Exam 1

PHYS 485: Fall 2017

This is an in-class exam lasting 1 hour and 10 minutes. You may use one "cheat sheet" that you have prepared and a calculator but no other resources. You will be graded on all questions. Give as many details as you can while retaining clarity to get full credit. Scores for each subsection are indicated in brackets and a list of useful formulae can be found at the end of the exam. Maximum score: 50 points

***Academic Integrity**: Giving assistance to or recieveing assistance from another student or using unauthorized materials, including unauthorized electronic devices, are ground for disciplinary action up to and including expulsion. Please turn off and put away all internet-capable electronic devices and refrain from sharing calculators.*

*If you have **any** questions please raise your hand and wait for a TA to assist you.*

I. PARTICLE IN A DOUBLE POTENTIAL WELL [14]

A quantum particle of mass m is subject to the potential plotted below. Consider three energy eigenstates that it can occupy, as marked on the plot.

(a) For the three energy states, draw the corresponding wavefunctions. You can pick which states these are (ground state, first excited state etc.) as long as you keep in mind that $E_3 > E_2 > E_1$. [4]

(b) For the wavefunction having energy E_3, where (in region I, II and so on) would you expect the wavefunction to

i) be oscillating versus decaying (for each region, give the appropriate answer)?

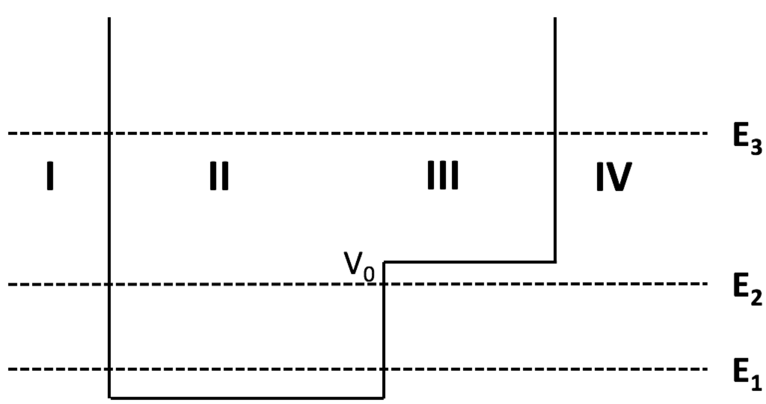
ii) have the shortest wavelenght of oscillation?

iii) have the greatest amplitude?

Give reasons for your answers. [5]

(c) Explain what happens to the wavefunction in the situations where $V > E$. Does this behavior agree with the predictions of classical physics? Which are these situations in the cases shown below? [3]

(d) For the wavefunction with E_3, give the wavelength of oscillation in the regions where it does oscillate in terms of particle's mass, energy, etc. Recall Schrodinger's equation and its solutions. [2]



II. INFINITE SQUARE WELL POTENTIAL [20]

An electron in an infinite square well potential of length L has an initial wavefunction (at time $t = 0$) given by

$$\Psi(x, t = 0) = A(\psi_1(x) + 2\psi_3(x))$$

where $\psi_1(x)$ and $\psi_3(x)$ correspond to the normalized eigenstate wavefunctions having energies E_1 and E_3 respectively.

(a) What is the normalization constant A (give a number)? [1]

(b) What is the probability of finding the electron in

i) The ground state?

ii) The 2nd excited state? [3]

(c) i) What is the expectation value of the energy of this state? Give the answer either in terms of the ground state energy E_1 or in terms of the length L.

ii) Assume that the length is $L = 6$ nm. What is the average energy in electronvolts?

iii) If you were to measure the energy of the particle, would you ever obtain this average value. Explain your answer. [4]

(d) i) Given an expression for the time-dependent wavefunction $\Psi(x, t)$

ii) Find the time-dependent probability density $|\Psi(x, t)|^2$ (expressed in terms of ψ_1, ψ_3, time, etc.) [4]

(e) (i) Find the smallest time T such that $|\Psi(x, t)|^2 = |\Psi(x, 0)|^2$ i.e. the time period of probability density oscillation. Express T in terms of E_1 etc.

(ii) For the well of length $L = 6$ nm, give the value of this period in seconds. [4]

Hint: Use integration by parts and the following identity

$$\int x \sin(\alpha x) dx = \frac{\sin(\alpha x)}{\alpha^2} - \frac{x \cos(\alpha x)}{\alpha}.$$

(f) Now, consider the wavefunction

$$\phi(x) = \begin{cases} \sqrt{\frac{30}{L^5}}\, x(L - x) & 0 < x < L \\ 0 & \text{elsewhere} \end{cases}$$

which is not an eigenstate of the system but may overlap with $\psi_3(x)$ (and other eigenstates of the time independent Schrodinger equation). If the particle is described by $\phi(x)$, what is the probability that a measurement of its energy will yield E_3? [4]

III. MATTER WAVES [16]

In a standard two-slit experiment, we use a single electron having a de Broglie wavelength $\lambda_{\mathrm{dB}} = 633$ nm. The distance between the slits and the detector is $L = 10$ m and the two slits are separated by $d = 0.1$ mm.

(a) What is the momentum of this electron? What is its kinetic energy in electronvolts? [3]

(b) Draw a figure of the set-up and label all given quantities. Denote the direction from the slits to the detector screen as x and the distance along the detector screen as y. Let $y = 0$ correspond to the point on the screen equidistant to both slits. [2]

(c) What is the separation between two adjacent interference maxima? *Use the small angle approximation.* [3]

(d) Consider a point on the screen whose distances to the two slits are d_1 and d_2 respectively.

i) What is the probability amplitude for the electron to arrive at this point from each of the slits in terms of d_1 and d_2? (Hint: Your solution will feature exponential functions) *You do not have to normalize these amplitudes.*

ii) What is the net probability amplitude at this point?

iii) What is the probability of detecting an electron at this point? [6]

(e) In another experiment, an electron having the same de Broglie wavelength has an uncertainty in its speed of 1 m/s. What is the uncertainty in its position?

IV. USEFUL FORMULAE

1. Planck constant: $h = 6.63 \times 10^{-34}\mathrm{J} \cdot \mathrm{s}$; $\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34}\mathrm{J} \cdot \mathrm{s}$.
2. Mass of electron: $m_e = 9.11 \times 10^{-31}$ kg
3. Charge of electron: $e = 1.60 \times 10^{-19}$ C
4. $1\mathrm{eV} = 1.60 \times 10^{-19}$ J
5. $1\mathrm{m} = 10^9\mathrm{nm} = 10^6\mu\mathrm{m}$
6. Kinetic energy of an electron (as used here):
$$K = \frac{1}{2}m_e v_e^2 = \frac{p_e^2}{2m_e} \tag{1}$$
7. de Broglie wavelength: $\lambda_{dB} = h/p$ for p the momentum of the particle
8. Uncertainty relationship: $\Delta x \Delta p \geqslant \frac{\hbar}{2}$
9. Schrodinger equation(1-d):
   (a) Time-dependent
$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi \tag{2}$$
   (b) Time-independent
$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi \tag{3}$$
10. Energy and eigen-wavefunctions for a particle in the infinite square well
$$E_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2$$
$$\psi_n = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$
11. Time evolution of energy eigenstates:
$$\psi_n(x,t) = e^{-iE_n t/\hbar}\psi_n(x,0) \tag{4}$$