1.) 3pts

Mechanosensitive channels respond to stimuli in the environment such as touch or temperature. When they open, they let sodium ions in and may raise the membrane potential. If it passes a threshold potential an action potential is initiated.

2.) 7pts

Potassium leakage channels differ from those involved in action potentials because they open randomly and not in response to membrane voltage. They are useful in maintaining resting potential.

3.) 7pts

The refractory period is the period after the sodium channels have depolarized and another action potential cannot be started. The refractory period typically takes 2-3 ms.

4.) 3pts

Myelin insulates the axon in segments. Action potentials can only propagate in regions called the nodes of Ranvier where there is no myelin present. This speeds up the propagation of the action potential.

5.) 14pts

a.) The charge in question is the gating charge of the channel. This has nothing to do with the charge of the ion.

b.) We are assuming the channel is voltage gated and is either completely open or completely closed.

c.) The slope tells you how responsive the channel is to changes in voltage. The voltage at which \( P(\text{open}) = 50\% \) is the natural zero point for the channel. If \( V_m \) gets larger or smaller, the graph shifts to the left or right.
a) Using $[C_{in}] = 145$ mM, $[C_{out}] = 10$ mM, $q = 1.6 \times 10^{-19}$ C, $k_B = 1.38 \times 10^{-23}$ J/K and $T = 300$ K, we can calculate $V_n$:

$$V_n = \frac{-k_B T}{q} \ln \frac{[C_{in}]}{[C_{out}]} = -0.07 \, V = -70 \, mV$$

b) $Q = C V_n$

For parallel plate capacitor,

$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon 2\pi a L}{b}$$

Thus, the total charge that is separated across the axon membrane:

$$Q = \frac{2\pi a e L}{b} V_n$$

c) Using the values $a = 2 \times 10^{-6}$ m, $\varepsilon = 9\varepsilon_0 = 9 \times 8.85 \times 10^{-12}$ F/m, $L = 10^{-4}$ m, $b = 5 \times 10^{-9}$ m, and $V_n = -0.07 \, V$, we can calculate $Q$:

$$Q = \frac{2\pi a e L}{b} V_n = 1.4 \cdot 10^{-13} \, C$$

The number of $K^+$ ions that moved across the membrane can then be calculated:

$$No. \, of \, K^+ \, ions = \frac{1.4 \cdot 10^{-13} \, C}{1.6 \cdot 10^{-19} \, C} = 8.7 \cdot 10^5 \, ions$$

The original number of $K^+$ ions in cell can be calculated by using the concentration of $K^+$ in the cell (145 mM) and the volume of the inner axon with radius $a$.

$$No. \, of \, K^+ \, ions \, in \, cell = [K^+] \cdot \pi r^2 L \cdot N_A$$

$$No. \, of \, K^+ \, ions \, in \, cell = \frac{145 \, mol}{1000 \, L} \cdot \pi (2 \cdot 10^{-6} m)^2 (10 \cdot 10^{-6} m) \cdot 6.02 \cdot 10^{23} \, mols$$

$$= 1.1 \cdot 10^{10} \, ions$$

The percentage of $K^+$ rushing out can then be calculated:

$$\% K^+ = \frac{8.7 \cdot 10^5}{1.1 \cdot 10^{10}} = 0.008\%$$

This is a very small percentage. The amount of potassium ion rushing in and out is very small compared to the total amount of potassium ion in the cell. There is not a significant change in the concentration of $K^+$ ions when an action potential fires.
0) **Resting state.** The -70 mV resting potential is caused by the permeability of the cell membrane to certain ions and also the different concentration of these ions inside and outside the cell. In most cells, K\(^+\) makes the largest contribution to the resting potential, followed by Na\(^+\) and Cl\(^-\). The resting potential is thus close to but not exactly the Nernst potential of K\(^+\) ion. It can be calculated using the Goldman-Hodgkin-Katz voltage equation:

\[
E_m = \frac{RT}{F} \ln \left( \frac{P_{Na^+} [Na^+]_o + P_{K^+} [K^+]_o + P_{Cl^-} [Cl^-]_o}{P_{Na^+} [Na^+]_i + P_{K^+} [K^+]_i + P_{Cl^-} [Cl^-]_o} \right)
\]

where \(R\) is universal gas constant, \(T\) is absolute temperature, \(F\) is Faraday constant. \(P_{rel}\) is the relative permeability of ions (K\(^+\) being the most permeable at rest state, followed by Cl\(^-\) then Na\(^+\)). In squid axon, the relative permeability of K\(^+\) : Na\(^+\) : Cl\(^-\) = 1 : 0.04 : 0.45). [Ion] is the concentration of ions. Below is a table of relative ion concentrations in and out of the cell (Source: Animal Physiology 2e, Table 11.1):

<table>
<thead>
<tr>
<th>Ion type</th>
<th>Squid axon</th>
<th>Mammalian muscle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Out (mM)</td>
<td>In (mM)</td>
</tr>
<tr>
<td>Na(^+)</td>
<td>440</td>
<td>50</td>
</tr>
<tr>
<td>K(^+)</td>
<td>20</td>
<td>400</td>
</tr>
<tr>
<td>Cl(^-)</td>
<td>560</td>
<td>60</td>
</tr>
<tr>
<td>A(^-) (organic anions)</td>
<td>—</td>
<td>270</td>
</tr>
</tbody>
</table>
1) **Depolarization.** Certain stimulus (e.g. Ca\(^{2+}\)) causes sodium channel to open, letting an influx of sodium ions into the cell that depolarize the membrane from -70 mV to +30 mV, which is close to the Nernst potential of sodium ion.

2) **Repolarization.** At +30 mV, Na\(^+\) channel closes and K\(^+\) channel opens. K\(^+\) ions flow out of the cell, repolarizing the membrane towards the rest potential.

3) **Hyperpolarization.** Some K\(^+\) channel is still opened when the resting potential (-70 mV) is reached. This slow closing of K\(^+\) channel causes the membrane potential to drop further to around -90 mV. Na\(^+\)/K\(^+\) transporter then restores the membrane potential to its resting potential (-70 mV).

The concentration of Na\(^+\) and K\(^+\) do not change appreciably during the action potential. To set up a potential, only a small percentage of ions have to be transported across the membrane.

The first action potential on a nerve is initiated by a neurotransmitter received by the dendrite. Examples of neurotransmitters include glutamate, dopamine, carbon monoxide and acetylcholine. Ligand gated ion channels are transmembrane channels which open or close to allow ions like Na\(^+\), K\(^+\), Ca\(^{2+}\), or Cl\(^-\) to pass in response to binding of a ligand like a neurotransmitter.

8.) 36pts

a) The up and down movement of positive charged S4 segments contributes to gating current (it moves outward upon depolarization)

 Ionic current is caused by the inward or outward flow of positive or negative ions through ion channels upon activation. Based on the definition we made above, ionic current will change when comparing a calcium channel to a sodium channel

b) Boltzmann factor closed = \(\exp\left(-\frac{qV}{k_B T}\right)\)

c) Boltzmann factor opened = \(\exp\left(-\frac{q(0)}{k_B T}\right)\) = 1

d.)

\[
P_{open} = \frac{1}{1 + e^{-\frac{qV}{k_B T}}}
\]
If $V \to \infty$, \( e^{-\frac{qV}{k_BT}} \) goes to zero, so $P_{\text{open}} = 1$

If $V \to -\infty$, \( e^{-\frac{qV}{k_BT}} \) goes to $\infty$, so $P_{\text{open}} = 0$

For the open probability to be $\frac{1}{2}$:

\[
\frac{1}{1 + e^{-\frac{qV}{k_BT}}} = \frac{1}{2}
\]

\[
1 + e^{-\frac{qV}{k_BT}} = 2
\]

\[
e^{-\frac{qV}{k_BT}} = 1
\]

In order for \( e^{-\frac{qV}{k_BT}} \) to be 1, $V$ must be 0. Thus $V_{\text{midpoint}} = 0$
Boltzmann factor closed = $e^{-\frac{qV}{k_BT}}$

Boltzmann factor opened = $e^{-\frac{q(0)+\frac{1}{2}kdx^2}{k_BT}} = e^{\frac{kdx^2}{2k_BT}}$

$$P_{open} = e^{-\frac{kdx^2}{2k_BT}}$$

$$e^{-\frac{kdx^2}{2k_BT}} + e^{-\frac{qV}{k_BT}} = 1 + e^{\frac{(qV-\frac{1}{2}kdx^2)}{k_BT}}$$

The midterm potential, $V_0$, is thus:

$$V_0 = \frac{kdx^2}{2q}$$

If spring is relaxed when opened, then

B-F closed: $e^{-\frac{qV-\frac{1}{2}kdx^2}{k_BT}}$

$V_0$ is positive
B-F opened: \( e^{-\frac{q(0)}{k_B T}} = 1 \)

\[
\rho_{open} = \frac{1}{1 + e^{\frac{(qV^2 + \frac{1}{2}kdx^2)}{k_B T}}} = \frac{1}{2}
\]

\[V_0 = -\frac{kdx^2}{2q}\]

\( V_0 \) is negative