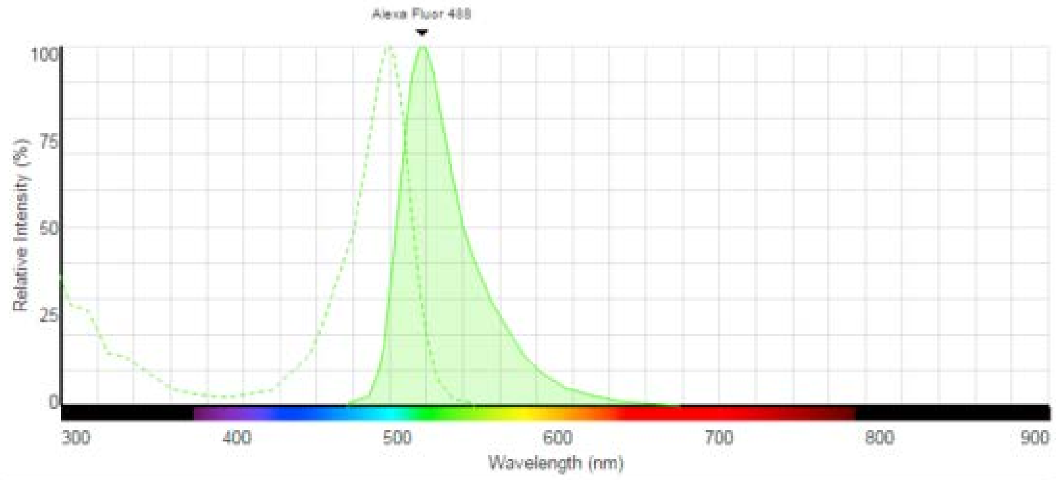
1 **9 pts.**

a-b



(from Thermofisher scientific SpectraViewer)

c.) They are close to mirror images because the energy spacing between the ground and excited states is roughly the same. The emission is redshifted because of thermal relaxation.

2.) **11pts.**

Case1: $400/\sqrt{400} = 20 \;\; 20/\sqrt{400+400} \approx 14$

Case2: $400/\sqrt{400+400} = 14.14$

Case3: $400/\sqrt{400+4000} = 6.03$

In all cases, the signal can be seen.

Signal/Fluctuations must be greater than 1.

3.) **29 pts.**

a. $\frac{P_f}{P_b} = \frac{e^{-\frac{E_f}{k_BT}}}{e^{-\frac{E_b}{k_BT}}} = e^{-\frac{E_f - E_b}{k_BT}}$

b. $Z = e^{-\frac{E_f}{k_BT}} + e^{-\frac{E_b}{k_BT}} = 1 + e^{-\frac{\varepsilon}{k_BT}}$

$$P_f = \frac{e^{-\frac{E_f}{k_BT}}}{Z} = \frac{1}{Z} = \frac{1}{1 + e^{-\frac{\varepsilon}{k_BT}}}$$

$P_b = 1 - P_f$

c. $W_{net} = P_f W_f - P_b W_b = F\Delta x\,(P_f - P_b) = F\Delta x\,\left(P_f - (1 - P_f)\right) = F\Delta x\,(2P_f - 1)$

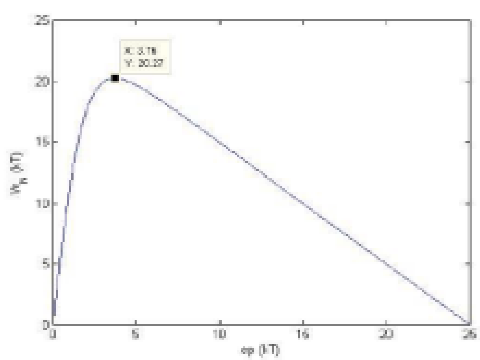
d. $F\Delta x = E_{ATP} - \varepsilon = 25k_BT - \varepsilon$

e. $W_{net} = (E_{ATP} - \varepsilon)\left[\frac{2}{1+e^{-\frac{\varepsilon}{k_BT}}} - 1\right] = (E_{ATP} - \varepsilon)\left[\frac{1-e^{-\frac{\varepsilon}{k_BT}}}{1+e^{-\frac{\varepsilon}{k_BT}}}\right]$

f. To find the value of ε that maximizes W_{net}, we can either take the derivative of W_{net} with respect to ε:

$$\frac{dW_{net}}{d\varepsilon} = 0$$

or we can simply plot the graph W_{net} vs ε using Matlab, Mathematica, EXCEL, etc



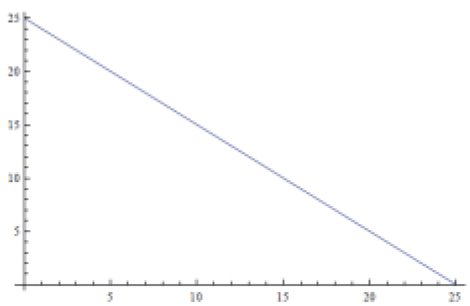
Both methods will give the same answer. We find that W_{net} is maximum (20.27 k_BT) when $\varepsilon = 3.76$ k_BT.

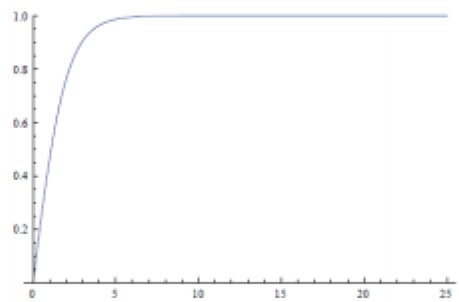
g. $P_f = \frac{1}{1+e^{-\frac{\varepsilon}{k_BT}}} = 0.977 = 97.7\%$

$P_b = 1 - P_f = 0.023 = 2.3\%$

h. When $\varepsilon = 0$, $P_f = 50\%$. When $\varepsilon = E_{ATP}$, P_f is approximately 100%.

We can compare the two components of W_{net} to figure out why the maximum W_{net} is not $E_{ATP}/2$. The first graph below (left) comes from the term $(E_{ATP} - \varepsilon)$, which can be thought of as the energy for doing work. When this term is larger, kinesin can carry heavier load. The second graph below (right) comes from the term $\left[\frac{1-e^{-\frac{\varepsilon}{k_BT}}}{1+e^{-\frac{\varepsilon}{k_BT}}}\right]$, which can be thought of as biasing energy. We can see that W_{net} is maximized at $\varepsilon = 0$ for the first term and at $\varepsilon = E_{ATP}$ for the second term. In order to bias the movement forward, a certain amount of energy needs to be invested to overcome thermal energy k_BT. We notice from the second graph that the biasing energy need not be big. With only 2 k_BT, 70% of steps are biased forward. With 5 k_BT, almost 100% of the steps are biased forward. Since the first term is linear, we can conclude that the maximum W_{net} will lie between 2 k_BT and 5 k_BT, and not at $E_{ATP}/2$ (or 12.5 k_BT)





4.) **9pts**

a-b.) Estimate Kinesin's body length at 8-16nm.

640/12=53 body length's per second.

If a car is around 15ft, then a similar car would move 15*53 ft/s.

This translates into roughly 550 mph.

5.) **23pts.**

a.)

$$\frac{d[ATP](t)}{dt} = -k_1[ATP](t)$$

$$[ATP](t) = Ae^{-k_1 t}$$

b.)

$$\int_0^\infty A\, e^{-k_1 t} dt = 1$$

$$A\left(-\frac{1}{k_1}e^{-k_1 t}\right)\Big|_0^\infty = 1$$

$$A\left(-\frac{1}{k_1}(0-1)\right)\Big| = 1$$

$$A = k_1$$

$$P(t) = k_1 e^{-k_1 t}$$

c.)

$$f(t) = k_1 e^{-k_1 t}$$

$$g(t) = k_2 e^{-k_2 t}$$

d.)

$$P(t) = \int_0^t k_1 e^{-k_1 u}\ k_2 e^{-k_2(t-u)} du = k_1\, k_2 \int_0^t e^{-k_1 u + k_2 u}\ e^{-k_2 t} du$$

$$P(t) = k_1\, k_2\, e^{-k_2 t} \int_0^t e^{(k_2 - k_1)u}\, du$$

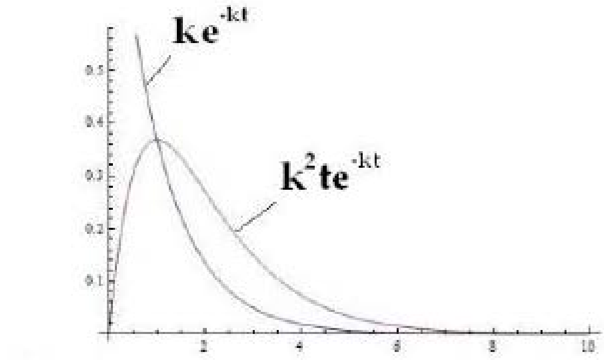
$$P(t) = k_1\, k_2\, e^{-k_2 t}\left(\frac{1}{k_2 - k_1} e^{(k_2-k_1)u}\Big|_0^t\right) = \frac{k_1\, k_2}{k_2 - k_1} e^{-k_2 t}\left(e^{(k_2-k_1)t} - 1\right)$$

$$P(t) = \frac{k_1\, k_2}{k_2 - k_1}\left(e^{-k_1 t} - e^{-k_2 t}\right)$$

e.)

$$P(t) = \int_0^t f(u) g(t-u)\, du = \int_0^t k e^{-ku}\, k e^{-k(t-u)} du = k^2 e^{-kt} \int_0^t e^0\, du = k^2 t e^{-kt}$$

f.)



ATP synthase has a dwell time similar to the exponential function.

6.) **11 pts.**

a.)If you label on the foot, then you get alternating stepsize of 72nm and 0 nm. This is the red graph.

b.) As you label further up the leg, the step size will become alternating. The big step will be less than 72nm, but greater than if it were labeled at the center (36nm). The smaller step will be 72nm minus the big step.

Since we see every step, the dwell time histogram will be the green graph.

7.) **9pts.**

As numerical aperature is increased, more wavelets are available. Light has to travel less far to meet up with another wavelet that is a path length difference of lambda/2 to destructively interfere.
Therefore, a larger numerical aperture is better for resolving small structures.