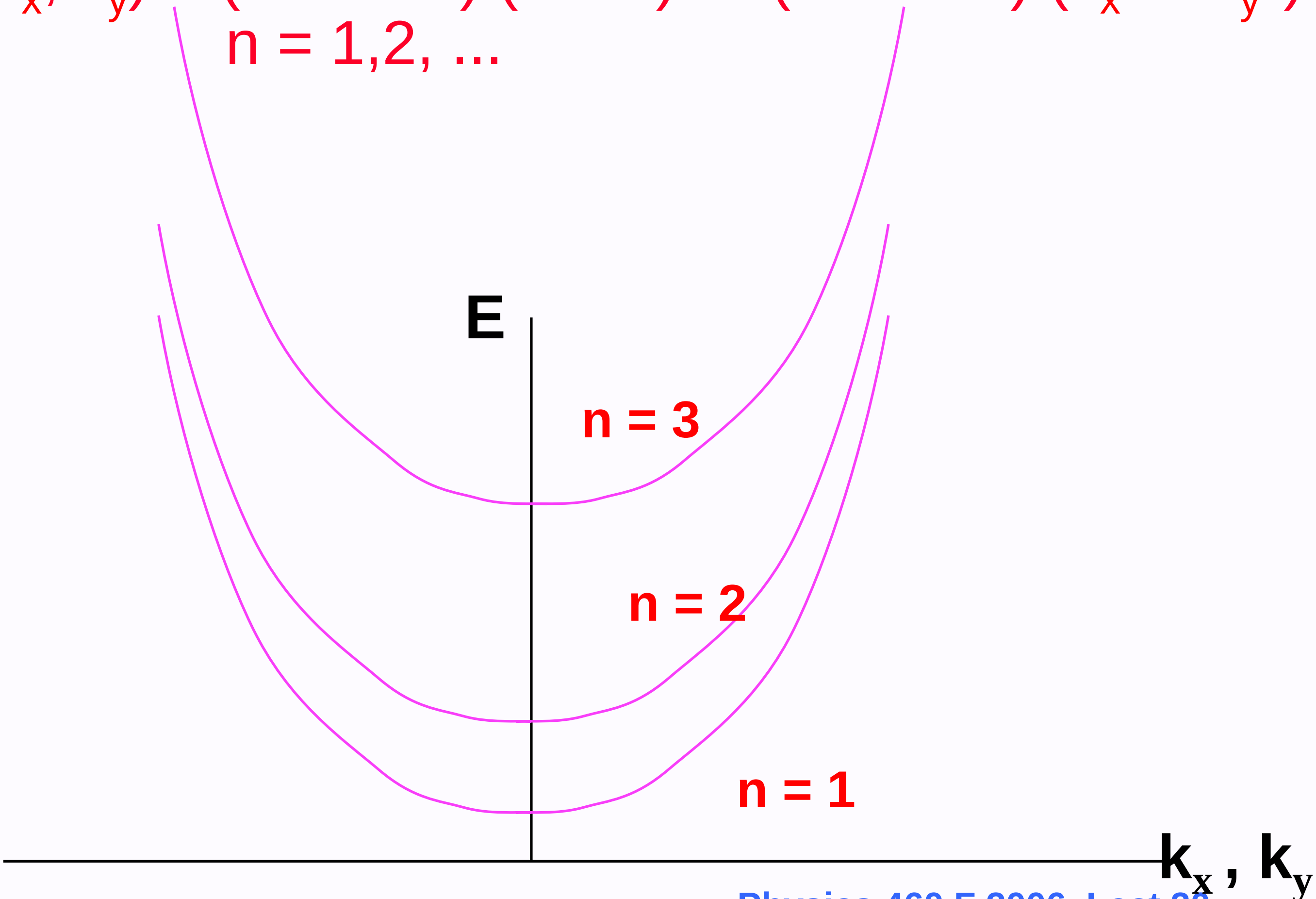
$$= (\hbar^2/2m^*)(n\pi/L)^2 + (\hbar^2/2m^*)(k_x^2 + k_y^2)$$

$$n = 1,2, \ldots$$

- This is just a set of two-dimensional free electron bands (with m = m*) each shifted by the constant $(\hbar^2/2m^*)(n\pi/L)^2$, $n = 1,2, \ldots$

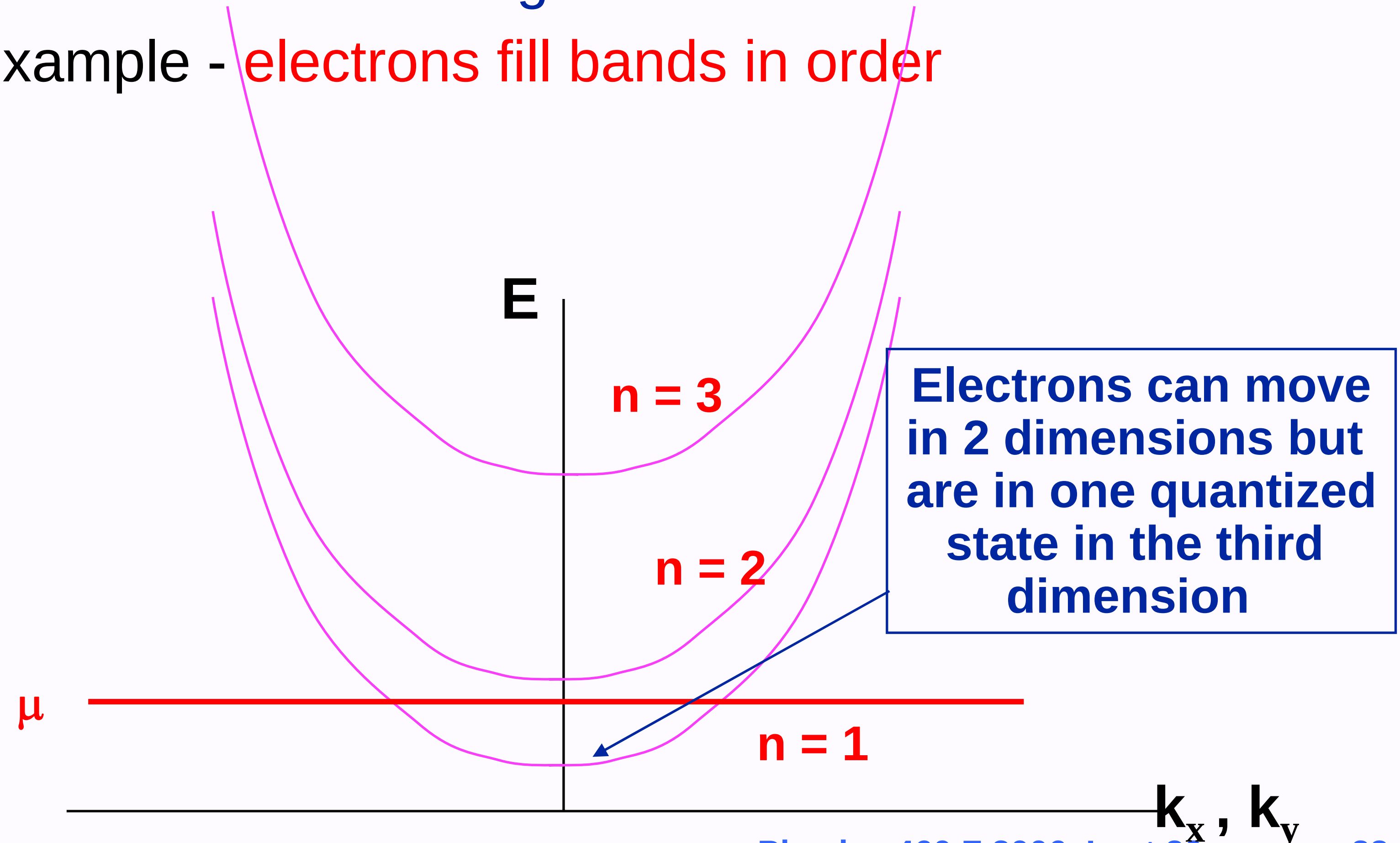
Quantized two-dimensional bands

- $E_n(k_x, k_y) = (\hbar^2/2m^*)(n\pi/L)^2 + (\hbar^2/2m^*)(k_x^2 + k_y^2)$
  $n = 1,2, \ldots$



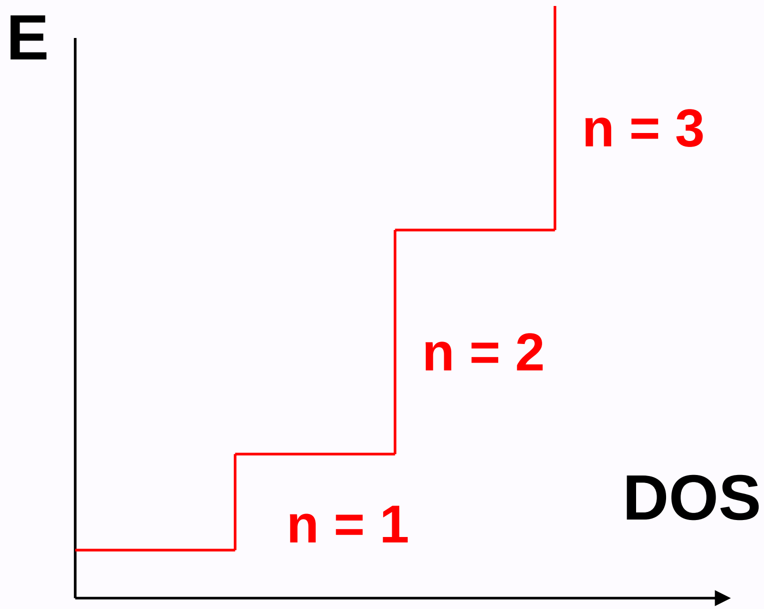
Quantized two-dimensional bands

- What does this mean? One can make two-dimensional electron gas in a semiconductor!
- Example - electrons fill bands in order



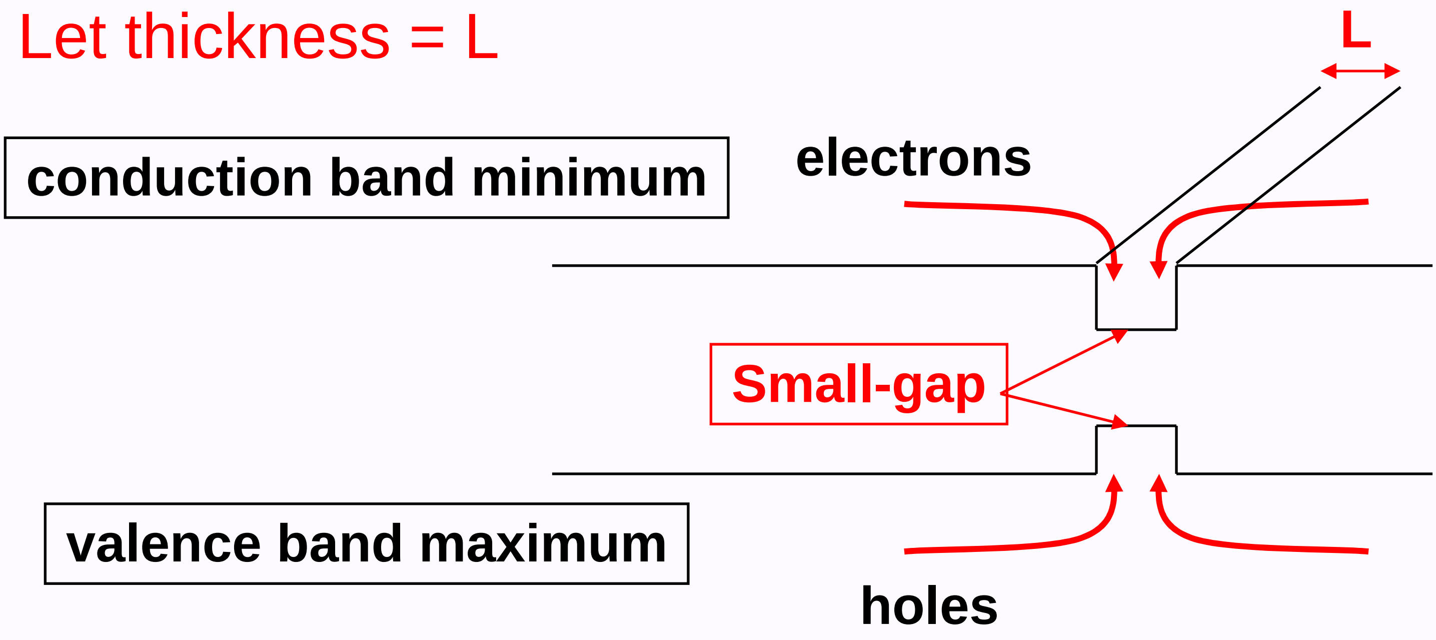
Density of States in two-dimensions

- Density of states (DOS) for each band is constant
- Example - electrons fill bands in order



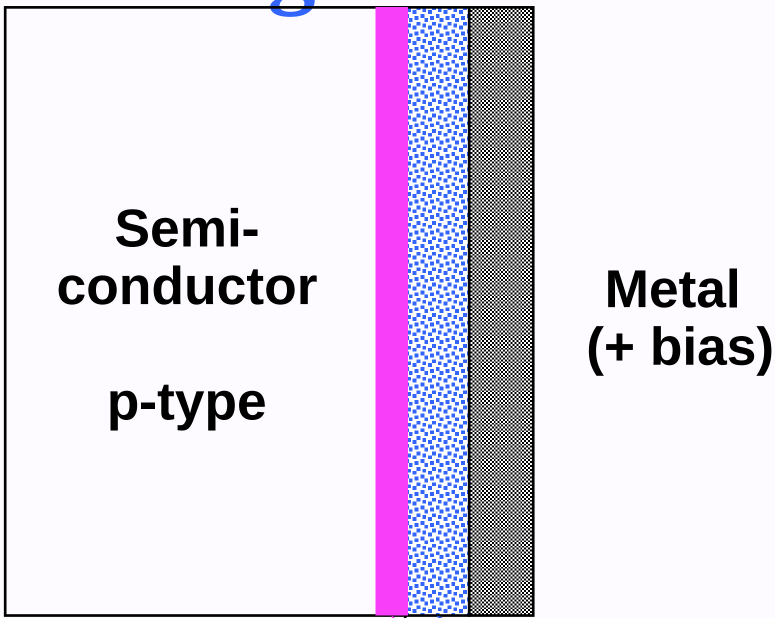
Quantum Layered Structures

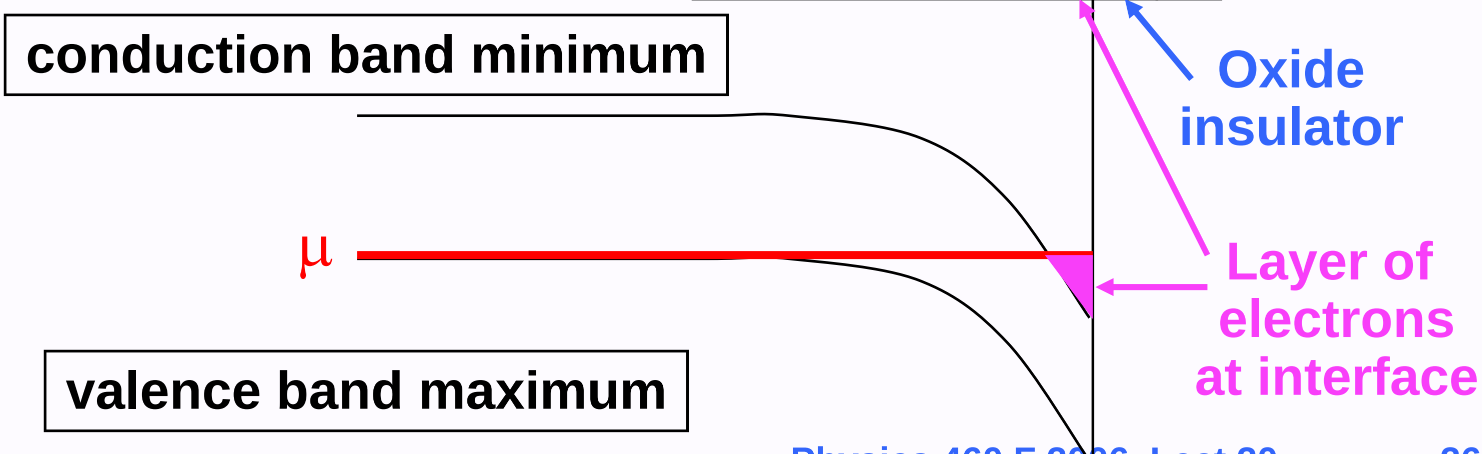
- If wells are very thin one gets quantization of the levels and they are called "quantum wells"
- Confined in one direction - free to move in the other two directions
- Let thickness = L



MOS Structure - Again

- Electrons form layer
  Mobile in two dimensions
  Confined in 3rd dimension
- If layer is thin enough, can have quantization of levels due to confinement
- Similar to layer structures discussed next

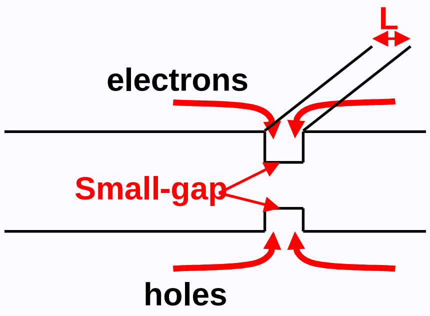




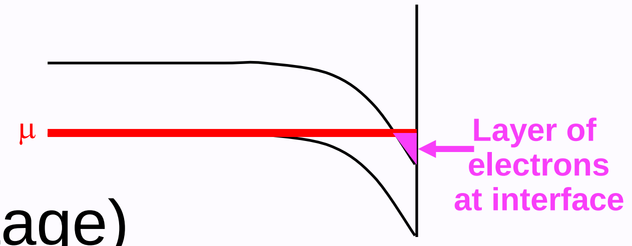
Electrons in two dimensions

- If the layer is think enough all electrons are in the lowest quantum state in the direction perpendicular to the layer but they are free to move in the other two directions
- $E(\underline{\mathbf{k}}) = (\hbar^2/2m^*)(n\pi/L)^2 + (\hbar^2/2m^*)(k_x^2 + k_y^2)$
  $n = 1,2, \ldots$

- This can happen in a heterostructure
  (the density of electrons is controlled by doping)



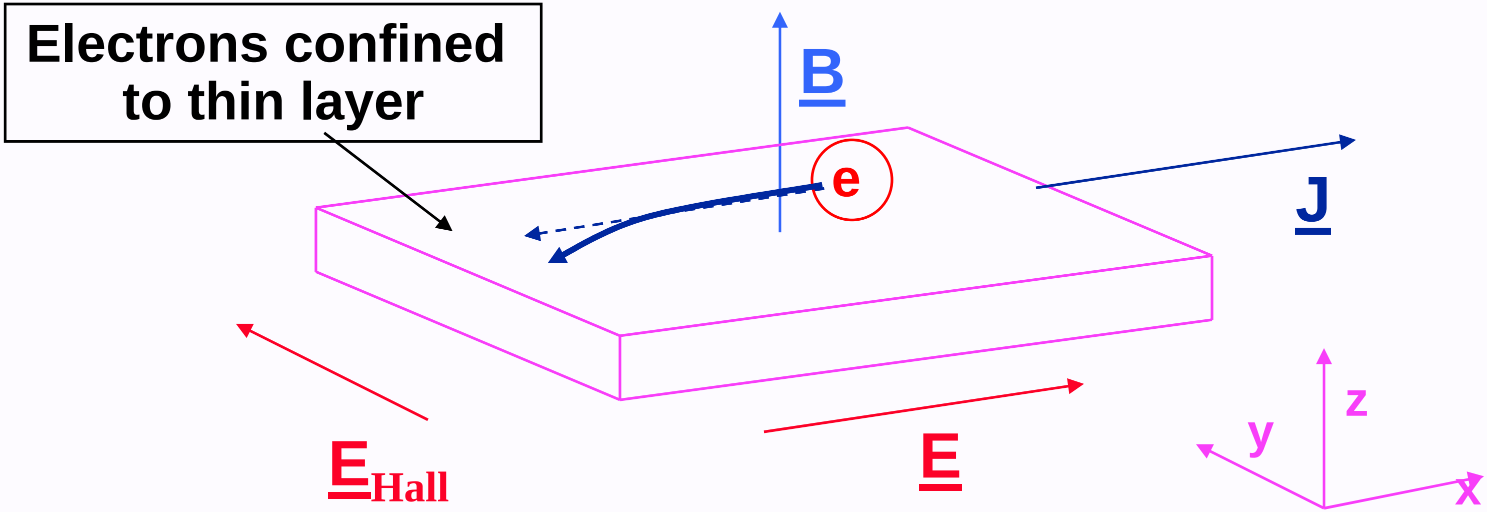
- Or a MOS structure
  (the density of electrons is controlled by the applied voltage)



Hall Effect

- See lecture 18 – here we consider only electrons of density n = #/area
- The Hall effect is given by

$$\rho_{Hall} = E_{Hall} / J\, B = -(1/ne) \qquad \text{(SI units)}$$

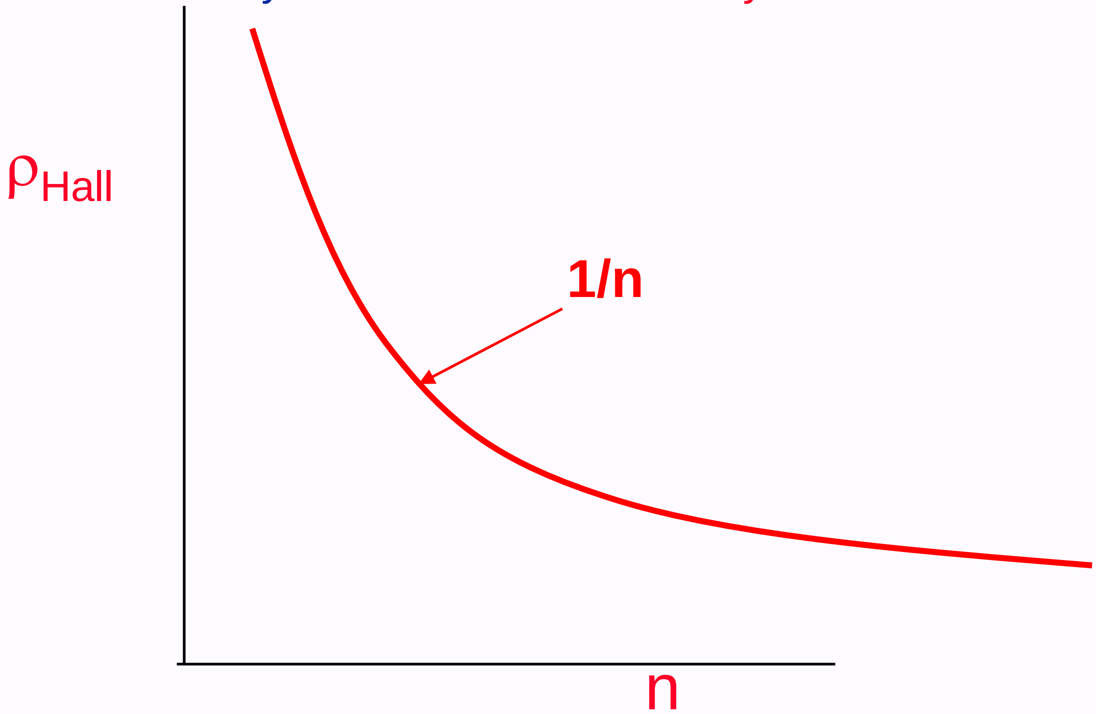


Hall Effect

- Expected result as the density n is changed
- The Hall constant = is given by

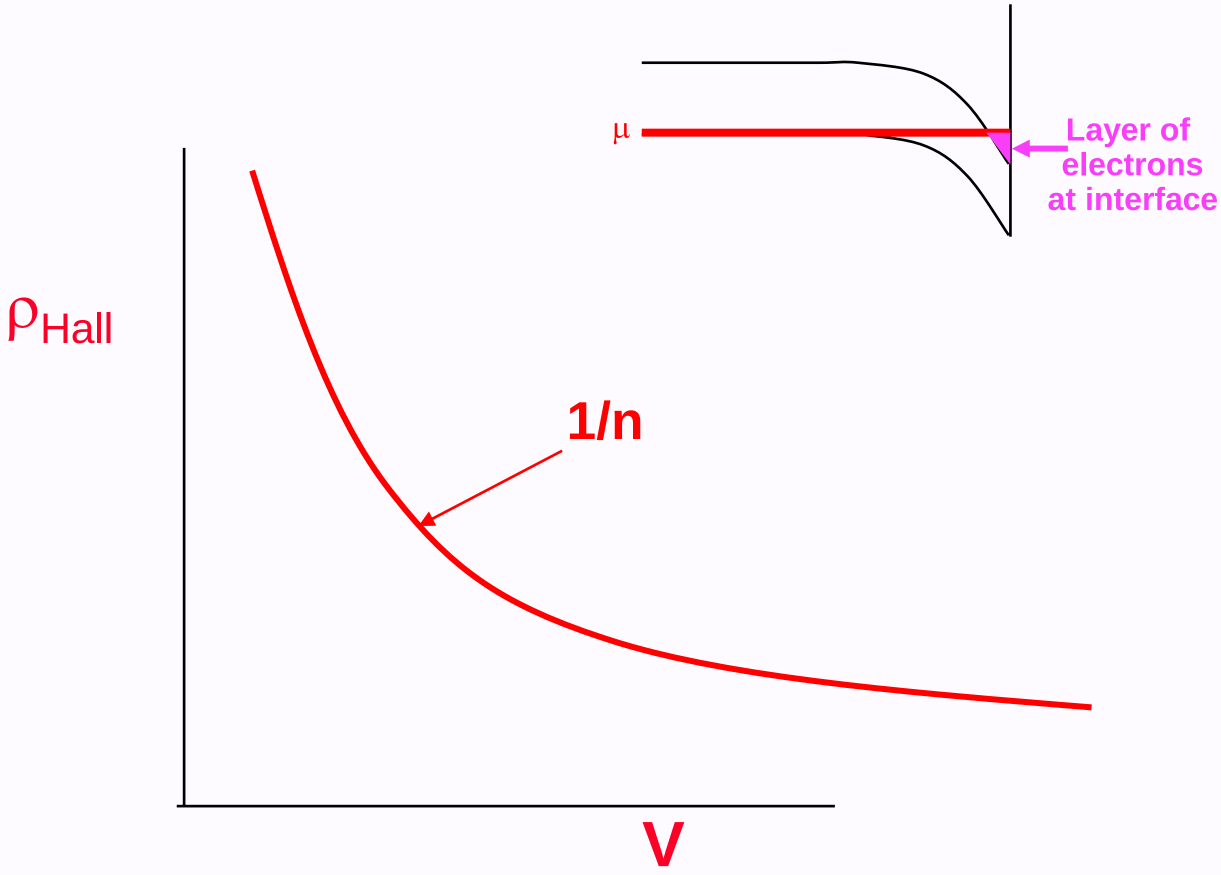
$$\rho_{Hall} = E_{Hall} / J B = V_{Hall}/IB = -(1/ne)$$

where $I = J \times L_y$, $V_{Hall} = E_{Hall} \times L_y$



Hall Effect

Consider a MOS device in which we expect n to be proportional to the applied voltage



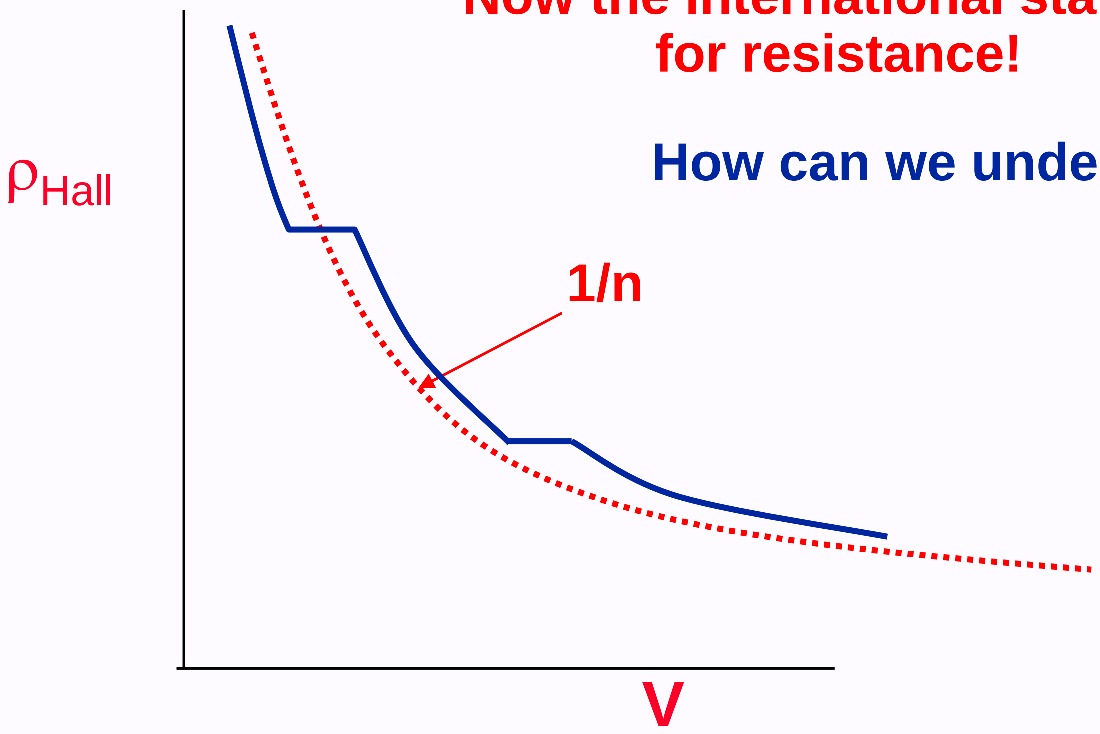
Quantized Hall Effect (QHE)

- What really happens is -----
- Quantized values at the plateaus

$$\rho_{Hall} = (h/e^2)(1/s), \quad s = \text{integer}$$

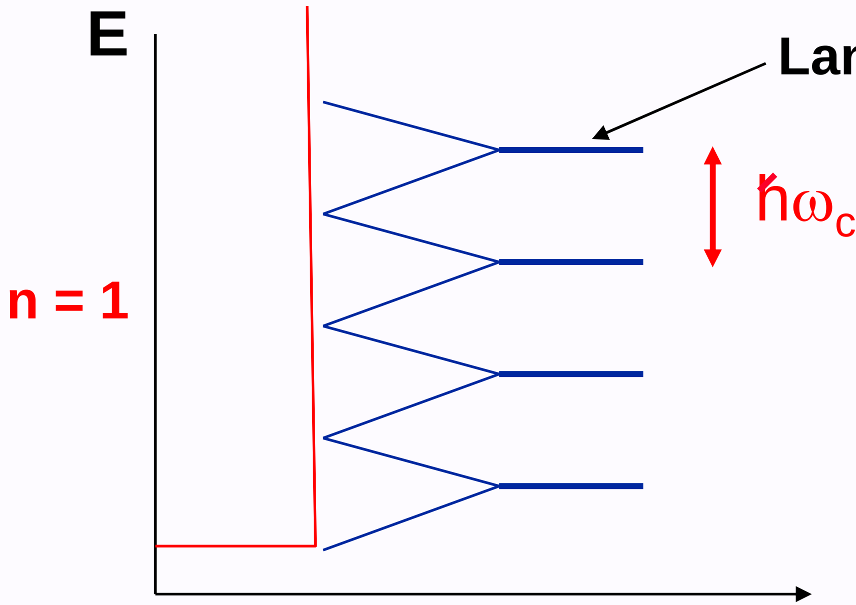
Now the international standard for resistance!

How can we understand this?



Quantized Hall Effect

- In a magnetic field, electrons in two dimensions have a very interesting behavior
- The energies of the states are quantized at values $\hbar\omega_c$ (ω_c = qB/m* = cyclotron frequency from before)
- (Similar to figure 10, Ch 17 in Kittel)

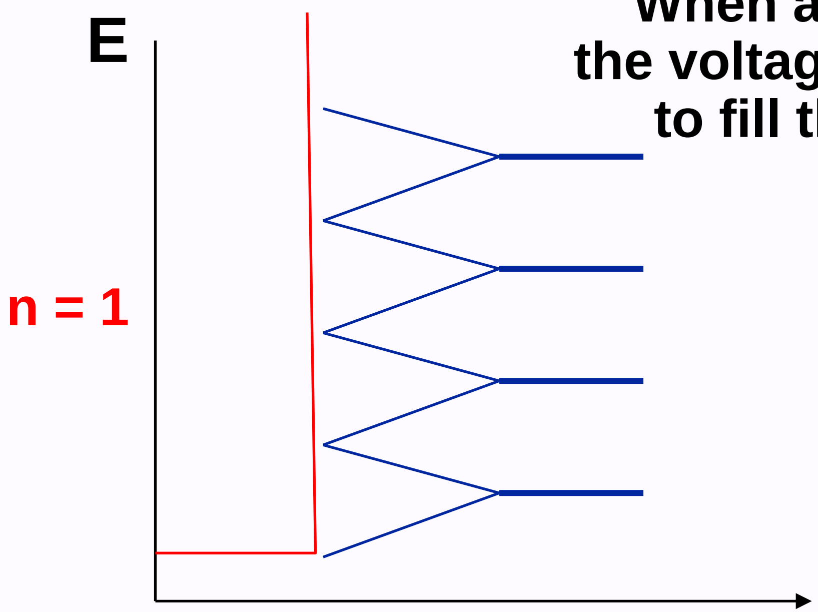


Quantized Hall Effect

- Now what do you expect for the Hall effect, given by $R_H = 1/(nec)$

As a function of the applied voltage V, the electrons fill the Landau levels

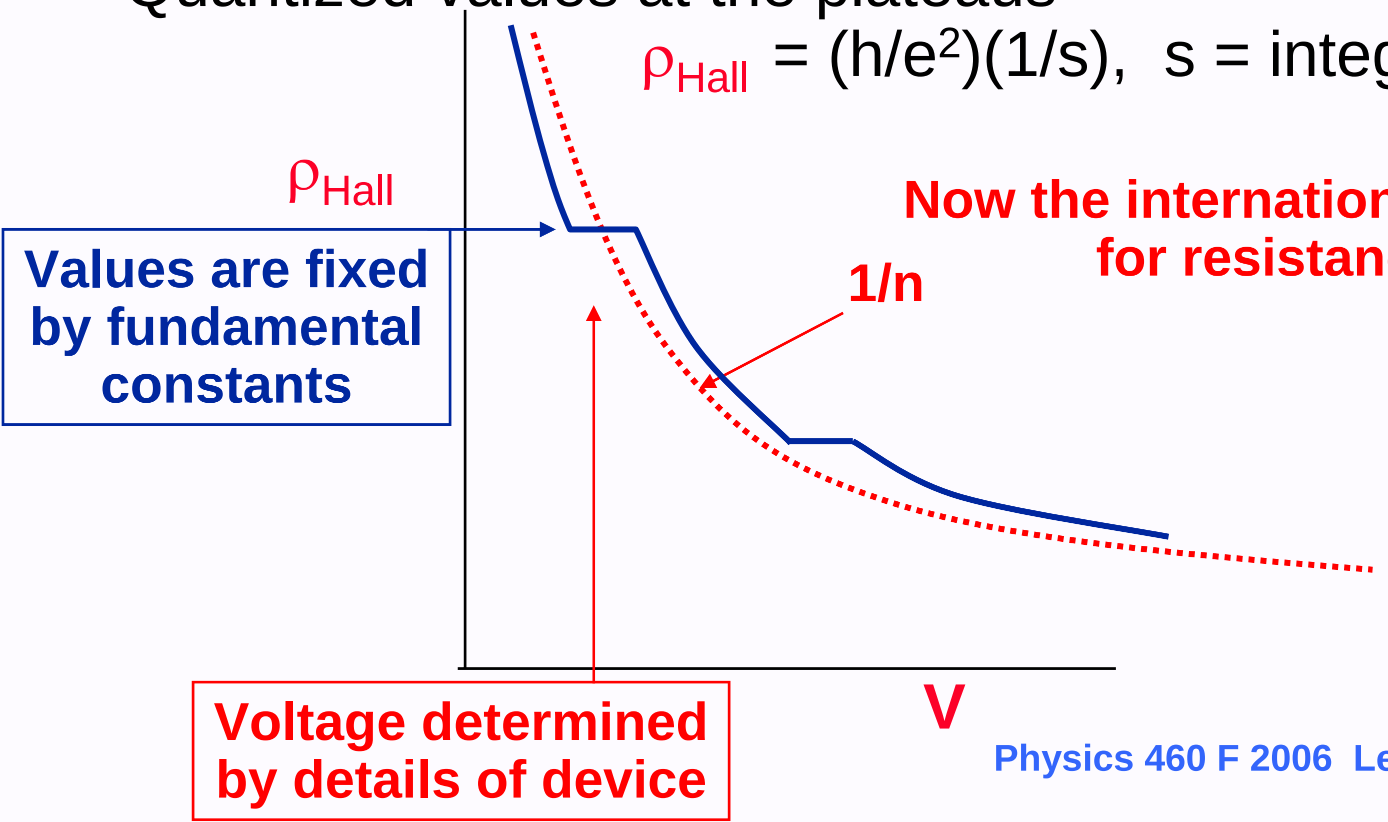
When a level is filled the voltage must increase to fill the next level



Quantized Hall Effect (QHE)

- The Hall constant is constant when the levels are filled
- **Elegant argument due to Laughlin that it work in a dirty ordinary semiconductor!**
- Quantized values at the plateaus

$$\rho_{Hall} = (h/e^2)(1/s), \quad s = \text{integer}$$



Summary

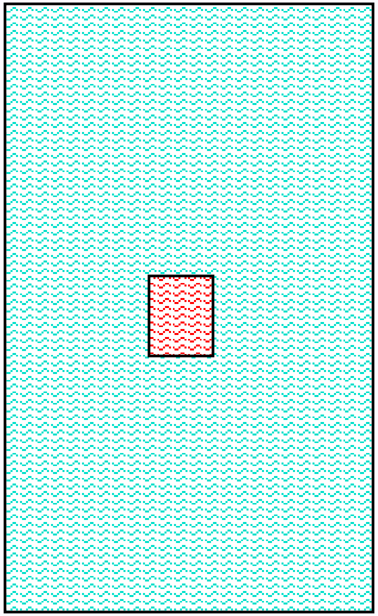
- What is a semiconductor Structure?
- Created by Applied Voltages
  - Conducting channels near surfaces
  - Controlled by gate voltages
  - MOSFET
- Created by material growth
  - Layered semiconductors
  - Grown with atomic layer control by “MBE”
  - Confinement of carriers
    - High mobility devices
    - 2-d electron gas
    - Quantized Hall Effect
  - Lasers
- Covered briefly in Kittel Ch 17, p 494-503, 507- 511
  - added material in the lecture notes

Next time

- Semiconductor nanostructures

Semiconductor Quantum Dots

- Structures with electrons (holes) confined in all three directions
- Now states are completely discrete
- "Artificial Atoms"

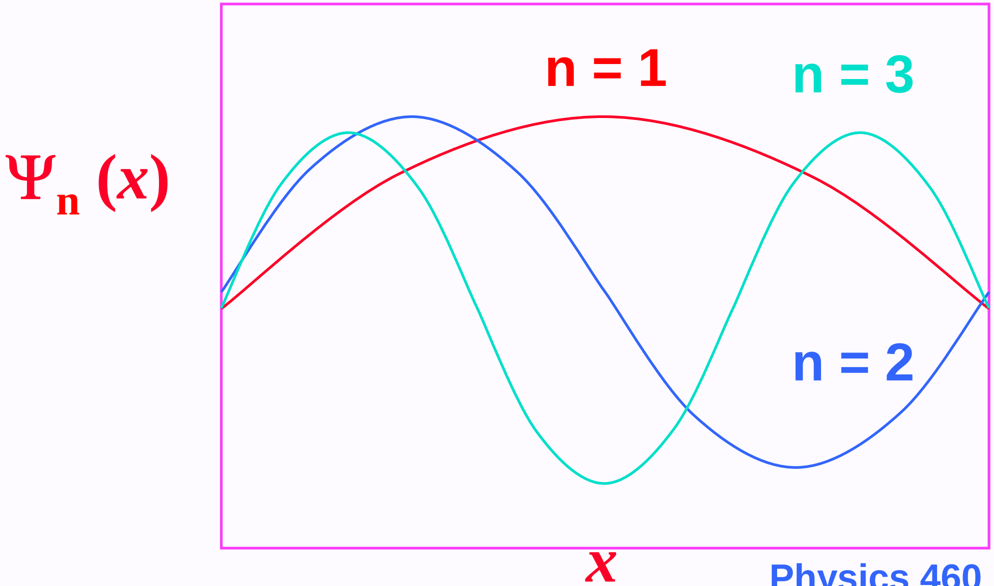


Semi-conductor Small-gap e.g. GaAs

Semi-conductor Large-gap e.g. AlAs

Quantization in all directions

- Now we must quantize the k values in each of the 3 directions
  $E = (h^2/2m^*) [(n_x \pi/L_x)^2 + (n_y \pi/L_y)^2 + (n_z \pi/L_z)^2]$
  $n_x , n_y , n_x = 1,2, ...$
- Lowest energy solutions with $\Psi_n (x,y,z) = 0$ at $x = 0,L_x$, $y = 0,L_y$, $z = 0,L_z$ has behavior like that below in all three directions



Confinement energies of Electrons

- The motion of the electrons is exactly like the "electron in a box" problems discussed in Kittel, ch. 6
- Except the electrons have an effective mass m*
- And in this case, the box has length L in one direction (call this the z direction - $L = L_z$) and very large in the other two directions (L_x, L_y very large)

- Key Point: For ALL cases, the energy

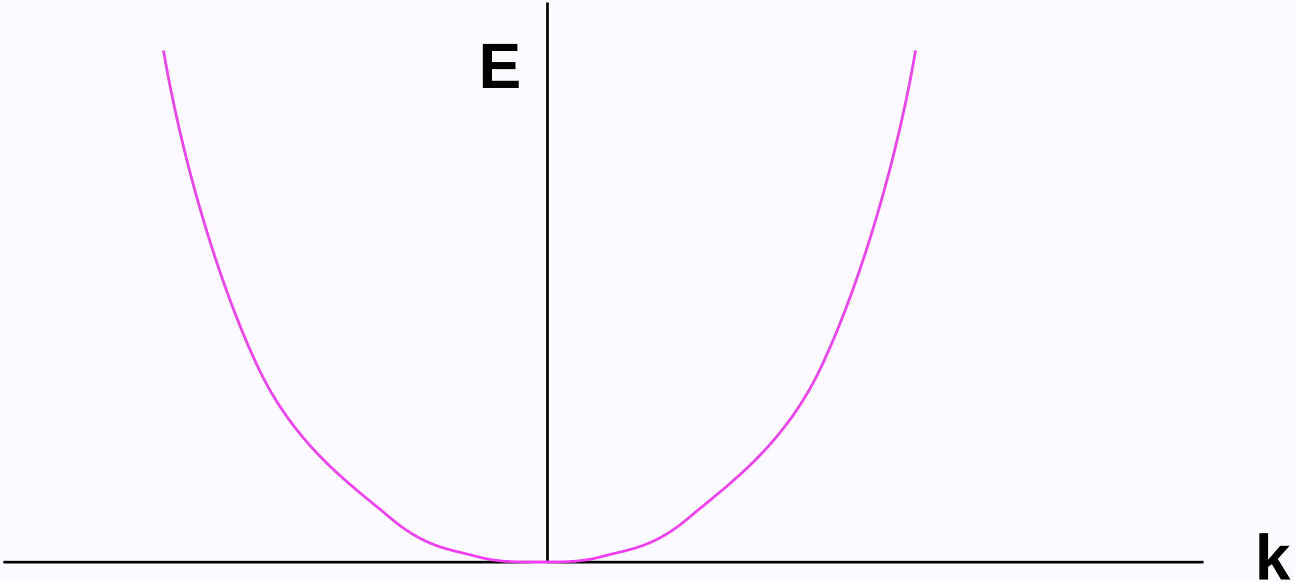
$$E(\mathbf{k}) = (\hbar^2/2m^*)(k_x^2 + k_y^2 + k_z^2)$$

- We just have to figure out what k_x, k_y, k_z are!

Quantization in the confined dimension

- For electrons in a box, the energy is quantized because the waves must fit into the box (Here we assume the box walls are infinitely high - not true but a good starting point)

$E(k_z) = (\hbar^2/2m^*)\, k_z^{\,2}$, $k_z = n\,\pi/L$, $n = 1, 2, \ldots$



Electrons in a thin layer

- To describe a thin layer, we consider a box with length L in one direction (call this the z direction and define $L = L_z$) and very large in the other two directions (L_x, L_y very large)
- Solution

$$\Psi = 2^{3/2} L^{-3/2} \sin(k_x x) \sin(k_y y) \sin(k_z z) ,$$

$k_x = m\,\pi/L$, m = 1,2, …, same for y,z

$$E(k) = (\hbar^2/2m)(k_x^2 + k_y^2 + k_z^2) = (\hbar^2/2m)\,k^2$$