PHYS 460 - LECTURE 18

MAGNETISM

So far in this class we have focussed our attention almost entirely on the CHARGE of electrons and phenomena associated with electric fields (e.g. bonding, currents in response to voltages). For the next couple of weeks, we are going to turn our attention to the other major property of electrons: SPIN!

As it turns out, spin interactions are primarily mediated through charges and electron fields, but the results are a series of phenomena associated with magnetic fields comprising one of the most subtle & complex manifestations of quantum mechanics in many body systems, we are going to first study the ~~[illegible]~~ ~~[illegible]~~ meaning and origin of magnetism, and use these phenomena as a ~~[illegible]~~ jumping off point to study the topics of order parameters and phase transitions

Start with some definitions

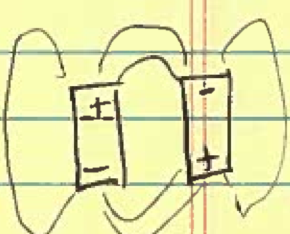
MAGNETISM: the class of physical phenomena that either create or are mediated by magnetic fields

MAGNETISM (IN MATERIALS): phenomena associated with magnetic moments of constituent material particles

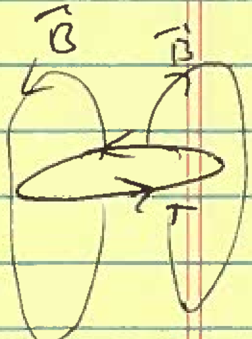
SUCH DEFINITIONS ARE ALWAYS A BIT UNSATISFACTORY AS THEY APPEAR SELF REFERENTIAL. REMEMBER THOUGH THAT THE MAGNETIC FORCE IS A FUNDAMENTAL FORCE OF NATURE. WE ARE NOT SETTING OUT TO DERIVE IT (NOR CAN WE), BUT TO DESCRIBE IT. THESE DEFINITIONS OF MAGNETS ARE MEANT TO REMIND YOU OF THE LONG HISTORY OF CONNECTED EXPERIMENTAL OBSERVATIONS ASSOCIATED WITH MATERIALS AND MAGNETIC FIELDS. (YOU ENCOUNTER THE SAME ISSUE WHEN TRYING TO DEFINE "ELECTRIC CHARGE".)

SHORT HISTORY OF MAGNETISM

- ~600BC - DISCOVERY OF LODESTONE (Fe_3O_4) IN GREECE (AND INDIA AND CHINA)
  - * ARISTOTLE AND GREEK PHILOSOPHERS DISCUSS ATTRACTION, POLES, BASIC PROPERTIES



- 1200-1600 - ADVENT OF COMPASS, DISCOVERY OF EARTH AS A MAGNET



- 1819/20 - OERSTED AND AMPERE DISCOVER ELECTRIC CURRENTS CREATE MAGNETIC FIELDS → SOLENOID

- 1820 - BIOT-SAVART LAW

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int_C I \frac{d\vec{l} \times \vec{r}'}{|\vec{r}'|^3}$$

- 1831 – FARADAY DISCOVERS TIME-VARYING $\vec{B}$ CREATES $\vec{E}$

- 1861 – MAXWELL'S EQUATIONS
  $\longrightarrow$ ELECTROMAGNETISM

- 1905 – EINSTEIN SHOWS DIFFERENCE BETWEEN $\vec{E}$ & $\vec{B}$ SIMPLY A MATTER REFERENCE FRAMES

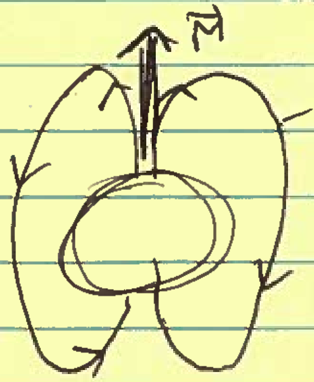
- 1919 – BOHR-VAN LEEUWEN THEOREM
  $\longrightarrow$ CLASSICAL SOLIDS CANNOT SUPPORT A NET MAGNETIC MOMENT (i.e. MAGNETS CAN'T EXIST)

- 1920s ADVENT OF QUANTUM MECHANICS, QUANTIZATION OF ANGULAR MOMENTUM, DISCOVERY OF "SPIN" (INTRINSIC ANG. MOM.)

- 1930s-1940s DEVELOPMENT OF MICROSCOPIC IDEAS OF MAGNETIC ORDER

MAGNETISM IN MATERIALS

OF FUNDAMENTAL INTEREST WHEN STUDY MAGNETISM IN MATERIALS, IS THE MAGNETIC MOMENT, WHICH IS LITERALLY THE STRENGTH OF THE DIPOLE FIELD EMERGING FROM A MATERIAL



$$\vec{M} = \frac{\text{moment}}{\text{Volume}} = \text{"MAGNETIZATION"}$$

SOME MATERIALS (LIKE LODESTONE) DEVELOP SPONTANEOUS DIPOLE MOMENTS WHEN TEMPERATURE IS DECREASED BELOW A CERTAIN TEMPERATURE, T_c

DEF. FERROMAGNET: A MATERIAL WHICH DEVELOPS (OR CAN DEVELOP) A SPONTANEOUS MOMENT, EVEN WITHOUT AN APPLIED FIELD.

WE WILL LATER SEE THAT THESE MATERIALS HAVE A DEFINITION IN TERMS OF LOCAL PARTICLES, WHICH WILL SHED MORE LIGHT ON THE SITUATION.

MOST MATERIALS ARE NOT FERROMAGNETS, CLEARLY, BUT STILL HAVE INTERESTING MAGNETIC PROPERTIES. IN THESE CASES, $\vec{M}$ IS USUALLY A RESPONSE OF THE MATERIAL TO AN APPLIED FIELD, $\vec{H}$

$$\vec{M} = \chi \vec{H}$$

WHERE WE HAVE ASSUMED THE RESPONSE IS PROPORTIONAL (i.e. "LINEAR RESPONSE"), AND HAVE DEFINED

$$\chi = \frac{|\vec{M}|}{|\vec{H}|} = \text{"MAGNETIC SUSCEPTIBILITY"}$$

SIDE NOTE: $\vec{H}$ = APPLIED FIELD

$\vec{M}$ = INDUCED FIELD

$\vec{B}$ = TOTAL LOCAL FIELD

$\rightarrow \vec{B} = \mu_0(\vec{H} + \vec{M})$ IN S.I. UNITS

$\Rightarrow$ WHEN $|\vec{M}| << |\vec{H}|$, $\chi = \mu_0 \frac{|\vec{M}|}{|\vec{B}|}$

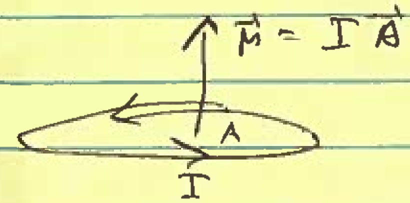
χ IS A FUNDAMENTAL MATERIAL PROPERTY AND QUANTIFIES HOW A MATERIAL "RESPONDS" TO THE PRESENCE OF A FIELD, AND IS USUALLY THE FIRST MEASUREMENT ONE MAKES OF A NEW MAGNETIC MATERIAL. FOR NON-FERROMAGNETS, MATERIALS ARE LOOSELY CATEGORIZES ACCORDINDING TO THE SIGN OF χ.

DEF: PARAMAGNET - MATERIAL WHERE $\chi > 0$, MEANING FIELDS ENCOURAGE RESPONSE IN SAME DIRECTION

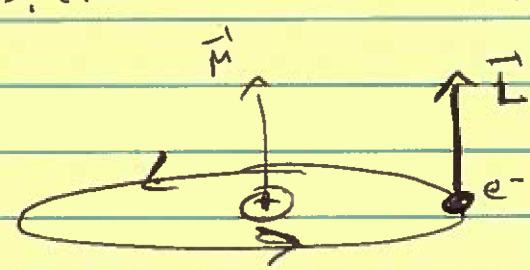
DEF: DIAMAGNET - MATERIAL WHERE $\chi < 0$, AND RESPONSE OPPOSE APPLIED FIELDS (LOOK UP FLOATING FROGS!)

THESE ARE EXPERIMENTAL DEFINITIONS AND BOTH, WE WILL SEE, ARE VERY COMMON.

BUT WHAT DICTATES THIS RESPONSE? THE ANSWER, OF COURSE, ARE THE ELECTRONS! WE KNOW A LOOP OF CURRENT CREATES A MAGNETIC DIPOLE MOMENT (AMPERE'S LAW)



COMPARE TO AN ELECTRON IN ORBIT AROUND A NUCLEUS. (PARDON THE INCORRECT PLANETARY ANALOGY.)



$\vec{\mu} = \gamma \vec{L}$

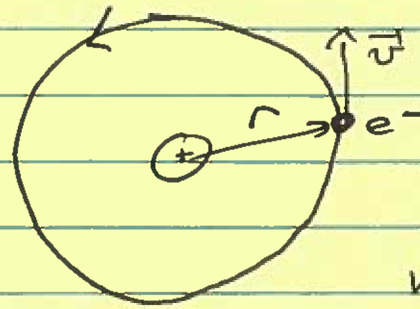
GYROMAGNETIC RATIO

SO MAGNETIC MOMENTS RESULT FROM <u>ORBITAL ANGULAR MOMENTUM</u>. MAGNETISM RESULTS FROM HOW ELECTRONS MOVE.

ELECTRONS ALSO CAN CARRYING <u>SPIN</u>, WHICH IS A INTRINSIC ANGULAR MOMENTUM OFTEN INCORRECTLY THOUGHT OF AS AN ELECTRON SPINNING LIKE A TOP ABOUT SOME AXIS.

NET MAGNETIC MOMENT IS THE <u>SUM OF MOMENTS DUE TO THE COLLECTIVE ANGULAR MOMENTUM IN THE ENTIRE SYSTEM</u>

AN ESTIMATE OF THE SIZE OF AN ELECTRON MAGNETIC MOMENT CAN BE OBTAINED BY CONSIDERING AN IDEALIZED HYDROGEN ORBITAL



$$\mu = AI$$

$$= (\pi r^2)\left(\frac{-e}{\tau}\right)$$

WHERE $\tau = \frac{2\pi r}{v}$

$$\rightarrow \mu = -\pi r^2 \frac{ev}{2\pi r} = -\frac{rev}{2} = \frac{e\hbar}{2m_e}$$

SINCE $\hbar$ IS THE QUANTUM OF ANGULAR MOMENTUM $|L| = |m_e v r| = \hbar$

THIS ALLOWS US TO DEFINE THE "BOHR MAGNETON"

$$\mu_B \equiv \frac{e\hbar}{2m_e} = 9.274 \times 10^{-24}\ Am^2$$

$$= 5.788 \times 10^{-5}\ \frac{eV}{T}$$

QUANTUM MECHANICALLY, AN ATOM WITH QUANTUM NUMBERS l, m_l, s, m_s HAS

$$|L| = \sqrt{l(l+1)}\,\hbar \qquad |S| = \sqrt{s(s+1)}\,\hbar g$$

$$L_z = m_l \hbar \qquad S_z = m_s g \hbar$$

$g \approx 2$ FOR e^-

NOTE THAT NUCLEI ALSO HAVE SPINS, BUT THE NUCLEAR MAGNETON IS MUCH WEAKER

$\mu_n = \frac{m_e}{m_p}\mu_B \approx \frac{\mu_B}{2000}$ $\rightarrow$ WE WILL NEGLECT

PAULI PARAMAGNETISM

A SIMPLE EXAMPLE OF MAGNETIC RESPONSE COMES FROM THE MATH FRAMEWORK WE HAVE ALREADY ENCOUNTERED TO DESCRIBE METALS: SOMMERFELD THEORY

CONSIDER A FREE ELECTRON METAL IN THE PRESENCE OF A MAGNETIC FIELD, $\vec{B} = B_z \hat{z}$. FOR SMALL FIELDS WE CAN IGNORE ORBITAL ANGULAR MOMENTUM, AND THE HAMILTONIAN BECOMES

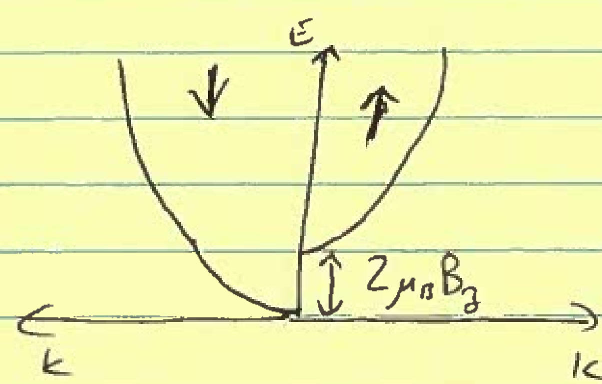
$$H = \frac{p^2}{2m} + g \mu_B S_z B_z$$

WHERE $g \approx 2$
AND $S_z = \pm \frac{1}{2}$

THE FIRST TERM ONLY INVOLVES $\vec{k}$ AND THE SECOND m_s, SO THE TWO COMMUTE. THE SOLUTIONS HAVE DIFFERENT ENERGIES FOR SPIN UP ($+\frac{1}{2}$) AND SPIN DOWN ($-\frac{1}{2}$) ELECTRONS

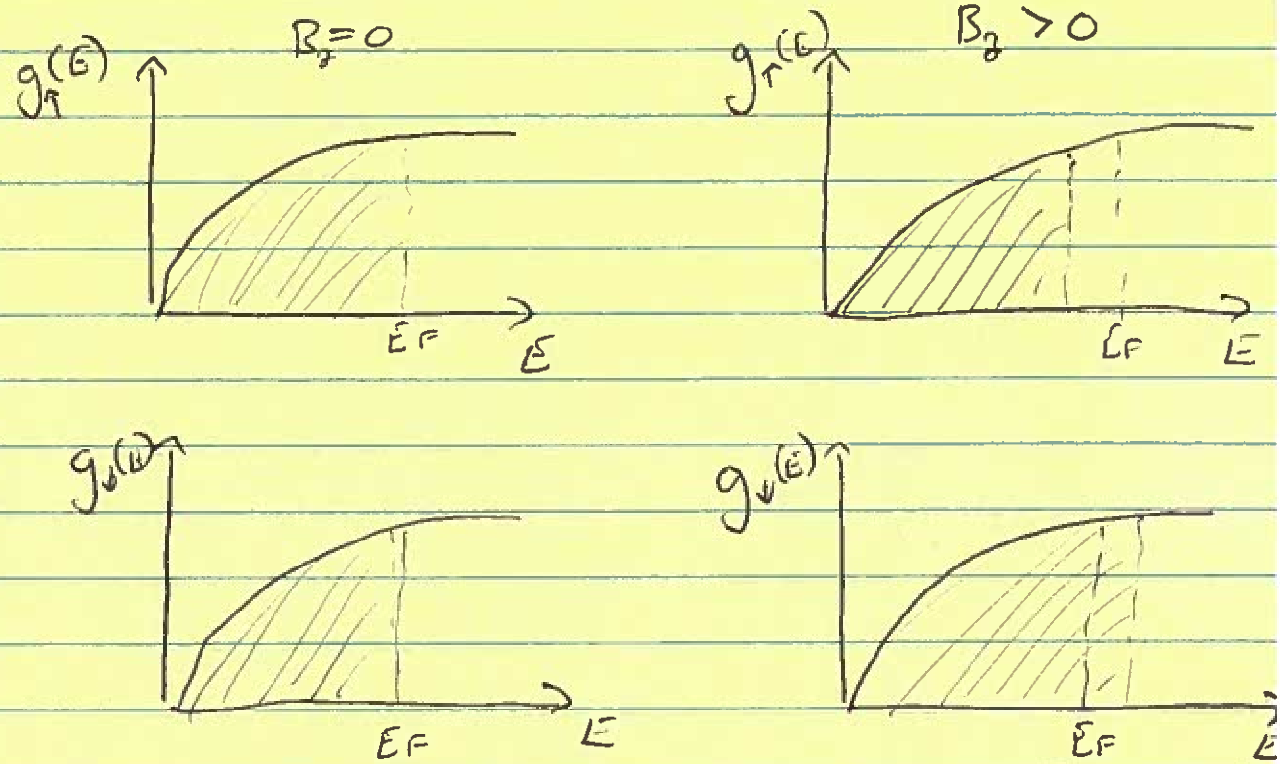
$$\varepsilon(\vec{k}, \uparrow) = \frac{\hbar^2 k^2}{2m} + \mu_B B_z$$

$$\varepsilon(\vec{k}, \downarrow) = \frac{\hbar^2 k^2}{2m} - \mu_B B_z$$



RECALL, FOR $B_z = 0$ THE NUMBER OF STATES PER dE WAS $g(E) = \frac{(2m)^{3/2}}{2\pi^2\hbar^3} E^{1/2}$

IN THE PRESENCE OF FIELD, THE LOWER ENERGY OF THE $\downarrow$ BAND MEANS MORE STATES WILL BE OCCUPIED THERE, AT THE EXPENSE OF THE $\uparrow$ BAND



WHICH GIVES A NET MAGNETIZATION

$$M = -\frac{1}{V}\frac{dE}{dB} = -\frac{\mu_B}{V}\left(\left[\#\text{ SPINS UP}\right] - \left[\#\text{SPINS DOWN}\right]\right)$$

NOW RECALL

$$n = \frac{N}{V} = \int_0^\infty g(E)\, f(E)\, dE$$

AT $T=0$ $f(E)$ = STEP FUNCTION AT $E=E_F$ AND $n_\uparrow = n_\downarrow \equiv \frac{n}{2}$

FOR SMALL B_z, ONE CAN ASSUME THE DENSITY OF STATES AND CHEMICAL POTENTIAL REMAIN UNCHANGED. THUS ONLY CHANGE IS IN OCCUPANCY FACTOR (FERMI-DIRAC) $f(\varepsilon) \longrightarrow f(\varepsilon \pm \mu_B B_z)$

$$n_\pm = \int_0^\infty g(\epsilon) f(\varepsilon \pm \mu_B B_z)\, d\varepsilon$$

$$\approx \int_0^\infty g(\epsilon) f(\epsilon)\, d\epsilon \pm \frac{\mu_B B_z}{2} \int_0^\infty g(\epsilon) \frac{\partial f(\epsilon)}{\partial \varepsilon}\, d\epsilon$$

BUT $\frac{\partial f(\epsilon)}{\partial \epsilon} = \delta(\epsilon - \epsilon_F)$ AT $T=0$

$$\rightarrow \quad n_\pm = \frac{n}{2} \pm \frac{\mu_B B_z}{2} g(\epsilon_F)$$

$$\Rightarrow \quad M = \mu_B (n_+ - n_-) = \mu_B^2 B_z\, g(\epsilon_F)$$

$$\boxed{\chi_p = \frac{M}{H} = \frac{\mu_0 M}{B_z} = \mu_0 \mu_B^2\, g(\epsilon_F)}$$

AS A FIRST APPROXIMATION, METALS ARE PREDICTED TO HAVE <u>WEAK</u> <u>TEMPERATURE INDEPENDENT PARAMAGNETISM</u>, ALL OF WHICH IS SEEN IN EXPERIMENT

$\longrightarrow$ PAULI PARAMAGNETISM