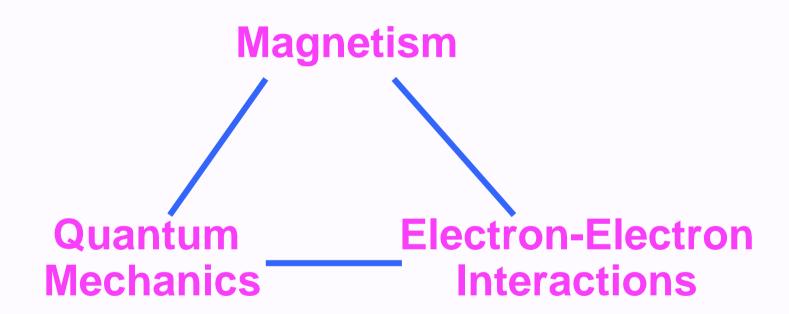
Lecture 24: Magnetism (Kittel Ch. 11-12)



Outline

- Magnetism is a purely quantum phenomenon!
 Totally at variance with the laws of classical physics
 (Bohr, 1911)
- Diamagnetism
- Spin paramagnetism (Pauli paramagnetism)
- Effects of electron-electron interactions
 Hund's rules for atoms examples: Mn, Fe
 Atoms in a magnetic field Curie Law
 Atomic-like local moments in solids
 Explains magnetism in transition metals, rare earths
- Magnetic order and cooperative effects in solids
 Transition temperature Tc
 Curie-Weiss law
- Magnetism: example of an "order parameter"
- (Kittel Ch. 11-12 only selected parts)

Magnetism and Quantum Mechanics

- Why is magnetism a quantum effect?
- In classical physics the change in energy of a particle per unit time is <u>F</u> · <u>v</u> (<u>F</u> = force, <u>v</u> = velocity <u>vectors</u>).
- In a magnetic field the force is always perpendicular to velocity - therefore the energy of a system of particles cannot change in a magnetic field <u>B</u>
- Similarly the equilibrium free energy cannot change with applied B field
- Since the change in energy is dB · M , there must be no total magnetic moment M !

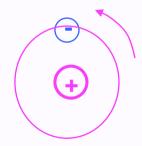
Definitions

- <u>B</u> = μ₀ (<u>H</u> + <u>M</u>)
 (μ₀ is the permeability of free space,
 <u>B</u> is the field that causes forces on particles)
- If the magnetization is proportional to field, $\underline{\mathbf{M}} = \chi \ \underline{\mathbf{H}}$ and $\underline{\mathbf{B}} = \mu \ \underline{\mathbf{H}}$, $\mu = \mu_0 \ (1 + \chi)$
- Diamagnetic material: χ < 0; μ < μ_0
- Paramagnetic material: $\chi > 0$; $\mu > \mu_0$

• Ferromagnetic material: $\underline{\mathbf{M}} \neq 0$ even if $\underline{\mathbf{H}} = 0$

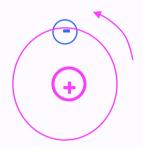
Diamagnetism

- Consider a single "closed shell" atom in a magnetic field (In a closed shell atom, spins are paired and the electrons are distributed spherically around the atom - there is no total angular momentum.)
- Diamagnetism results from current set up in atom due to magnetic field
- Like Lenz's law current acts to oppose the external field and "shield" the inside of the atom from the field (like a dielectric)



"Classical" Theory of Diamagnetism

- If the field **B** is small compared to the quantum energy level separation, the closed shell atom may be considered to rotate rigidly due to the field **B**
- This is like a classical current -BUT it occurs only because the atom is in a quantized state



- The entire electron system rotates together with the frequency $\omega = eB/2m$
- Like Lenz's law the current acts to oppose the external field

"Classical" Theory of Diamagnetism

• Total current = charge/time = $I = (-Ze) (1/2\pi)(eB/2m)$

Susceptibility =

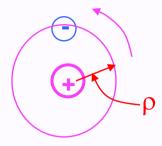
$$\chi = \mu_0 \text{ M/B} = -\mu_0 \text{NZe}^2/4\text{m} < \rho^2 > = -\mu_0 \text{NZe}^2/6\text{m} < r^2 >$$

where $M = N\mu$, N = density of atoms, and for a spherical atom $\langle r^2 \rangle = 2/3 \langle \rho^2 \rangle$, where r is the radius in 3 dimensions

"Classical" Theory of Diamagnetism

• From previous slide - for closed shell atoms

$$\chi = \mu_0 \text{ M/B} = - \mu_0 \text{NZe}^2/6\text{m}) < r^2 >$$



- Results: VERY small diamagnetism!
 For rare gasses in a solid, magnetic susceptibility is only VERY slightly less than in vacuum
- Similar results are found for typical "closed shell" insulators -- like Si, diamond, NaCl, SiO₂, because they have paired spins and filled bands like a closed shell atom -- VERY weak diamagnetism

Spin Paramagnetism

- What about spin?
- Unpaired spins are affected by magnetic field!
- The energy in a field is given by $U = \underline{\mu} \cdot \underline{B} = m g \ \mu_B B$ where $m = component of spin = \pm 1/2,$ g = "g factor" = 2, and $\mu_B = Bohr magneton = e\hbar/2m$



- Any atom with an unpaired spin (e.g. and odd number of spins) must have this effect
- At temperature = 0, the spin will line up with the field in a paramagnetic way - i.e. to increase the field

Spin Paramagnetism in a metal

- What happens in a metal?
- Spin up electrons
 (parallel to field)
 are shifted opposite
 to spin down electrons
 (antiparallel to field)
- Energies shift by $\Delta E = \pm \mu_B B$
- Magnetization

$$M = \mu_B (N \uparrow - N \downarrow)$$

$$= \mu_B (1/2) D(E_F) 2 \mu_B B = \mu_B^2 D(E_F) B$$

Density of states for both spins

N↑ - **N**↓

Free electron gas (see previous notes + Kittel)

$$M = (3/2) N \mu_B^2 B/(k_B T_F)$$

Density of states

Ε

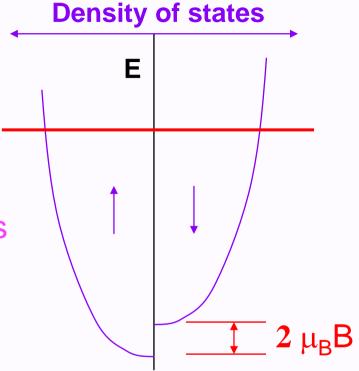
 $2 \mu_B B$

Spin Paramagnetism in a metal

Result for a metal:

• $M = \mu_B^2 D(E_F) B$ or $\chi = \mu_B^2 D(E_F)$

 This is a way to measure the density of states! (Note: There are corrections from the electron-electron interactions.)



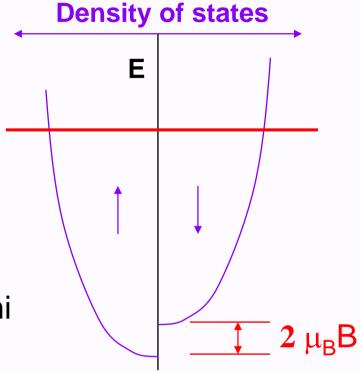
Paramagnetic

Tends to align with the field to increase the magnetization

Spin Paramagnetism in a metal

- Early success of quantum mechanics
- Explained by Pauli

• The magnitude is greatly reduced by the factor $\mu_B B/(k_B T_F)$ due to the fact that the Fermi energy $E_F = k_B T_F >> \mu_B B$ for any reasonable B



 The same reason that the heat capacity is very small compared to the classical result

Magnetic materials

- What causes some materials (e.g. Fe) to be ferromagnetic?
- Others like Cr are antiferromagnetic (what is this?)
- Magnetic materials tend to be in particular places in the periodic table: transition metals, rare earths Why?
- Starting point for understanding: the fact that open shell atoms have moments. Why?
- Leads us to a re-analysis of our picture of electron bands in materials. The band picture is not the whole story!

Questions for understanding materials:

 Why are most magnetic materials composed of the 3d transition and 4f rare earth elements

Group I	Group II					Transition	elements					Group III	Group IV	Group V	Group VI	Group VII	Group	
H 1 1.0079 1s ¹																	He 2 4.0026 1s ²	
Li 3 6.94 2x1	Be 4 9.012 2s ³	Symbol C 6 Atomic mass* 12.011 $2\rho^2$ Electron configuration										B 5 10.81 2p1	C 6 12.011 2p ² 2	N 7 14.007 2p ³	O 8 15.999 2p4	F 9 18.998 2p ⁵	Ne 10 20.18 2p ⁶	
Nn 11 22.990 3a ¹	Mg 12 24.305 3s ²												Si 14 28.086 3p [‡]	P 15 30.974 3p ³	S 16 32.06 3p4	CI 17 35.453	Ar 18 39.948 3p ⁶	
K 19 39.098 4s ¹	Ca 20 40.08 4s ²	Se 21 44.956 3d ¹ 4s ²	Ti 22 47.90 3d ² 4s ²	V 23 50.94 3d ¹ 4s ²	Cr 24 51.996 3d ⁵ 4s ¹	Mn 25 54.938 3d ⁵ 4s ²	Fe 26 55.847 3d*4s ²	Ce 27 58.933 3d ² 4s ²	Ni 28 58.71 3/84/2	Cu 29 63.546 3d ¹⁹ 4s ¹	Zn 30 65.38 3d ¹⁰ 4s ²	Ga 31 69.72 4p ¹	Ge 32 72.59 4p ²	As 33 74.922 4p ³	Se 34 78.96 4p4	Br 35 79.904 4p ⁵	Kr 36 83.80 4p5	
Rb 37 85.467 5s ²	Sr 38 87.62 5s ²	Y 39 88.906 4d ¹ 5a ²	Zr 40 91.22 4d ² 5s ²	Nb 41 92,906 4d*5s1	95.94 4d ⁶ 4s ¹	98.9 4d°5s2	101.07 4d°5s1	102.906 4d ⁶ 5s ²	106.4 4gen	Ag 47 107,868 4d*5s1	Cd 48 112.41 4d ¹⁹⁵ s ²	In 49 114.82 5p ¹	Sn 50 118.69 5p:	Sb 51 121.75 5p ³	Te 52 127.60 5p ⁴	I 53 126.90 5p ³	Xe 54 131.30 5p5	
Cs 55 132.905	Ba 56 137.33 602	57-71†	Hr 72 178.49 5d ² 6s ²	Ta 73 180.95 5d ¹ 6s ²	W 74 183.85 5d*6s2	Re 75 186.207 5d%s ²	Os 76 190.2 5d*6s2	Ir 77 192.22 5d ¹ 6s ²	Pt 78 195.09 5d ⁹ 6s ¹	Au 79 196.966 5d ¹⁰ 6s ¹	Hg 80 200.59 5d ¹⁰ 6s ²	T1 81 204.37 6p ¹	Pb 82 207.2 6p ²	Bi 83 208.980 60°	Po 84 (209) 6g*	At 85 (210) 6p ²	Rn 86 (222) 6pt	
Fr 87 (223) 7s ¹	Ra 88 226.025 7s ²	89-103‡	Rf 104 (261) 6d ² 7x ²	Ha 105 (260) 6d ⁹ 7z ²	106 (263)	107 (262)	108 (265)	109 (266)									.,	,
† LANTHANIDE SERIES		ES	La 57 139.906 5d ⁶ 6s ²	Ce 58 140.12 4/26s2	Pr 59 140.908 4/ ³ 6a ²	Nd 60 144.24 4f ⁴ 6x ²	Pm 61 (145) 4f ³ 6r ²	Sm 62 150.4 4/*6r2	Eu 63 151.96 4/16s ³	Gd 64 157.25 5d/4/76s ²	Tb 65 158.925 4/%62	Dy 66 162.50 4/ ¹⁸ 6s ²	Ho 67 164.930 4/1/6s ²	Er 68 167.26 4/**6r*	Tm 69 168.934 4/**6a2	Yb 70 173.04 4/146s2	Lu 71 174,967 5d*4f**6s*	(Lanthani
: ACTINIDE SERIES			Ac 89 (227) 6d ⁴ 7s ²	Th 90 232.038 6s ⁰ 7s ²	Pn 91 231.039 5/26d*7x2	U 92 238.029 50'6d'7s2	Np 93 237.048 5/46d ² 7s ²	Pu 94 (244) 5(*7;2	Am 95 (243) 56*762	Cm 96 (247) 5/*6d*7s2	Bk 97 (247) 5/%d*7s ²	Cf 98 (251) 5f ¹⁸⁷ s ²	Es 99 (253) 5/**752	Fm 100 (257) 50°27s ²	Md 101 (258) 5f ¹⁰ 7s ²	No 102 (259) 55 ^{rs} 73 ²	Lr 103 (260) 6d*7s2	(Actinic

Average value based on the relative abundance of isotopes on earth. For unstable elements, the mass of the most stable isotope is given in brackets.

The first step in understanding magnetic materials

- Magnetic moments of atoms
- In most magnetic materials (Fe, Ni,) the first step in understanding magnetism is the consider the material as a collection of atoms
- Of course the atoms change in the solid, but this gives a good starting point – qualitatively correct

When are atoms magnetic?

- An atom MUST have a magnetic moment if there are and odd number of electrons – spin ½ (at least)
- "Open shell" atoms have moments Hund's rules –
 1st rule: maximum spin for electrons in a given shell
 2nd rule: maximum angular momentum possible for the
 given spin orientation

• Fe²⁺ - 6 d electrons

$$m_L = -2$$
 -1 0 1 2Physics 460 F 2006 Lect 24

Hund's Rules & Electron Interactions

Hund's rules -

1st rule: maximum spin for electrons in a given shell

Reason – parallel-spin electrons are kept apart because they must obey the exclusion principle – thus the repulsive interaction between electrons is reduced for parallel spins!

2nd rule: maximum angular momentum possible for the given spin orientation

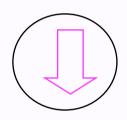
Reason – maximum angulat momentum means electrons are going the same direction around the nucleus – stay apart – lower energy!

Electron-Electron Interactions!

Magnetic atoms in free space

- Curie Law (Kittel p 305)
- Consider N isolated atoms, each with two states (spin 1/2) that have the same energy with no magnetic field, but are split in a field into $E_1 = -\mu B$, $E_2 = \mu B$





• In the field B, the populations are:

$$N_1 / N = \exp(\mu B/k_B T) / [\exp(-\mu B/k_B T) + \exp(\mu B/k_B T)]$$

 $N_2 / N = \exp(-\mu B/k_B T) / [\exp(-\mu B/k_B T) + \exp(\mu B/k_B T)]$

• So the magnetization M is $M = \mu (N_1 - N_2) = \mu N \tanh(x), \quad x = \mu B/k_B T$

Magnetic atoms in free space

- Curie Law -- continued
- Similar laws hold for any spin

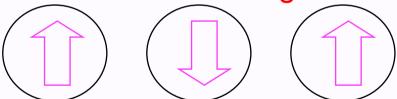
$$M = gJ\mu_B N B_J(x), \quad x = gJ\mu_B B / k_B T$$
 where $B_J(x) = Brillouin Function$ (Kittel p 304)

$$\chi = M/B = C/T$$
, where C = Curie constant

• For large x (large B or T small compared to $gJ\mu_BB/k_B$) M saturates and $\chi \to constant$

When do solids act like an array of magnetic moments?

- Consider a solid made from atoms with magnetic moments
- If the atoms are widely spaced, they retain their atomic character -- insulators because electron-electron interactions prevent electrons from moving freely
- Thus the material can be magnetic and insulating!



 OPPOSITE to what we said before! Real materials can be metallic and non-magnetic (like Na) or magnetic insulators (see later)

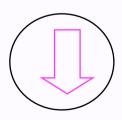
When are atoms magnetic?
Which atoms are most likely to keep their atomic like properties in a solid?

Transition metals and rare earths Why?

Because the electrons act like they have partially filled shells even in the solid!

This is why they have a special place in the periodic table - the elements in the transition series have similar chemical properties as the electrons fill the 3d or 4f shell







Questions for understanding materials:

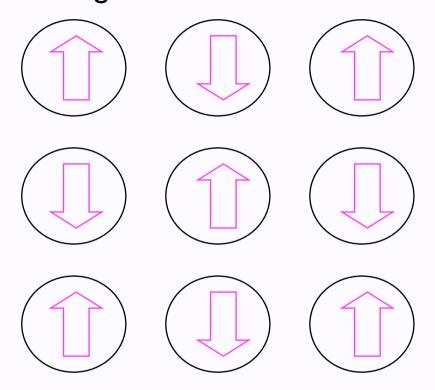
3d transition 4f rare earth

PPENI	DIX D:]	PERIOD	C TABL	E OF T	HE ELEN	MENTS												
Group	Group											Group	Group	Group	Group	Group	Group	
1		II Transition elements										Ш	IV	v	VI	VII	0	1
H 1 1.0079																	He 2 4.0026	
1.0079																	152	
U 3	Be 4	l			Symbol	C 6	**:					B 5	C 6	N 7	0 8	F 9	Ne 10	
6.94	9.012	Symbol C 6 Atomic number Atomic mass* 12.011										10.81	12.011	14.007	15.999	18,998	20.18	
$2x^{1}$	$2x^2$	2p ² Electron configuration										2p1	2p ¹ 2	$2p^{3}$	2,04	$2p^{s}$	2p6	
Na 11	Mg 12										Al 13	Si 14	P 15	S 16	Cl 17	Ar 18		
22.990	24.305											26.982	28.086	30.974	32.06	35.453	39.948	
la ¹	3,2			,						-		3p:	3p‡	3p3	$3p^4$	lp ¹)p*	
K 19	Ca 20	Sc 21	Ti 22	V 23	Cr 24	Mn 25	Fe 26	Co 27	Ni 28	Cu 29	Zn 30	Ga 31	Ge 32	As 33	Se 34	Br 35	Kr 36	
39.098 4s ¹	40.08 4s ²	44.956 3d ¹ 4s ²	47.90 3d ² 4x ²	50.94 3d ¹ 4x ²	51.996	54.938 3d ⁵ 4s ²	55.847 3d*4s2	58.933	58.71 3,842	63.546	65.38	69.72	72.59	74.922	78.96	79.904	83.80	
					3a ⁶ 4s ¹			3d"4s2		3d**4s*	3d ¹⁰ 4s ²	4p1	4p2	4p3	4p4	4p ⁵	495	
Rb 37 85.467	Sr 38 87.62	Y 39 88.906	Zr 40 91.22	Nb 41 92,906	95.94	98.9	101.07	102.906	106.4	Ag 47 107.868	Cd 48 112.41	In 49 114.82	Sn 50 118.69	Sb 51 121.75	Te 52 127.60	I 53 126.90	Xe 54 131.30	Į.
592	552	4d15s2	4d ² 5s ²	44 ⁴ 5s1	4d ⁶ 4s ¹	4d°5s2	4d°5s1	4a ⁶ 5s ²	4df0	4d**5s1	4d**5s2	5p1	5p:	5p3	5p+	5p3	5p5	l
Cs 55	Ba 56	57-71†	Hf 72	Ta 73	W 74	Re 75	Os 76	lr 77	Pt 78	Au 79	Hg 80	T1 81	Pb 82	Bi 83	Po 84	At 85	Rn 86	i
132.905	137.33		178.49	180.95	183.85	186.207	190.2	192.22	195.09	196.966	200.59	204.37	207.2	208.980	(209)	(210)	(222)	l
Sat	6x2		5a ¹² 6s ²	541651	5d46s2	3d ⁹ 6s ²	5d*6s2	5d16s1	5d%s1	5d**6s1	5d186s2	6ρ1	6p [‡]	6p3	6g*	6p ²	6p*	l
Fr 87	Ra 88	89-103‡	Rf 104	Ha 105	106	107	108	109										
(223)	226.025		(261)	(260)	(263)	(262)	(265)	(266)										
7.51	752		6d ² 7g ²	6a ¹ 7z ²	L													
			La 57	Ce 58	Pr 59	Nd 60	Pm 61	Sm 62	Eu 63	Gd 64	Tb 65	Dy 66	Ho 67	Er 68	Tm 69	Yb 70	Lu 71	1
+ LANTHANIDE SERIES		E5	139.906	140.12	140.908	144.24	(145)	150.4	151.96	157.25	158.925	162.50	164.930	167.26	168.934		174,967	(Lanth:
			5d66s2	4/26s2	4/1602	454622	4f16s2	4/^46r2	4/*6x1	5814/7652	4/2652	4f186s2	4/1/602	45°161°	4/13/602	4/14602	5414/14602	
			Ac 89	Th 90	Pa 91	U 92	Np 93	Pu 94	Am 95	Cm 96	Bk 97	Cf 98	Es 99	Fm 100	Md 101	No 102	Lr 103	1
ACTINIDE SERIES			(227)	232.038	231.039	238.029	237.048	(244)	(243)	(247)	(247)	(251)	(253)	(257)	(258)	(259)	(260)	(Act
			605752	642712	5/26/17/2	5/36d*7s2	5/46417s2	5/**712	55*752	5f*6d*7s2	5/16d17s2	5/18732	5/11742	50'27/2	5f**7x2	5/147x2	6d 17s2	

Average value based on the relative abundance of isotopes on earth. For unstable elements, the mass of the most stable isotope is given in brackets.

Magnetic solid

"Localized" magnetic moments on the atoms



How do the atoms decide to order?

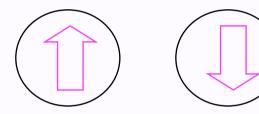
How do magnetic atoms affect each other?

- Curie-Weiss Law (Kittel p 324)
- The simplest approximation is to assume each atom acts like it is in an effective magnetic field B_E due to the neighbors

- One expects $\underline{\mathbf{B}}_{\mathsf{E}} = \lambda \ \underline{\mathbf{M}}$ where λ is some factor see next slide
- At high temperature we do not expect any net order, i.e., $\underline{\mathbf{B}}_{E} = 0$ and $\underline{\mathbf{M}} = 0$ unless one applies an external field $\underline{\mathbf{B}}_{A}$.
- What happens as the temperature is lowered?

How do magnetic atoms affect each other?

 Note: The "effective magnetic field B_E" is NOT really a "magnetic field" as in Maxwell's Equations



- The "effective magnetic field <u>B</u>_E" is due to the exclusion principle and electron-electron interactions that depend upon the relative spin of nearby electrons
- The "effective magnetic field B_E" can favor parallel spins or antiparallel spins depends upon many details simplest approximation is B_F = λ M

Curie-Weiss Law

At high temperature we have

$$\underline{\mathbf{M}} = \chi \ (\underline{\mathbf{B}}_{\mathsf{E}} + \underline{\mathbf{B}}_{\mathsf{A}}) \quad \text{or} \quad \underline{\mathbf{M}} \ (1 - \lambda \chi) = \chi \quad \underline{\mathbf{B}}_{\mathsf{A}}$$
 or $\underline{\mathbf{M}} = \underline{\mathbf{B}}_{\mathsf{A}} \ \chi \ / \ (1 - \lambda \chi)$

• Using the form of $\chi = C/T$,

Diverges as T is reduced

to $T = T_c = \lambda C$

$$\chi / (1 - \lambda \chi) = 1/(T/C - \lambda)$$

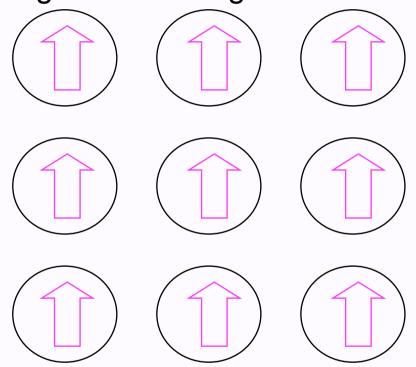
or

$$\chi_{eff} = \chi / (1 - \lambda \chi) = C/(T - T_c), T_c = C \lambda$$

 What does this mean? The magnetic moments all allign to make a ferromagnet without any external field below a critical temperature T_c

Ferromagnetic solid

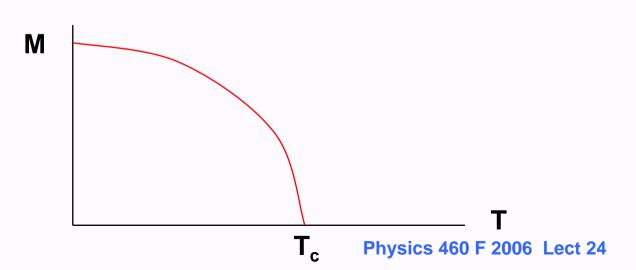
 "Localized" magnetic moments on the atoms aligned together to give a net magnetic moment



• Although there is some thermal disorder, there is a net moment at finite temperature.

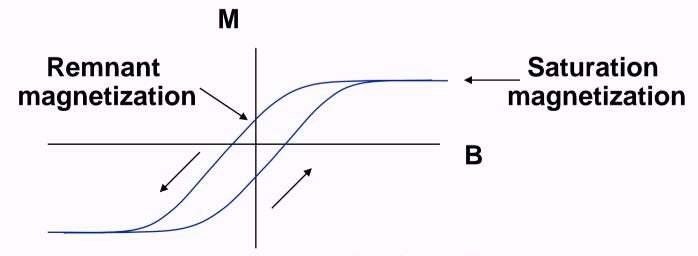
Example of a phase transition to a state of new order

- At high temperature, the material is paramagnetic
 Magnetic moments on each atom are disordered
- At a critical temperature T_c the moments order Total magnetization M is an "Order Parameter"
- Transition temperatures:
 T_c = 1043 K in Fe, 627 K in Ni, 292 K in Gd



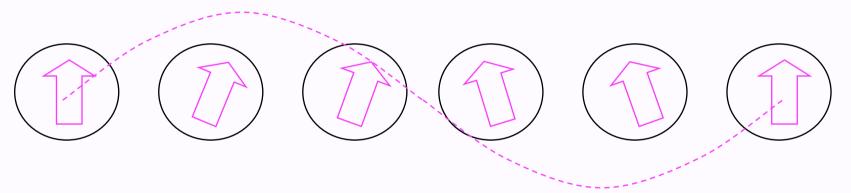
Real Magnetic materials

- Domains and Hysteresis
- A magnet usual breaks up into domains unless it is "poled" - an external field applied to allign the domains
- A real magnet has "hysteresis" it does not change the direction of its magnetization unless a large enough field is applied - irreversibility



Magnons

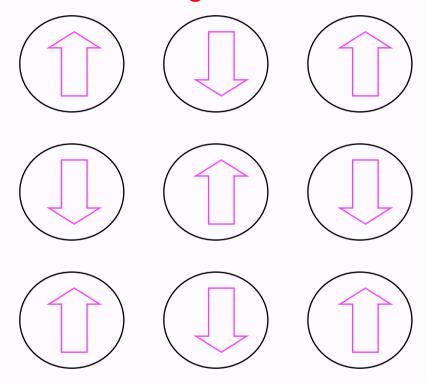
- Whenever there is an order, there can be variations in the order as function of position
- "Magnons" are quanta of magnetic vibrations very much like "phonons"



Can be observed directly by neutron scattering

Antiferromagnetic solid

 Magnetic moments can also order to give no net moment - antiferromagnet



 Transition temperature T_{transition} = T_{Neel} (named for Louis Neel)

Summary

- Magnetism is a purely quantum phenomenon!
 Totally at variance with the laws of classical physics (Bohr, 1911)
- $\underline{\mathbf{B}} = \mu_0 \left(\underline{\mathbf{H}} + \underline{\mathbf{M}} \right)$, $\underline{\mathbf{M}} = \chi \underline{\mathbf{H}}$ and $\underline{\mathbf{B}} = \mu \underline{\mathbf{H}}$
- Diamagnetism M opposite to H
 Closed shell atoms
 Insulators like Si, NaCl, ...
 Very weak
- Spin paramagnetism M adds to H
 Example of metal measures density of states
- How does ferromagnetism happen? Other forms of magnetism?
- Why does magnetism occur in transition metals, rare earths?

Summary

- Open shell atoms have magnetic moments
 Controlled by electron-electron interactions
 Hund's Rules
- Curie Law for atoms in a magnetic field
- Atomic-like effects (local magnetic moments) can occur in solids – transition metals, rare earths
- Magnetism is cooperative phenomenon whereby all the moments together go through a phase transition to form an ordered state
 Curie-Weiss Law

Curie-Weiss Law
Ferromagnetism
Antiferromagnetism
Magnetism as an "order parameter"

Curie-Weiss Law
Only
mentioned

Magnons
Domains, irreversibility

briefly

(Kittel - parts of Ch 11-12)

Next time

- Special presentation Raffi Budakian
 Magnetic Resonance Force Spectroscopy
 see Kittel, p. 356
- Start Surfaces and Scanning Tunneling Microsope (STM)