PHYS 460 - LECTURE 25

LAST LECTURE I SPENT A FAIR AMOUNT OF TIME INTRODUCING SUPERCONDUCTIVITY, AN ORDERED STATE STABILIZED BY ELECTRON-ELECTRON INTERACTIONS IN METALS, CONCENTRATING SPECIFICALLY ON KEY EXPERIMENTAL OBSERVATIONS: PERFECT CONDUCTIVITY, PERFECT DIAMAGNETISM, CRITICAL FIELDS AND ENERGY GAPS.

TODAY, I AM GOING TO PRESENT AN OVERVIEW OF THE THEORY OF SUPERCONDUCTIVITY. AS YOU MIGHT IMAGINE, THIS IS A BIG TOPIC, SO I AM GOING TO CONCENTRATE ON THREE PARTICULAR THEORIES/IDEAS OF MAJOR HISTORICAL IMPORTANCE:

1. LONDON EQUATIONS
2. GINZBURG-LANDAU THEORY
3. BCS THEORY

BCS THEORY IS CONSIDERED "THE ANSWER", AND THE OTHER TWO CAN BE DERIVED FROM BCS AS LIMITING CASES. IT IS ALSO THE MOST COMPLEX HOWEVER, AND THUS I WILL ONLY PRESENT BASIC IDEAS AND PREDICTIONS, NOR WILL I FULLY DERIVE THE OTHER TWO, BUT WILL TALK ABOUT THEM IN A BIT MORE DETAIL IN ORDER TO EXPLORE SOME KEY IDEAS:

1. LONDON PENETRATION DEPTH
2. COHERENCE LENGTHS
3. FLUX QUANTIZATION

WE BEGIN WITH THE FIRST WRITTEN DOWN

THE LONDON EQUATIONS

FRITZ AND HEINZ LONDON WROTE DOWN THEIR EQUATIONS SOON AFTER DISCOVERY OF THE MEISSNER EFFECT AS A FIRST ATTEMPT TO CAPTURE THE PROPERTIES OF SUPERCONDUCTIVITY BY DESCRIBING LOCAL ELECTRON MOTION.

THEIR ANALYSIS BEGAN WITH THE IDEA THAT THERE WERE TWO TYPES OF ELECTRONS: "NORMAL" ELECTRONS THAT BEHAVED AS THEY DO IN METALS AND "SUPERFLUID" FRACTION, n_s, THAT IS RESPONSIBLE FOR ALL NOVEL EFFECTS. IF n IS TOTAL ELECTRON DENSITY,

$$n = n_e + n_s$$

THE EQUATIONS DESCRIBING THE SUPERFLUID DENSITY ARE

$$\vec{E} = \frac{\partial}{\partial t}\left(\Lambda \vec{J}_s\right) \quad \text{AND} \quad \vec{B} = -\mu_0 \vec{\nabla} \times \left(\Lambda \vec{J}_s\right)$$

WHERE Λ IS A CONSTANT WITH UNITS OF LENGTH SQUARED

THE FIRST EQUATION IS ASSOCIATED WITH PERFECT CONDUCTIVITY AND THE SECOND PERFECT DIAMAGNETISM, AS WE WILL SEE. I.E. "SHOULD BE"

THESE EQUATIONS CAN BE DERIVED FROM BCS THEORY, BUT THE BASIC MOTIVATION CAN BE SEEN BY CONSIDERING THE DRUDE MODEL

DRUDE MOTIVATION: RECALL $m\frac{d\vec{v}}{dt} = e\vec{E} - \frac{m\vec{v}}{\tau}$

IN THE STEADY STATE, $\frac{d\vec{v}}{dt}=0$, AND THIS LEADS TO $\sigma = \frac{ne^2\tau}{m}$. IN A S.C HOWEVER, THERE IS NO STEADY STATE $(\tau \to \infty)$ AND

$$\frac{d\vec{v}_s}{dt} = \frac{e\vec{E}}{m}$$

$\Lambda = \frac{m}{n_s e^2}$

$$\Rightarrow \boxed{\frac{d\vec{J}_s}{dt} = n_s e \frac{d\vec{v}_s}{dt} = \frac{n_s e^2}{m}\vec{E} \equiv \frac{\vec{E}}{\Lambda}}$$

FIRST LONDON EQ

OR SINCE $\vec{p} = m\vec{v} + e\vec{A}$ (CANONICAL MOMENTUM)

$$\langle \vec{p} \rangle = 0 \Rightarrow \langle \vec{v}_s \rangle = -\frac{e\vec{A}}{m}$$

$$\Rightarrow \vec{J}_s = n_s e \langle \vec{v}_s \rangle = -\frac{n_s e^2}{m}\vec{A} = -\frac{\vec{A}}{\Lambda}$$

TIME DERIVATIVE

$$\boxed{\frac{d\vec{J}_s}{dt} = \frac{\vec{E}}{\Lambda}} \qquad \left(\vec{E} = -\frac{d\vec{A}}{dt}\right)$$

FIRST LONDON

CURL

$$\boxed{\vec{\nabla}\times\vec{J}_s = -\frac{\cancel{\mu_0}\vec{B}}{\mu_0\Lambda}} \qquad \left(\vec{B} = \cancel{\mu}\,\mu_0\vec{\nabla}\times\vec{A}\right)$$

SECOND LONDON

THE ASSOCIATION OF THE FIRST EQ WITH PERFECT CONDUCTIVITY IS OBVIOUS, BUT IT TAKES A COUPLE STEPS TO SEE HOW MEISSNER EFFECT ALSO ARISES.

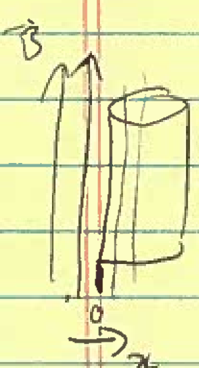
TAKE CURL OF MAXWELL EQUATION WITH $\vec{E}=0$

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}_s$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = -\nabla^2\vec{B} = \mu_0\, \vec{\nabla}\times\vec{J}_s$$

$$= -\frac{\vec{B}}{\Lambda}$$

THUS IN THE SCENARIO SHOWN ON THE LEFT



$$\vec{B}(x) = \vec{B}(x=0)\, e^{-\frac{x}{\lambda_L}}$$

WHERE $\lambda_L = \Lambda^{1/2} = \left(\frac{m}{n_s e^2}\right)^{1/2}$

IS THE "LONDON PENETRATION DEPTH"

THIS SHOWS THAT FIELD IS EXPELLED EXPONENTIALLY BY SUPERCURRENTS AND DEFINES A CHARACTERISTIC LENGTHSCALE $\longrightarrow$ MEISSNER EFFECT

NOTE THAT THIS DEPENDED ON THE EQUATION $\vec{J}_s = -\frac{\vec{A}}{\lambda_L^2}$ WHICH IS NOT GAUGE INVARIANT! $\longrightarrow$ THIS IS OUR BROKEN SYMMETRY

$\vec{\nabla}\cdot\vec{A}=0$ "LONDON GAUGE"

$\lambda_L \approx 200$ Å FOR Al

COHERENCE LENGTH

THE LONDON PENETRATION DEPTH IS CONSIDERED A PRIMARY MATERIAL PROPERTY, BUT IT IS DERIVED ON THE LONDON EQUATIONS, WHICH IS A SEMI-CLASSICAL LOCAL PICTURE. THE NEXT STEP IS TO GENERALIZE TO A QUANTUM PICTURE, WHERE

$$n_s = \psi^* \psi$$

FOR A SUPERFLUID QUANTUM WAVEFUNCTION. THIS WAS EXACTLY THE STEP TAKEN BY GINZBURG AND LANDAU, AND IMMEDIATELY RESULTED IN THE ARTICULATION OF A SECOND LENGTHSCALE, ξ, WHICH ROUGHLY SPEAKING IS THE DISTANCE OVER WHICH ONE CAN VARY n_s WITHOUT DESTROYING THE S.C. STATE.

BEFORE WRITING THE G-L EQUATIONS, IT IS WORTH TAKING A MOMENT TO UNDERSTAND WHY ξ, OR "COHERENCE LENGTH" MIGHT EXIST. IT IS ASSOCIATED WITH THE IDEA THAT SPATIALLY VARYING $\psi(x)$ IS ASSOCIATED WITH KINETIC ENERGY OF ELECTRONS.

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RECALL

$$KE = \frac{p^2}{2m} = -\frac{\hbar^2}{2m}\nabla^2\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}$$

IN 1D

COMPARE THEN $\psi(x) = e^{ikx}$ FOR A NORMAL METAL

$$\Rightarrow \psi^*\psi = 1$$

AND $KE = \dfrac{\hbar^2 k^2}{2m}$

TO SOME HEAVILY MODULATED WAVEFUNCTION

$$\phi(x) = \frac{1}{\sqrt{2}}\left(e^{i(k+q)x} + e^{ikx}\right)$$

$$\Rightarrow \quad \phi^*\phi = 1 + \cos(qx)$$

$$KE \approx \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2}{2m} kq \qquad \text{FOR } q \ll k$$

THUS IF THE SUPERCONDUCTING WAVEFUNCTION IS VARIED TOO MUCH, AN ADDITIONAL KINETIC ENERGY ON THE ORDER

$$\frac{\hbar^2}{2m} k_F q > \Delta$$

COULD DESTROY THE S.C. STATE

IT IS NATURAL THEN TO DEFINE A LENGTHSCALE, $\xi = \frac{1}{q_{max}}$, SHORTER THAN WHICH ONE CANNOT ALTER THE STATE. THIS IS THE "COHERENCE LENGTH"

$$\xi = \frac{1}{q_0} = \frac{\hbar^2 k_F}{2m\Delta} = \frac{\hbar v_F}{2\Delta}$$

AND IS CONSIDERED ANOTHER FUNDAMENTAL LENGTHSCALE.

NOTE, IN BCS, ξ IS PREDICTED TO BE

$$\xi_0 = \frac{2\hbar v_F}{\pi\Delta} \sim 100-1000\,\text{Å}$$

GINZBURG - LANDAU THEORY

AS I MENTIONED, THE IDEA OF COHERENCE LENGTH ARISES MORE NATURALLY IN THE PHENOMENOLOGICAL THEORY OF GINZBURG AND LANDAU. DEFINING THE WAVEFUNCTION

$$\psi(x) = n_s^{1/2} e^{i\theta}, \quad \psi^*(x) = n_s^{1/2} e^{-i\theta}$$

SO THAT $\psi^*\psi = n_s$, AND THEN USING A VARIATIONAL APPROACH TO DETERMINING FREE ENERGY, THEY CAME UP WITH THE EQUATION

$$\boxed{\frac{1}{2m^*}\left(-i\hbar\vec{\nabla} - q\vec{A}\right)^2 + \beta|\psi|^2\psi = -\alpha(T)\psi}$$

G-L EQUATION

WHICH LOOKS AN AWFUL LOT LIKE A MODIFIED SCHRODINGER EQUATION IN THE PRESENCE OF A MAGNETIC FIELD AND WITH A NON-LINEAR TERM THAT CAN ACCOUNT FOR CHANGING n_s

IN THIS CONTEXT, ONE DEFINES THE COHERENCE LENGTH AS.

$$\xi(T) = \frac{\hbar}{|2m^*\alpha(T)|^{1/2}}$$

A TEMPERATURE DEPENDENT QUANTITY, WHICH CAN BE SHOWN TO BE THE LENGTHSCALE OVER WHICH DEVIATIONS FROM EQUILIBRIUM $\psi_0(x)$ DIE OUT

TWO LENGTHSCALES

THE FUNDAMENTAL LENGTHSCALES, λ & ξ, REFLECT THE "ROBUSTNESS" OF THE S.C. STATE AND PLAY A CENTRAL ROLE. THIS CAN BE SEEN IN THEIR VARIATION WITH IMPURITY CONCENTRATION. THOUGH S.C. ITSELF IS LARGELY UNAFFECTED BY THE PRESENCE OF (NON-MAGNETIC) IMPURITIES, BOTH λ & ξ <u>DO</u> VARY WITH IMPURITY CONCENTRATION. FOR HIGH CONCENTRATION (THE "DIRTY LIMIT"), THIS VARIATION IS EMPIRICALLY FOUND TO BE:

$$\xi = (\xi_0 \ell)^{1/2} \quad , \quad \lambda = \lambda_L \left(\frac{\xi_0}{\ell}\right)^{1/2}$$

WHERE λ_L, ξ_0 ARE LONDON, G-L ESTIMATES AND ℓ IS THE MEAN FREE PATH OF ELECTRONS IN THE METALLIC STATE.

THUS, THOUGH S.C. CONTINUE TO CARRY CURRENT PERFECTLY AND T_c IS UNCHANGED, A DIRTY SYSTEM WILL SCREEN FIELDS LESS EFFECTIVELY AND WILL BE LESS TOLERANT OF SPATIAL VARIATIONS.

THE RATIO OF THE TWO

$$K = \frac{\lambda}{\xi} \approx \frac{\lambda_L}{\ell}$$

ALSO DEPENDS ON ℓ, WHICH CAN CHANGE S.C. "TYPE"

THE GINZBURG - LANDAU PARAMETER

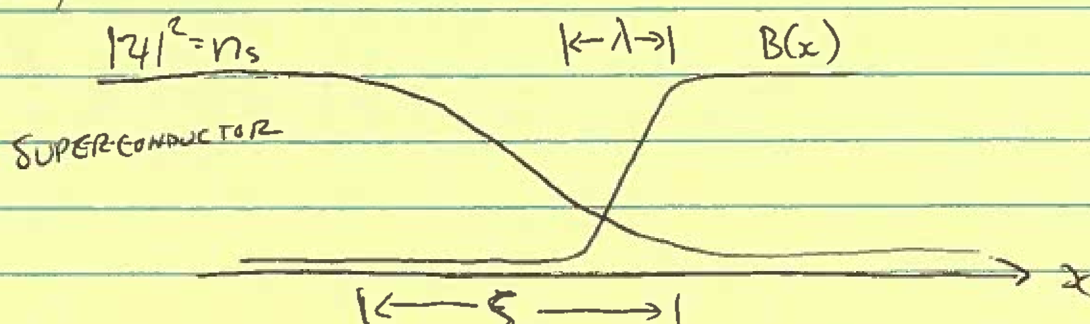
ONE MAJOR RESULT OF G-L THEORY THAT WAS NOT INITIALLY APPRECIATED FROM BCS WAS THE EXISTENCE OF S.C. "TYPES" AND THE ROLE OF THE RATIO

$$\kappa = \frac{\lambda}{\xi}$$

NOW KNOWN AS THE "G-L PARAMETER"

SPECIFICALLY, IT WAS SHOWN BY ABRIKOSOV IN PAPERS IN 1952 & 1957 THAT MATERIALS WITH $\kappa < \frac{1}{\sqrt{2}}$ ARE TYPE-I, AND $\kappa > \frac{1}{\sqrt{2}}$ TYPE-II. THE FULL PROOF IS QUITE INVOLVED, BUT BASIC ENERGY ARGUMENTS CAN BE MADE IN THE EXTREME LIMITS.

FOR $\kappa \ll 1$, CONSIDER THE INTERFACE BETWEEN S.C. AND NORMAL STATES IN THE PRESENCE OF FIELD, $B(x)$



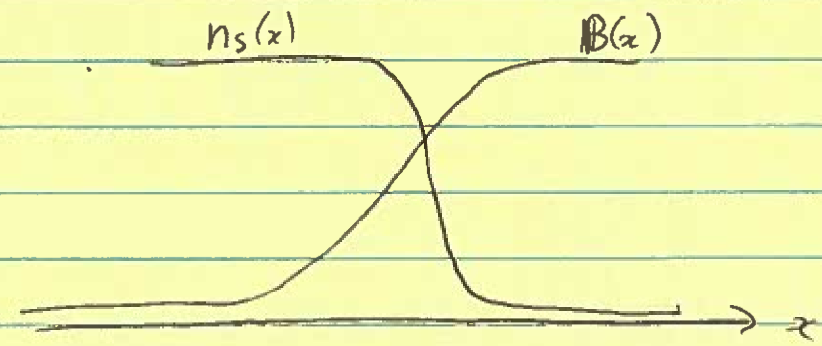
FIELDS ARE EXPELLED ON A LENGTHSCALE, λ, BUT S.C. ONLY VARIES ON LENGTHSCALE, ξ

THUS, WHEN $\kappa \ll 1$, THERE IS A REGION OF SIZE $\xi - \lambda$ WHERE FIELD HAS BEEN EXPELLED (ENERGY COST) BUT ONE IS NOT EXPERIENCING THE FULL BENEFITS OF THE S.C. CONDENSATION ENERGY

$\longrightarrow$ DOMAIN WALLS COST ENERGY

$\longrightarrow$ FM-LIKE DOMAIN STRUCTURE

FOR $\kappa \gg 1$

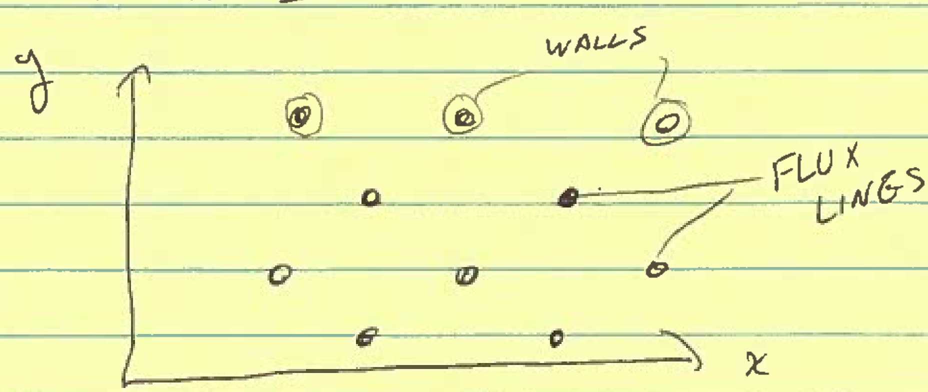


THE ARGUEMENT IS REVERSED: ALL BENEFIT WITHOUT PAYING COST IN REGION $\lambda - \xi$

$\longrightarrow$ DOMAIN WALLS ARE ENERGETICALLY FAVORED!

$\longrightarrow$ AS MANY WILL BE INTRODUCED AS POSSIBLE UNTIL LIMITED BY QUANTUM MECHANICS (FLUX QUANTIZATION)

$\longrightarrow$ TYPE-II



NOTE: CLOSE PACKED STRUCTURE OF FLUX LINES

FLUX QUANTIZATION

WHY DOESN'T INFINITELY MANY DOMAIN WALLS GET CREATED? BECAUSE THERE IS A MINIMUM AMOUNT (QUANTUM) OF FLUX ALLOWED TO PENETRATE THE S.C. CONDENSATE. ONE CAN REACH THIS CONCLUSION BY CONSIDERING G-L. IN A SC "RING"

CONSIDER A CURRENT DEEP INSIDE THE SC. CIRCUMNAVIGATING THE HOLE IN THIS RING.

$$\vec{J}_s = q\psi^* \vec{v} \psi = \frac{q}{m}\psi^* p \psi$$

YOU CAN SHOW →

$$= \frac{n_s q}{m}\left(\hbar \nabla\theta - q\vec{A}\right)$$

SINCE $\oint_c \vec{J}_s \cdot d\vec{l} = 0$ AROUND A CLOSED PATH, THIS IMPLIES

$$\oint_c \hbar \vec{\nabla}\theta \cdot d\vec{l} = \oint_c q \vec{A} \cdot d\vec{l}$$

$$= q \iint \vec{\nabla}\times\vec{A} \cdot d\vec{\sigma}$$

BY STOKES THM

$$= q \iint \vec{B} \cdot d\vec{\sigma}$$

$$= q \cdot \underset{\uparrow \text{FLUX}}{\Phi}$$

BUT SINCE $\theta(x)$ IS CONTINUOUS

$$\oint \vec{\nabla}\theta \cdot d\vec{l} = \Delta\theta = 2\pi s \qquad s \in \mathbb{Z}$$

WHERE THE LAST EQUALITY IS NECESSARY FOR ψ TO BE SINGLE VALUED. $\left(\psi = n_s^{1/2} e^{i\theta}\right)$

$$\Rightarrow \quad \Phi = \frac{2\pi\hbar}{q} \cdot s$$

$$= \frac{h}{2e} \cdot s$$

$$\equiv \Phi_0 \cdot s$$

WHERE Φ_0 IS THE FUNDAMENTAL QUANTUM OF FLUX.

THIS FLUX QUANTIZATION IS ALSO RESPONSIBLE FOR THE LONGEVITY OF PERSISTENT CURRENTS IN SOLENOIDS, SINCE CURRENT CANNOT CHANGE UNLESS A QUANTUM OF FLUX CAN TUNNEL OUT OF THE RING, WHICH ONE CAN SHOW WOULD TAKE AN AVERAGE TIME GREATER THAN THE LIFE OF THE UNIVERSE FOR $T \ll T_c$