

Atomic Numerology for Physics 485

Physics 485 Homework frequently involves numeric calculations of quantities with roughly atomic dimensions, and energies. The formulae in the lecture notes are in mks units. We will typically express energies in electron volts (eV), and distances in nanometers (nm):

Distance	$1 \text{ nm} = 1 \times 10^{-9} \text{ m}$
energy	$1 \text{ eV} \cong (1.6 \times 10^{-19} \text{ C}) \times 1 \text{ V} \cong 1.6 \times 10^{-19} \text{ Joule}$

Most of our examples will involve the electron mass and charge. Rather than using mks values of fundamental constants, we will typically calculate quantities using conveniently memorable combinations of constants.

electron rest energy μc^2	$\mu c^2 \cong 0.511 \text{ MeV} \cong 0.511 \times 10^6 \text{ eV}$
Planck Constant \hbar	$\hbar = \frac{h}{2\pi} \rightarrow \hbar c \cong 197 \text{ eV} \cdot \text{nm}$
Fine Structure constant α	$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \cong \frac{1}{137}$

We illustrate the use of $\hbar c$, μc^2 , and α in computing some useful atomic quantities. See if you can follow the algebra - usually involving adding appropriate powers of c (the speed of light).

1. Energy of a 600 nm (visible) photon:

$$E_\gamma = hf = h \frac{c}{\lambda} = \frac{2\pi\hbar c}{\lambda} \xrightarrow{\lambda=600\text{nm}} \frac{2\pi(197 \text{ eV} \cdot \text{nm})}{600 \text{ nm}} = 2.06 \text{ eV}$$

2. Energy of an electron in an $a = 0.1 \text{ nm}$ box

$$E_n = \frac{p^2}{2\mu} = \frac{(\hbar k)^2}{2\mu} = n^2 \frac{(\hbar\pi)^2}{2\mu a^2} = n^2 \frac{\pi^2 (\hbar c)^2}{2(\mu c^2) a^2} = n^2 \frac{\pi^2 (197 \text{ eV} \cdot \text{nm})^2}{2(0.511 \times 10^6 \text{ eV})(0.1 \text{ nm})^2} = n^2 (37.5 \text{ eV})$$

3. Wave function of a classically forbidden electron with $V-E = 10 \text{ eV}$

$$\psi = \mathcal{N} \exp(-\beta x) ; \beta = \frac{\sqrt{2\mu(V-E)}}{\hbar} = \frac{\sqrt{2\mu c^2(V-E)}}{\hbar c} = \frac{\sqrt{2(0.511 \times 10^6 \text{ eV})(10 \text{ eV})}}{197 \text{ eV} \cdot \text{nm}} = 16.22 \text{ nm}^{-1}$$

4. Bohr radius

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{e^2\mu} = \left[\frac{4\pi\epsilon_0\hbar c}{e^2} \right] \frac{\hbar c}{\mu c^2} = \frac{\hbar c}{\alpha\mu c^2} = [137] \frac{197 \text{ eV} \cdot \text{nm}}{0.511 \times 10^6 \text{ eV}} = 0.053 \text{ nm}$$

5. Rydberg energy

$$E_n = -\frac{E_1}{n^2} \text{ with } E_1 = \frac{e^4\mu}{2(4\pi\epsilon_0\hbar)^2} = \frac{e^4}{2(4\pi\epsilon_0\hbar c)^2} \mu c^2 = \frac{\alpha^2 \mu c^2}{2} = \frac{0.511 \times 10^6 \text{ eV}}{2(137)^2} = 13.6 \text{ eV}$$