Homework 3

PHYS 485: Fall 2017
Due Date: 09/27/2017

To recieve fully credit please write legibly and clearly, show all required steps and calculations and answer any explicit questions in full sentences.

I. QUANTUM DOT AND PARTICLE IN INFINITE SQUARE WELL [8]

Cadmium selenide nanoparticles can exhibit a property known as quantum confinement. One such nanoparticle is a cadmium selenide quantum dot which can tightly confine, or trap, an electron. The trapped electron inside the cadmium selenide quantum dot can be modeled as being in a one-dimensional infinite square well. To analyze this problem we can then take

$$V(x) = \begin{cases} 0 & 0 < x < L \\ \infty & \text{otherwise,} \end{cases} \tag{1}$$

where $L = 2.5$ nm, and use the Schrodinger equation to find the allowed energy states for this electron.

1. Solving the Schrodinger equation, as in Sec. 3.2 of Townsend, find the allowed energies and corresponding wavefunctions for this electron, as determined by n and L. [3]
2. Evaluate the first 3 energy levels in *electronvolts*. What wavelengths correspond to these 3 energies? [2]
3. Electrons can transition from one energy level to another by either absorbing a photon (if they are moving from a lower energy to a higher energy) or by emitting a photon (if they are moving from a higher energy to a lower energy). The energy of the photon has to be exactly equal to the energy difference between the two levels.

   An electron in its fifth excited state $n = 6$ will jump back to its groud state by emiting a photon. What is the energy and wavelength of this photon? What range of the electromagnetic spectrum does this wavelength correspond to? If visible, what is the closest color describing it? [3]

II. INFINITE SQUARE WELL II [11]

The wavefunction for a particle in a box is given by

$$\psi = \frac{i}{2}\psi_1 + \frac{\sqrt{3}}{2}\psi_2$$

where ψ_1 and ψ_2 are energy eigenfunctions with energy eigenvalues E_1 and E_2, respectively.

1. Show that ψ is normalized by explicitly calculating $\int |\psi(x)|^2 dx$ in the appropriate region. [1]
2. What is the probability that a measurement of energy yields the eigenvalue E_1? What is the probability that a measurement of energy yields the eigenvalue E_2? [3]
3. What are $\langle E \rangle$ and ΔE for this wavefunction? [2]
4. Calculate $|\psi(x,t)|^2$ as a function of time. Plot it for $t = 0$ and $t = \frac{2mL^2}{3\hbar\pi}$. [3]
5. Compute $\langle x(t) \rangle$. How does this average behave over time? Discuss. [2]

III. DECOMPOSING WAVEFUNCTIONS [9]

The wave function for a particle of mass m in an infinite square well where that $V = 0$ for $-L/2 < x < L/2$ and $V = \infty$ otherwise, is given by

$$\psi(x) = \begin{cases} A(x + L/2)(x - L/2) & -L/2 < x < L/2 \\ 0 & \text{otherwise.} \end{cases}$$

Important: for this infinite well potential energy states corresponding to odd n are proprotional to $\cos(nx\pi/L)$ while those with even n are proportional to $\sin(nx\pi/L)$. You will have to *normalize* these wavefunctions to obtain the correct answer.

1. Normalize these wavefunctions i.e. calculate A in terms of the length of the well L. [2]
2. What is the probability that a measurement of the energy of the particle yields the ground state energy? [2]
3. What is the probability that a measurement of the energy of the particle yields the energy of the first excited state? [2]
4. Determine $\langle E \rangle$ for $\psi(x)$. How does it compare to the ground state energy? Discuss. *Hint: See Example 3.5 in Townsend.* [3]

IV. PARTICLE IN A DOUBLE POTENTIAL WELL [12]

A particle is in the presence of the potential shown below as a function of space. In this problem you will use your knowledge of the properties of the Schrodinger equation and its solutions in order to draw the wavefunctions for this system. You do *not* have to perform any calculations in this problem.

1. For the three energy states shown above, draw the corresponding wavefunctions. You can pick which states these are (ground state, first excited state, etc.) as long as you keep in mind that $E_C > E_B > E_A$. [3]
2. Which energy states (ground state, first excited state, second excited state etc.) do your plots correspond to? Explain why these wavefunctions are consistent with $E_C > E_B > E_A$ based on the number of their nodes. [2]
3. For each of your plots in part (1), where would you expect the wavefunction to have the greatest curvature? Explain. [2]
4. For each of your plots in part (1), where would you expect the wavefunction to have the greatest amplitude? Explain. [2]
5. Explain what happens to the wavefunction when $V > E$. Does this behavior agree with the predictions of classical physics? Which of the energy levels that you have examined above are in this category? [3]