# Homework 2

PHYS 485: Fall 2017
Due Date: 09/20/2017

*To recieve fully credit please write legibly and clearly, show all required steps and calculations and answer any explicit questions in full sentences.*

## I. COMPLEX NUMBERS AND PROBABILITIES [7]

1. Suppose that the probability amplitude for a photon to arrive at a detector is $1/(1-i)$. What is the probability that the detector records a photon? What is the probability of detecting a photon if the probability amplitude equals $1/(1+i)+i$? [2]
2. Express the complex numbers $z_1 = (1+i)/\sqrt{2}$ and $z_2 = i$ in the form $re^{i\phi}$. [2]
3. Suppose that the probability amplitude is time-dependent: $z_3(t) = (1+e^{i\omega t})/2$. Plot this probability amplitude as an arrow in the complex plane for $t = 0, \pi/\omega, \pi/4\omega$, and $\pi/3\omega$. What is the *probability* at these points in time? [3]

## II. SCHRODINGER EQUATION [8]

1. Suppose that $\psi_1$ and $\psi_2$ are two orthogonal, normalized solutions of the Schrodinger equation. Given some complex numbers $\alpha$ and $\beta$, show that $\phi = \alpha\psi_1 - \beta\psi_2$ is also a solution of the Schrodinger equation. [1]
2. What does the relationship between $\alpha$ and $\beta$ have to be for $\phi$ to be normalized? [1]
3. If this relationship is satisfied, is $\phi' = \alpha\psi_1 + \beta\psi_2$ also a normalized solution of the Schrodinger equation? Explain. [1]
4. Normalize $\psi_3(x) = \sin(2\pi x/L)$ and $\psi_4(x) = \sin(6\pi x/L)$ assuming that both functions vanish for $x < 0$ and $x > L$. [2]
5. Verify that $\psi_5(x) = e^{i(kx-\omega t)}$ is a solution to the Schrodinger equation for a free particle by showing that

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi_5}{\partial x^2} = i\hbar\frac{\partial\psi_5}{\partial t}.$$

   Is $\psi_6 = A\cos(kx - \omega t)$ a solution as well? Plug it into the Schrodinger equation and corroborate your answer. [3]

## III. AVERAGES AND DEVIATIONS [9]

1. Consider a group of 8 students buying coffee at the Illini Union over the course of a busy workday. In this group 3 people only have one coffee a day, 3 order two coffees, and two students order three. Treating the number of coffees consumed as a discrete variable, sketch its probability distribution for this group. [1]
2. What is the expectation value for the number of coffees consumed in this group? Is there a person in this group that actually consumed that many cups of coffee i.e. if you perfomed a measurement such as asking each person how much coffee they had had on that day could your measurement ever produce this number? [2]
3. Calculate the standard deviation in this situation. [2]
4. Suppose another extremely sleep deprived student joins the group and [1] ends up consuming six coffees throughout the course of the day. How does the shape of the probability distribution change? Sketch it.

5. Calculate the expectation value $\langle x \rangle$ and the standard deviation $\Delta x$ for the normalized wavefunction [3]

$$\Psi_k(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\left(\frac{4\pi x}{L}\right) & 0 < x < L \\ 0 & \text{otherwise.} \end{cases}$$

## IV. DE BROGLIE WAVELENGTH, THE UNCERTAINTY PRINCIPLE AND FANTASY BASEBALL [6]

1. What is the de Broglie wavelength of a 0.3 kg baseball thrown at 25 m/s? [1]
2. What is the uncertainty in the position of the baseball if the uncertainty in its speed is 2 m/s? [1]
3. Now, imagine you are playing baseball in a parallel universe in which Planck's constant $h = 0.663$ ($10^{34}$ times larger than in our universe!). What is its de Broglie wavelength in this universe? Compare this to the de Broglie wavelength of the baseball in our universe and discuss. [2]
4. What is the uncertainty in the position of the baseball in this universe if the uncertainty in the velocity is 2 m/s? Compare this to the uncertainty in the position of the baseball in our universe and discuss. [2]