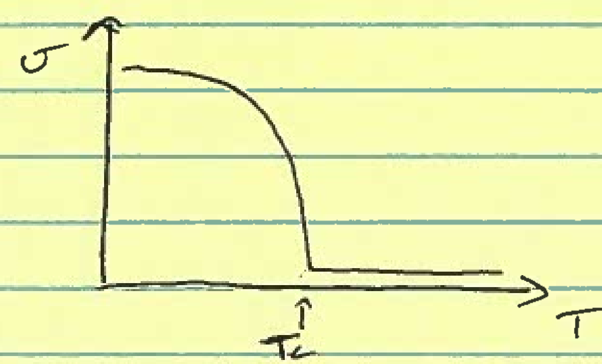
PHYS 460 - LECTURE 23

LAST LECTURE, WE ENCOUNTERED THE CONCEPT OF SPIN "ORDER" WHEN CONSIDERING THE FERROMAGNETIC ISING MODEL: AT LOW TEMPERATURES THE SYSTEM SPONTANEOUSLY ALIGNED SPINS SO THAT IT HAD A NET MAGNETIZATION, EVEN WITHOUT APPLIED FIELD. THIS SPONTANEOUS ALIGNMENT OCCURS BELOW A CRITICAL TEMPERATURE, T_c, WHERE ENERGY "WINS" OVER ENTROPY. THE TEMPERATURE DEPENDENCE OF $\langle S_z \rangle$ LOOKED LIKE THIS



WHERE HERE $\sigma = \langle S_z \rangle$ IS CALLED THE "ORDER PARAMETER". WHEN $\sigma \neq 0$, THE SYSTEM IS SAID TO BE "ORDERED"

WE DERIVED THIS CURVE IN THE CONTEXT OF $S=\frac{1}{2}$ FERROMAGNETISM USING MEAN FIELD THEORY. GOING BEYOND MEAN FIELD CHANGES SUBTLETIES WITH REGARDS TO EXACT TEMPERATURE DEPENDENCE, BUT THE BASIC BEHAVIOR IS THE SAME. FOR OTHER HAMILTONIANS, OTHER "ORDER PARAMETERS" CAN BE FOUND. (e.g. STAGGERED MAGNETIZATION FOR AF SPINS ON CUBIC LATTICE)

TODAY WE WILL EXPLORE DIFFERENT KINDS OF ORDER WHICH CAN OCCUR (OR NOT) AND HOW WE KNOW.

LET'S START BY CONSIDERING THE SAME HEISENBERG HAMILTONIAN

$$\mathcal{H} = -\frac{J}{2} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + g\mu_B \sum_i \vec{B} \cdot \vec{S}_i$$

WHERE THE FIRST SUM IS OVER NEAREST NEIGHBORS.

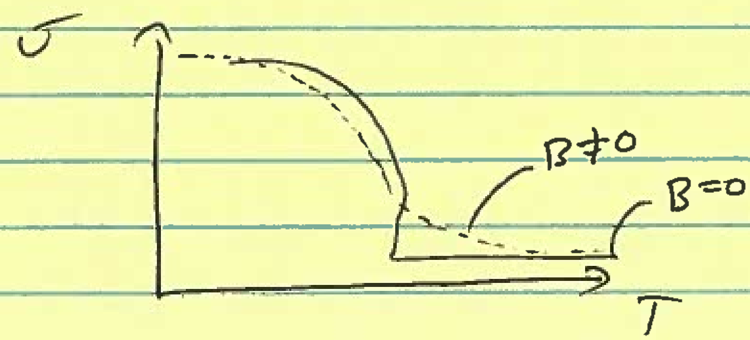
CASE (I): FERROMAGNETS $J > 0$

LAST CLASS, WE CONSIDERED THE $S = \frac{1}{2}$ CASE WITH ZERO APPLIED FIELD ($B = 0$). THE STATE WE FOUND AT LOW TEMPS HAD ALL THE MOMENTS ALIGNED, WHICH MAKES SENSE

NOW, I POINT OUT A FEW MORE THINGS OF NOTE:

- WE CAN GENERALIZE THIS ANALYSIS TO ANY SPIN AND (NOT SURPRISINGLY) REACH THE SAME ORDERED GROUND STATE. IN FACT, THE HEISENBERG FERROMAGNET WITH $B=0$ IS ONE OF THE FEW MODELS IN 3D WHERE THE EXACT SOLUTION HAS BEEN WRITTEN DOWN.

• AS SOON AS $B \neq 0$, NO MATTER HOW SMALL, THERE IS NOT AN ORDERING TRANSITION! THIS IS NOT TO SAY THAT SPINS DON'T LINE UP, BUT MERELY THERE IS NO FUNDAMENTAL DIFFERENCE BETWEEN SPINS LINING UP TO BE PARALLEL TO THE APPLIED FIELD AND SPINS LINING UP PARALLEL TO ONE ANOTHER $\longrightarrow$ TRANSITION IS "SMEARED OUT"

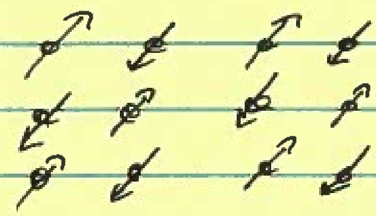


• YOU MAY HAVE NOTICED THAT I POINTED ALL THE SPINS OFF AN ANGLE $\nearrow$ ON THE LAST PAGE. LAST LECTURE THEY WERE ALL VERTICAL $\uparrow$. THIS WAS INTENTIONAL, AS I WANT TO POINT OUT THAT, THOUGH THE HAMILTONIAN SAYS SPINS SHOULD ALIGN, THERE IS NOTHING THAT SAYS IN WHAT DIRECTION.

THIS IS A SYMMETRY OF THE HAMILTONIAN THAT HAS TO BE BROKEN. MORE ON THIS LATER.

CASE (II): NÉEL ANTIFERROMAGNETS, $J<0$

LAST CLASS WE ALSO BRIEFLY CONSIDERED THE ANTIFERROMAGNETIC CASE, WHERE SPINS WANT TO BE ANTIPARALLEL. THOUGH THE QUANTUM MECHANICAL WAVEFUNCTION HAS NEVER BEEN WRITTEN DOWN, ONE CAN SHOW CLASSICALLY, AND IT IS REASONABLE TO EXPECT THAT THE GROUND STATE IS AS FOLLOWS



AS WAS FIRST ARGUED BY LOUIS NÉEL IN THE 1940'S, A FEW NOTES HERE TOO.

- AGAIN, THERE IS A ROTATIONAL SYMMETRY THAT IS BROKEN, WE WILL ADDRESS THIS

* IMPORTANT *

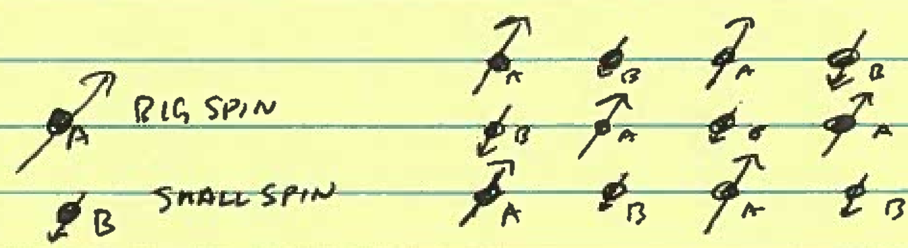
- THIS SOLUTION IS ONLY VALID IF THE LATTICE IS "BI-PARTITE" - i.e. IT CAN BE SEPARATED INTO TO EQUIVALENT SUBLATTICES. THE CUBIC LATTICE IS BI-PARTITE. THINGS GET MORE INTERESTING IF ONE CONSIDERS A TRIANGLE LATTICE.

- DESPITE THE "OBVIOUS" NATURE OF THIS STATE, ITS EXISTENCE WAS LONG CALLED INTO QUESTION BY LEV LANDAU AND OTHERS. WHY WOULDN'T $J<0$ JUST

FORCE THE SPINS TO PAIR UP FORM A BUNCH OF BONDING ORBITALS WITH SPIN SINGLETS (WHICH HAVE $\langle S_z \rangle = 0$ ALWAYS)? IT WASN'T UNTIL IT WAS OBSERVED WITH NEUTRON SCATTERING THAT ANTIFERROMAGNETISM WAS ACCEPTED AS REAL. MORE ON THIS LATER.

CASE III : FERRIMAGNETISM

THERE IS NO REASON THAT A MATERIAL CAN ONLY HAVE ONE TYPE OF MAGNETIC ATOM. MOST DON'T. IF THERE ARE TWO OR MORE, ONE CAN ENCOUNTER A STATE SUCH AS



WHERE SPINS ARE ANTI-PARALLEL TO NEAREST NEIGHBOURS, YET THE TOTAL MOMENT $M = -g\mu_B \sum_i \langle S_{zi} \rangle$ IS NOT ZERO. AGAIN FIRST SUGGESTED BY LOUIS NÉEL, THIS IS CALLED A "FERRIMAGNET" AND TREATED AS DISTINCT FROM A FERROMAGNET BY PEOPLE WHO MEASURE LOCAL EFFECTS IN MAGNETS.

MAGNETITE

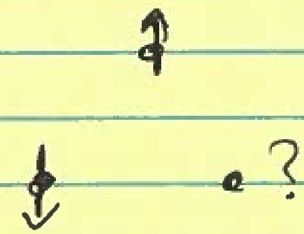
Fe_3O_4

$Fe^{2+}Fe_2^{3+}O_4$

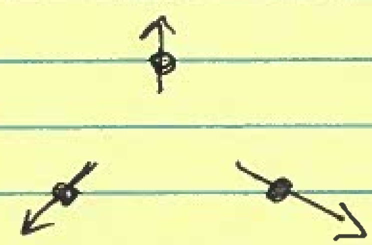
NOTE THOUGH THAT OUR MACROSCOPIC DEF OF FERROMAGNETS INCLUDES FERRIMAGNETS! THIS AMBIGUITY IN THE FIELD PERSISTS TODAY. IN FACT, MAGNETITE (THE FIRST FM) IS ACTUALLY A FERRIMAGNET!

CASE IV : NON-COLLINEAR ANTIFERROMAGNETS.

NOW CONSIDER THE CASE $J<0$ WITHOUT A BI-PARTITE LATTICE. CONSIDER THE TRIANGLE LATTICE



HERE, IF WE <u>INSIST</u> ON THE SPINS BEING ANTI-PARALLEL, THERE IS NO SOLUTION. TO MINIMIZE ENERGY THEN, SPINS ARE FORCED INTO A NON-COLLINEAR CONFIGURATION AS THE BEST COMPROMISE STATE. SOLUTION FOR THE TRIANGLE LATTICE IS AS FOLLOWS



MORE GENERALLY, NON-COLLINEAR SPIN STATES ARISE AS COMPROMISES STATES WHENEVER THERE ARE COMPETING INTERACTIONS. THIS COMES FROM MANY PLACES:

- NEXT-NEAREST NEIGHBOR J
- SPIN-ORBIT COUPLING
- SPIN-LATTICE COUPLING
- DIPOLE FIELDS
- METALLICITY
- CRYSTAL FIELD EFFECTS ...

SYMMETRY BREAKING

I'VE MENTIONED A FEW TIMES THAT THE HEISENBERG HAMILTONIAN HAS NAUGHT TO SAY ABOUT SPIN DIRECTIONS, AND THIS IS A SYMMETRY THAT HAS TO BE BROKEN. HOW? I HAVE ALREADY INTRODUCED YOU TO THE MOST COMMON MECHANISM: CRYSTAL FIELDS.

→ SMALL DIFFERENCES IN ATOM SPACING CAN FAVOR CERTAIN ORBITALS, WHICH ARE COUPLED TO SPINS. THIS CAN TYPE THE BALANCE IN FAVOR OF A DIRECTION NO MATTER HOW SMALL AN EFFECT! TO ACCOUNT FOR THIS, AN "ANISOTROPY TERM" ~~[illegible]~~ IS OFTEN ADDED TO THE HAMILTONIAN

COMMON SCENARIOS

A) TETRAGONAL PHASE WITH $c > a = b$

$$\rightarrow \mathcal{H} = -\frac{J}{2}\sum_{\langle i,j\rangle} \vec{S}_i \cdot \vec{S}_j - K \sum_i \left(S_i^z\right)^2$$

B) CUBIC PHASE WHERE SPINS POINT ALONG "PRIMARY" AXES

$$\rightarrow \mathcal{H} = -\frac{J}{2}\sum_{\langle i,j\rangle} \vec{S}_i \cdot \vec{S}_j - K \sum_i \left[\left(S_i^x\right)^4 + \left(S_i^y\right)^4 + \left(S_i^z\right)^4\right]$$

NOTE THIS STILL LEAVES A SMALLER SYMMETRY TO BE BROKEN (e.g. $+\hat{z}$ OR $-\hat{z}$ IN CASE OF FM). THESE SMALL DEGENERACIES CAN BE BROKEN SPONTANEOUSLY

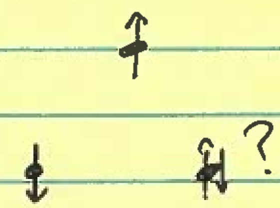
THE CASE A) FOR LARGE K IS KNOWN AS THE "ISING MODEL"

FRUSTRATION : ONE OF THE MOST ACTIVE AREAS IN THE FIELD OF MAGNETISM INVOLVES THE CASE OF "FRUSTRATION". THIS CAN OCCUR EITHER DUE TO THE GEOMETRY OF THE LATTICE OR COMPETING INTERACTIONS

$J<0$ GEOMETRIC FRUSTRATION : TRIANGLES + STRONG ANISOTROPY

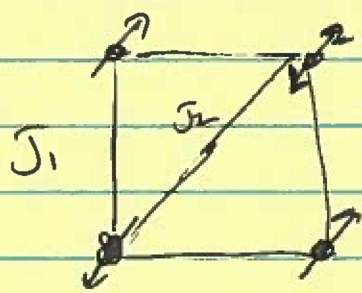
ANISOTROPY DOESN'T ALLOW NON-COLLINEAR COMPROMISE

EXCHANGE CAN'T CHOOSE BETWEEN ↑ AND ↓ ON 3rd SITE

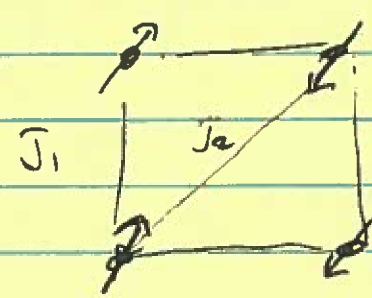


COMPETING INTERACTIONS : CONSIDER SQUARE LATTICE WITH NEAREST AND NEXT-NEAREST EXCHANGE

$|J_1| > |J_2|$



$|J_2| > |J_1|$



WHAT IF $J_1 = J_2$?

WHAT THESE CASES SAY IS THAT n.n. J IS NOT ENOUGH TO PICK AN ORDERED STATE. THUS A MORE EXPANSIVE HAMILTONIAN IS NEEDED, WHICH IS MUCH, MUCH HARDER ... AND INTERESTING!

NEUTRON DIFFRACTION

FINALLY, IT IS IMPORTANT TO ASK THE QUESTION: HOW DO WE KNOW WHAT THE SPINS ARE DOING? EXCEPT FOR THE FERROMAGNETIC CASE, ~~THE~~ THERE IS NO ~~T~~ OVERT SIGNATURE IN MAGNETIZATION, AND THE VAST MAJORITY OF MAGNETS ARE NOT FERROMAGNETS. (NOT EVEN MAGNETITE, THE ORIGINAL MOTIVATION FOR THE FIELD!)

SO, WHAT ELSE IS AVAILABLE TO US?

THE ANSWER IS SOMETHING WE HAVE ALREADY ENCOUNTERED: NEUTRON DIFFRACTION.

RECALL FROM LECTURE 6, I SHOWED THAT THE SCATTERING INTENSITY OF ANY WAVE FROM STATE $|\vec{k}\rangle \rightarrow |\vec{k}'\rangle$ WHEN SCATTERING FROM A SAMPLE WAS GIVEN BY

$$I(\vec{k}'-\vec{k}) \propto \delta(\vec{k}'-\vec{k}-\vec{G})\,|S(\vec{k}'-\vec{k})|^2$$

WHERE

$$S(\vec{k}'-\vec{k}) \equiv \int_{\substack{\text{UNIT}\\\text{CELL}}} d\vec{x}\; e^{i(\vec{k}'-\vec{k})\cdot\vec{x}}\, V(\vec{x})$$

HERE $\vec{G}$ IS THE RECIPROCAL LATTICE OF THE SAMPLE BEING STUDIED AND $V(\vec{x})$ IS THE INTERACTION POTENTIAL BETWEEN THE SAMPLE AND THE (PROBE) WAVE.

THIS EQUATION SAYS YOU GET BRAGG PEAKS WHEN $\Delta\vec{k} = \vec{G}$ (LAUE CONDITION), BUT WHAT IS $V(\vec{x})$? WE DIDN'T DISCUSS IN DETAIL

FOR X-RAYS, $V(\vec{x})$ IS THE COULOMB POTENTIAL

$\rightarrow$ X-RAY DIFFRACTION MOSTLY SEES CORE ELECTRON CHARGE DENSITY

$\rightarrow$ NO (STRONG) MAGNETIC SIGNATURE

NEUTRONS ARE SENSITIVE TO THE SAMPLE THROUGH TWO POTENTIALS:

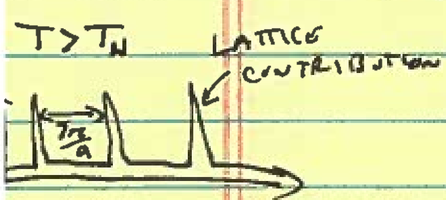
(1) STRONG NUCLEAR FORCE SEES ATOMIC NUCLEI

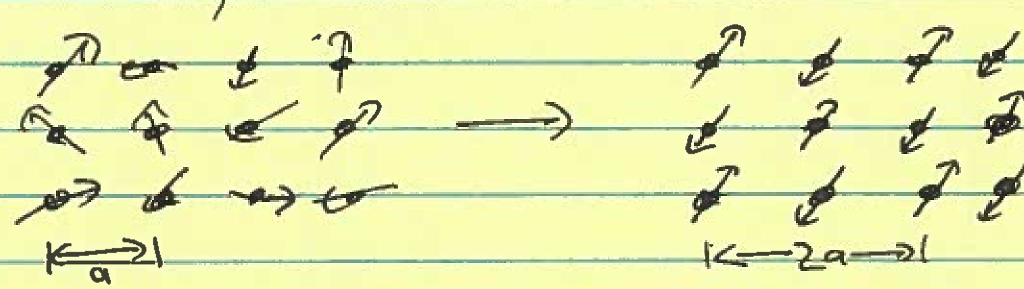
$\rightarrow$ SEES LATTICE

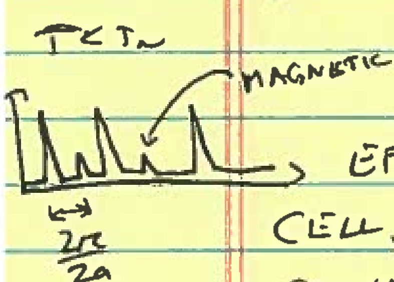
(2) NEUTRON <u>SPIN</u> SENSITIVE TO DIPOLE FIELDS CREATED BY LOCAL MOMENTS

$\rightarrow$ MAGNETISM!

SO, FOR EXAMPLE, AN ANTIFERROMAGNETIC TRANSITION







EFFECTIVELY DOUBLES THE SIZE OF THE UNIT CELL, SINCE THE SPINS DON'T CONTRIBUTE A NET SIGNAL IN THE DISORDERED STATE

$\rightarrow$ NEW BRAGG PEAKS!

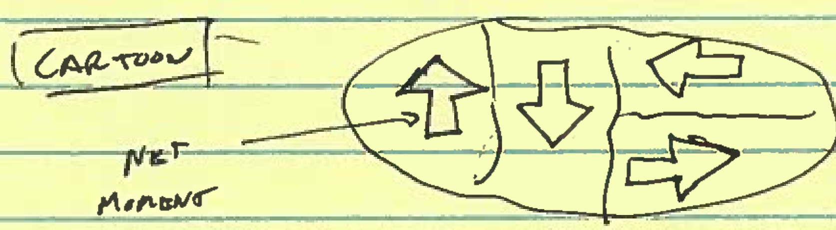
IN GENERAL, ONE CAN WORK OUT THE PERIODICITY AND DIRECTION OF ALL LOCAL SPINS WITH A COMPLETE NEUTRON DIFFRACTION PATTERN

DOMAINS – A BRIEF INTRO (SEE CH 2, SIMON FOR MORE)

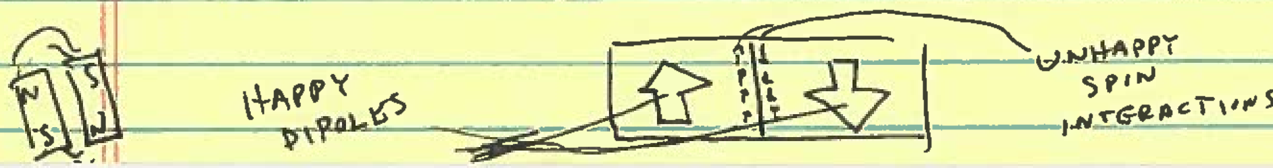
IT WOULD BE A STUNNING OMISSION TO CLOSE OUT THE STUDY OF MAGNETISM WITHOUT ADDRESSING ONE OBVIOUS QUESTION: IF FERROMAGNETS LIKE IRON CAN EXHIBIT A NET MAGNETIC MOMENT, WHY DON'T THEY ALWAYS EXHIBIT A NET MOMENT?

OR IN OTHER WORDS: "WHY DOESN'T IRON STICK TO MY FRIDGE?"

THE ANSWER, IN A WORD, IS DOMAINS. FM SAMPLES SEPARATE THEMSELVES INTO LOCAL VOLUMES WHERE THE SPIN DIRECTIONAL SYMMETRY IS BROKEN DIFFERENTLY, AND NET MOMENTS CANCEL

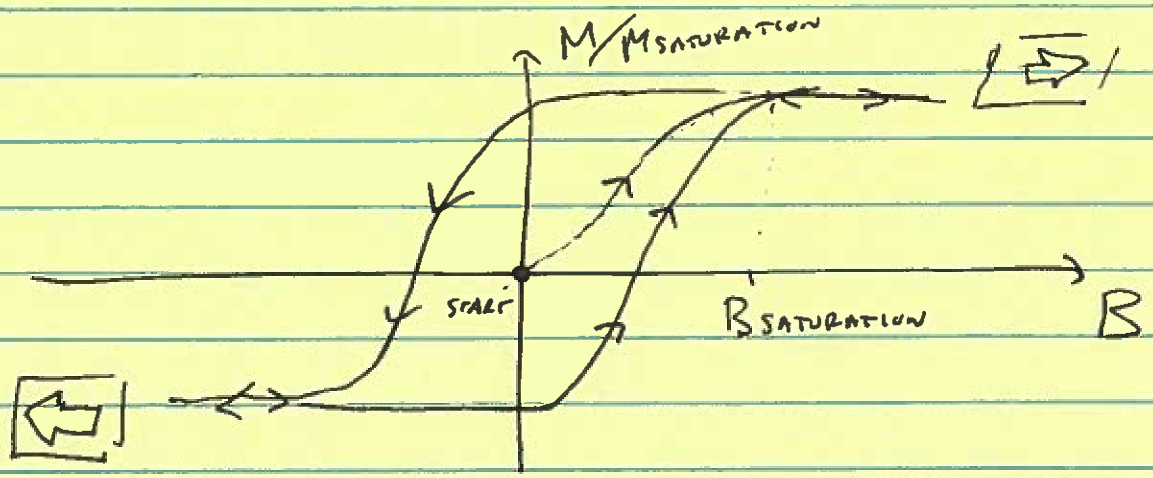


THIS IS NOT SIMPLY STATISTICAL RANDOMNESS. EVEN IF ONE PREPARED A SINGLE DOMAIN SAMPLE OF IRON, IT WOULD EVENTUALLY FORM DOMAINS. WHY? BECAUSE, MACROSCOPICALLY, DOMAINS MINIMIZE ENERGY DUE TO DIPOLE INTERACTIONS WHILE STILL SATISFYING AS MANY LOCAL EXCHANGE INTERACTIONS AS POSSIBLE → DOMAINS ARE A COMPROMISE STATE



THE BOUNDARY BETWEEN DOMAINS IS KNOWN AS A "DOMAIN WALL", AND A LOT OF THE PROPERTIES OF PERMANENT MAGNETS IS DETERMINED BY THE MOVEMENT, FLEXIBILITY A GREATER NATURE OF THESE WALLS.

THE MOST COMMON EXPERIMENTAL SIGNATURE IS THE "HYSTERISIS CURVE" IN MAGNETIZATION, IMAGINE COOLING A FM AN THEN RAMPING FIELD. THE RESPONSE LOOKS LIKE SO



ONE STARTS WITH A DOMAIN STRUCTURE → M=0 APPLYING A FIELD ALIGNS DOMAINS, PRIMARILY BY GROWING SOME DOMAINS AT THE EXPENSE OF OTHERS (WALLS MOVE!). WHEN B→0 NOW, M≠0, BUT YOU CAN SWITCH THE MOMENT BY APPLYING A BIG ENOUGH FIELD THE OTHER WAY.

THE SIZE OF $B_{saturation}$ DEPENDS ON HOW EASY IT IS TO MOVE WALLS, WHICH DEPENDS STRONGLY ON DISORDER (WALLS GET "PINNED" AT DEFECT SITES.) SINCE THE USEFULNESS OF A MAGNET DEPENDS ON $B_{saturation}$ BEING LARGE, THIS HAS BEEN STUDIED A LOT.