

PHYS 460 - LECTURE 1

- INTRODUCTION TO "CONDENSED MATTER", THE STUDY OF EMERGENT PROPERTIES OF MANY BODY SYSTEMS (SEE PPT SLIDES FOR DETAILS)

- FOCUS WILL BE ON "SOLID STATE PHYSICS"

- LARGEST AND OLDEST PART OF CM
- LABORATORY FOR EXPLORING MANY CONCEPTS IN THE FIELD
- CONCERNED WITH ~~THE~~ UNDERSTANDING ACHETYPAL MANY BODY SYSTEM: ELECTRONS INSIDE A MATERIAL (AND RESULTANT PHENOMENA)

- START WITH SIMPLE CLASSICAL MODELS, WHICH SERVES MANY PURPOSES:

- (1) ALLOWS INTRODUCTION OF COMMON AND IMPORTANT MEASUREMENTS AND TERMS
- (2) PROVIDES BACKGROUND ON MODELS WHICH ARE STILL USED AND DISCUSSED TODAY
- (3) PROVIDES A FRAMEWORK UPON WHICH WE CAN BUILD A DEEPER UNDERSTANDING OF MATERIALS

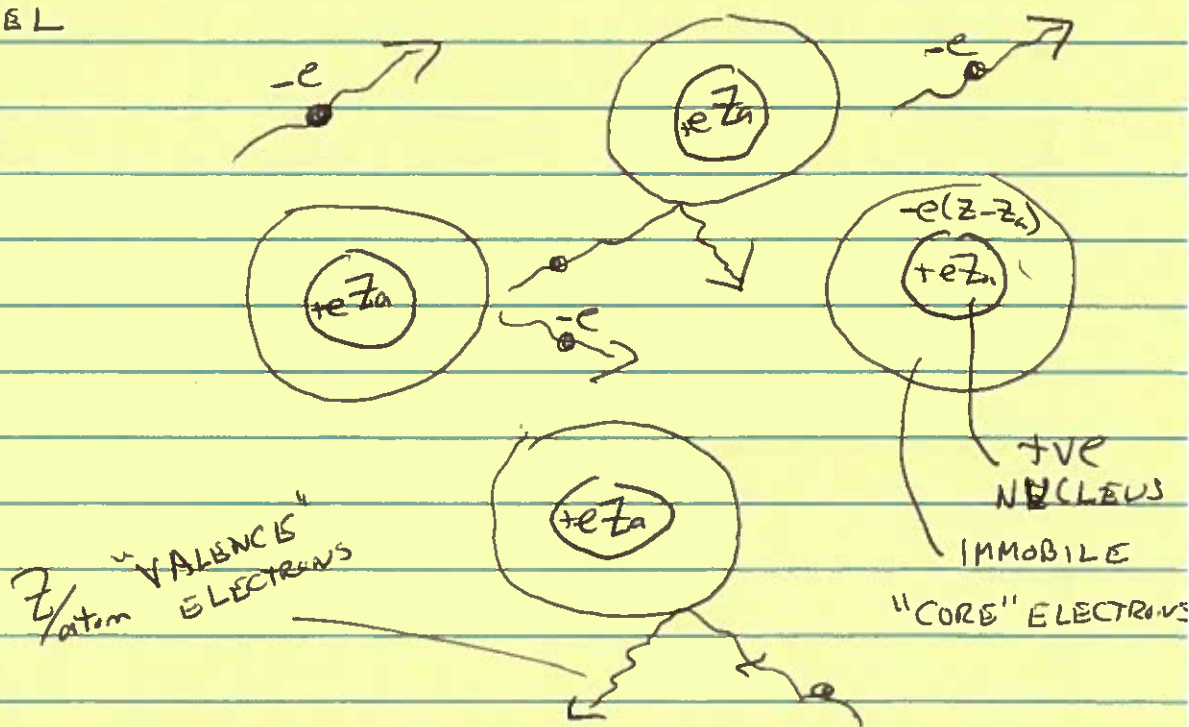
- NOTE - MANY CLASSICAL MODELS ARE WRONG IN SOME FUNDAMENTAL WAY, BUT UNDERSTANDING HOW THEY WRONG (OR WHY THEY WORK ANYWAY!) IS OFTEN AS USEFUL AS HAVING A WORKING THEORY

START AT THE BEGINNING : THE DRUDE MODEL OF METALS

- 1900
- FIRST QUANTITATIVE MODEL THAT ATTEMPTED TO EXPLAIN KNOWN PROPERTIES OF METALS (SHININESS, ELECTRICAL/HEAT CONDUCTION)
- APPLIED BOLTZMANN'S KINETIC THEORY OF GASES (LATE 1800S) TO THE MOVEMENT OF RECENTLY DISCOVERED ELECTRONS (J.J. THOMPSON, 1897) INSIDE MATERIALS

THE MODEL

e = fundamental charge
 Z_a = atomic number
 Z = # electrons which are "free" to move



CAN ESTIMATE

$$n = \frac{N}{V} \quad \begin{matrix} \text{(\# ELECTRONS)} \\ \text{(VOLUME)} \end{matrix}$$

$$= 6.02 \times 10^{23} \times \frac{\text{mass density}}{\text{atomic mass}}$$

$$\sim 10^{28} \text{ electrons/m}^3$$

SEE ASHCROFT
 FOR TABLE
 OF REAL VALUES

ANOTHER COMMON MEASURE OF DENSITY IS TO CALCULATE MEAN SEPARATION

- TAKE VOLUME N , AND CALCULATE RADIUS OF SPHERE WITH THAT VOLUME (r_s)

$$\Rightarrow r_s = \left(\frac{3}{4\pi n} \right)^{1/3} \sim 1 \text{ \AA} \equiv 10^{-10} \text{ m}$$

THIS IS $\sim 10^3$ TIMES SMALLER THAN MEAN ATOM SEPARATION IN GASES! (THUS WE SHOULD BEGIN TO QUESTION LEGITIMACY OF THE MODEL)

FOR DRUDS ASSUMES:

- ① INDEPENDENT AND FREE ELECTRONS
- ② UNDERGO COLLISIONS WITH PROBABILITY $p(dt) = \frac{dt}{\tau}$ IN SOME TIME dt
- ③ COLLISIONS ARE INSTANTANEOUS, AND COMPLETELY RANDOMIZE VELOCITIES
- ④ BETWEEN COLLISIONS ELECTRONS ARE ACCELERATED ACCORDING TO: $m \frac{d\vec{v}}{dt} = -e\vec{E} - e\vec{v} \times \vec{B}$

USING ONLY THIS, ONE CAN DERIVE PREDICTIONS FOR OHM'S LAW, THE HALL EFFECT, THE PLASMA FREQUENCY, THERMAL CONDUCTIVITY, AMONG OTHER THINGS.

PREDICTION 1

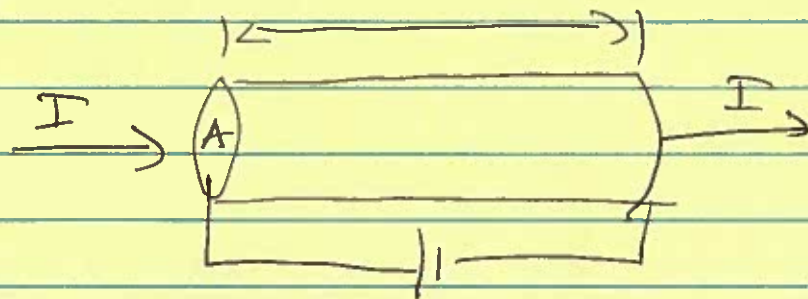
OHM'S LAW ? WE ALL KNOW

VOLTAGE $\swarrow$ CURRENT $\swarrow$ RESISTANCE
 $V = IR$

AS DID DRUDE, THESE QUANTITIES DEPEND ON SAMPLE GEOMETRY, HOWEVER. TO GET "INTRINSIC" PROPERTIES, WE INSTEAD ~~CONSIDER~~ CONSIDER RESISTIVITY, ρ , DEFINED BY

$\vec{E} = \rho \vec{j}$
FIELD $\swarrow$ "CURRENT DENSITY"

CONSIDER ROD OF LENGTH, L , CROSS-SECTIONAL AREA, A



THEN, WE DEFINE CURRENT DENSITY

$$\vec{j} = \frac{\text{CURRENT}}{\text{AREA}} = \frac{\vec{I}}{A}$$

ALSO

$$V = EL$$

$$\Rightarrow EL = jAR$$
$$R = \frac{EL}{jA}$$

OR

$$\rho = \frac{RA}{L} = \text{"RESISTIVITY"}$$

NOTE, IN GENERAL, ρ IS DEFINED BY THE EQUATION

$\vec{E} = \rho \vec{j}$ AND IS A TENSOR
OHM'S LAW

EQUALLY COMMON IS THE DEFINITION OF "CONDUCTIVITY"

$$\boxed{\vec{J} = \sigma \vec{E}}$$

CLEARLY $\sigma = \frac{1}{\rho}$ IF $\vec{J} \parallel \vec{E}$

WE CAN DERIVE THIS EQUATION AND GET AN EXPRESSION FOR σ USING DRUDE MODEL

NOTE, THAT THE CURRENT CROSSING AN AREA A PER TIME IS GIVEN BY

$$\underbrace{\vec{J}}_{\text{CURRENT AREA}} = -ne \underbrace{\vec{v}}_{\text{AVERAGE VELOCITY OF ELECTRONS}}$$

IF CONSTANT $\vec{E}$ IS APPLIED, THE ACCELERATION OF PARTICLES AFTER ~~WHEN~~ EXPERIENCING A COLLISION IS

$$\vec{v}(t) = \vec{v}(t=0) - \frac{e\vec{E}t}{m}$$

$\vec{v}(0) = 0$ BY ASSUMPTION $\uparrow$ NEWTON'S 2nd law

IF AVERAGE TIME BETWEEN COLLISIONS IS $t = \tau$, THEN

$$\langle \vec{v} \rangle = -\frac{e\vec{E}\tau}{m}$$

$\Rightarrow$

$$\vec{J} = \frac{ne^2\tau}{m} \vec{E} = \sigma \vec{E}$$

WITH

$$\sigma = \frac{ne^2\tau}{m}$$

OHM'S LAW!

THIS RELATIONSHIP ALLOWS US TO ESTIMATE τ FROM MEASURED ρ

$$\tau = \frac{m}{\rho n e^2} \sim 10^{-14} \text{ s}$$

USING EXPERIMENTAL DATA

$$(\rho \sim \mu\Omega\text{-cm})$$

DRUDE FURTHER ~~USED~~ USED THE EQUIPARTITION THM, $\frac{1}{2} m \bar{v}^2 = \frac{3}{2} k_B T$, FOR CLASSICAL PARTICLES TO ESTIMATE $\langle v \rangle \sim 10^5 \text{ m/s}$

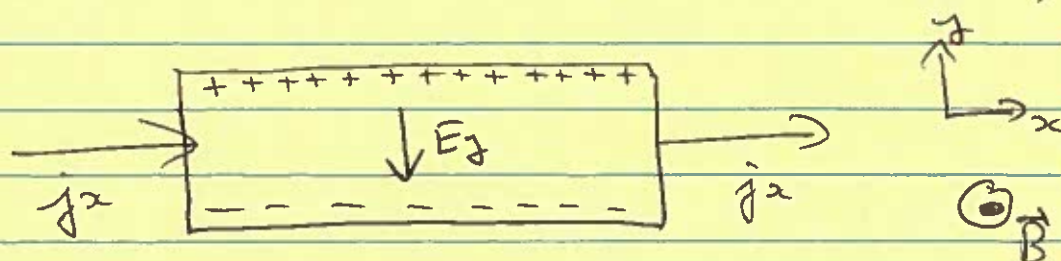
$$\Rightarrow l_0 \approx \langle v \rangle \tau = \text{"MEAN FREE PATH"} \sim 1-10 \text{ \AA}$$

REINFORCING THE IDEA THAT ELECTRONS SCATTER FROM NUCLEI.

WE WILL SEE THAT THESE ESTIMATES OF $\langle v \rangle$ ARE WAY OFF, AND IN MODERN MATERIALS, ONE CAN GET $l_0 \sim 10^8 - 10^{10} \text{ \AA}$

PREDICTION 2 / THE HALL EFFECT

IT WAS KNOWN FOR DECADES PRECEDING DRUDE, THAT DRIVING A CURRENT IN THE PRESENCE OF A MAGNETIC FIELD INDUCES A TRANSVERSE ELECTRIC POTENTIAL (HALL, 1879)



WE CAN ~~SEE~~ UNDERSTAND THIS (AS MODERN PHYSICISTS) AS AN EFFECT OF ELECTRONS ATTEMPTING TO SPIRAL AROUND THE APPLIED FIELD AND BUILDING UP AT THE WALLS.

DRUDE WAS THE FIRST TO DEVELOP THIS PICTURE AND CAME UP WITH A PREDICTION FOR THE "HALL CONSTANT"

$$R_H = \frac{E_y}{j_x B}$$

LET'S SEE HOW. START BY FINDING THE FULL PREDICTION FOR TIME DEPENDENCE OF ELECTRON MOMENTUM, $\vec{p}(t) = m\vec{v}(t)$
USE ASSUMPTIONS

$$\vec{p}(t+dt) = \underbrace{\frac{dt}{\tau}}_{\substack{\uparrow \\ \text{PROB OF} \\ \text{COLLISION IN} \\ dt}} \times 0 + \left(1 - \frac{dt}{\tau}\right) \underbrace{\left[\vec{p}(t) - e\vec{E} - e\vec{v} \times \vec{B}\right]}_{\substack{\uparrow \\ \text{PROB OF} \\ \text{NO COLLISION}}}$$

$$\Rightarrow \vec{p}(t+dt) - \vec{p}(t) \approx -\frac{dt}{\tau} (e\vec{E} + e\vec{v} \times \vec{B}) + \mathcal{O}\left(\left(\frac{dt}{\tau}\right)^2\right) - \frac{dt}{\tau} p(t)$$

$$\Rightarrow \boxed{\frac{d\vec{p}(t)}{dt} = -\frac{p(t)}{\tau} - e\vec{E} - e\vec{v} \times \vec{B}}$$

CONSIDER THE STEADY STATE, WHERE $\frac{d\vec{p}}{dt} = 0$. THEN, THE ABOVE EQUATION FOR x & y COMPONENTS GIVE

$$0 = -\frac{p_x}{\tau} - eE_x - \frac{eB}{m} p_y$$

$$\text{AND } 0 = -\frac{p_y}{\tau} - eE_y + \frac{eB}{m} p_x$$

MULTIPLYING THROUGH BY $-\frac{n e \tau}{m}$ AND USING $\vec{j} = -n e \frac{\vec{p}}{m}$, THESE EQUATIONS BECOME

$$\text{AND } \boxed{\begin{aligned} \sigma_0 E_x &= \omega_c \tau j_y + j_x \\ \sigma_0 E_y &= -\omega_c \tau j_x + j_y \end{aligned}} \quad \begin{aligned} \text{WHERE } \sigma_0 &= \frac{n e^2 \tau}{m} \\ \text{AND } \omega_c &= \frac{eB}{m} \end{aligned}$$

NOTICE THAT IF $B=0$, THE FIRST EQUATION JUST GIVES US OUR OHM'S LAW RESULT. IF $j_y=0$ (NO FLOW IN y), THEN WE GET

$$\boxed{R_H \equiv \frac{E_y}{j_x B} = -\frac{e \tau}{m \frac{n e^2 \tau}{m}} = -\frac{1}{n e}}$$

R_H MEASURES DENSITY OF CARRIERS!

LOOKING AT DATA, ONE SEES MATCH TO ATOMIC DENSITIES

PREDICTION 3

AC CONDUCTIVITY

IF WE LET THE $\vec{E}$ -FIELD VARY WITH TIME (AS ONE MIGHT EXPERIENCE DUE TO THE PRESENCE OF LIGHT), WE CAN AGAIN USE DRUDE TO MAKE PREDICTIONS

$$\text{LET } \vec{E}(t) = \text{Re}(\vec{E}(\omega) e^{-i\omega t})$$

$$\text{USE } \frac{d\vec{p}}{dt} = -\frac{\vec{p}}{\tau} - e\vec{E}$$

$$\text{TO GET } \vec{p}(t) = \text{Re}(\vec{p}(\omega) e^{-i\omega t})$$

WITHOUT EXPLICITLY DERIVING IT, ONE CAN GET THE RELATIONSHIP

$$\vec{j}'(\omega) = \sigma(\omega) \vec{E}(\omega)$$

$$\text{WHERE } \vec{j}(t) = \text{Re}(\vec{j}(\omega) e^{-i\omega t}) \text{ AND } \boxed{\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}}$$

AC CONDUCTIVITY

THIS CAN BE USED FOR MANY THINGS, BUT ONE OF FIRST APPLICATIONS IS TO DERIVE THE PLASMA FREQUENCY, ω_p , WHERE METALS BECOME TRANSPARENT TO LIGHT

$$\boxed{\omega_p = \frac{4\pi n e^2}{m}}$$

PREDICTION 4

WEIDENMAN-FRANZ LAW

THIS WILL BE COVERED IN NEXT LECTURE, BUT BASICALLY SHOWS THAT IF THE PARTICLES CARRYING CHARGE (i.e. ELECTRONS) ALSO CARRY HEAT, ONE CAN PREDICT

$$\begin{array}{l} \text{THERMAL} \\ \text{CONDUCTIVITY} \end{array} \rightarrow K = \frac{1}{3} C v m v^2$$
$$\begin{array}{l} \text{ELECTRICAL} \\ \text{COND.} \end{array} \rightarrow \sigma = \frac{n e^2}{m}$$

IF ONE ASSUME CLASSICAL PARTICLES (WE WILL SEE), ONE GETS

$$\frac{K}{\sigma} = \frac{3}{2} \left(\frac{k_B}{e} \right)^2 T$$

$$\equiv L T$$

$\hat{=}$ LORENZ NUMBER

$$\text{WITH } L = 1.11 \times 10^{-8} \frac{\text{W} \cdot \Omega}{\text{K}^2}$$

THIS IS CLOSE TO THE EXPERIMENTAL "UNIVERSAL VALUE" OF $L = 2.2 \times 10^{-8} \frac{\text{W} \cdot \Omega}{\text{K}^2}$, WHICH WAS CONSIDERED A SUCCESS, BUT IT HAS PROBLEMS,

IN GENERAL, DESPITE SUCCESSSES OF DRUDE, THERE WERE SEVERAL SHORTCOMINGS (TOO HIGH DENSITY, WRONG SCATTERING MECHANISM, TOO HIGH C_v , WRONG SIGN OF R_H SOMETIMES, TOO LOW ~~HEAT~~ v) WHICH REQUIRED THE ADVENT OF QUANTUM MECHANICS TO RESOLVE

→ SOMMERFELD THEORY

(NEXT LECTURE)