Homework 5

PHYS 485: Fall 2017
Due Date: 10/30/2017

To recieve fully credit please write legibly and clearly, show all required steps and calculations and answer any explicit questions in full sentences.

I. SIMPLE HARMONIC OSCILLATORS AND DIATOMIC MOLECULES [10]

Sodium Hydride is a diatomic molecule composed of ionically bound sodium and hydrogen. Vibrational motion about their equilibrium position can be approximated by simple harmonic oscillation. The reduced mass m of the molecule is about 1.6×10^{-27} kg and the effective spring constant k for the harmonic motion is about 77 N/m.

(a) What is the angular frequency ω of oscillations? [1]

(b) (i) What is the wavelength of a photon that can induce a transition from the $n = 0$ to $n = 1$ state of the harmonic vibrational motion of this molecule? [2]

(ii) What range of the electromagnetic spectrum does this wavelength correspond to (e.g. ultra-violet)? [1]

(iii) How would your answers for i) and ii) change if the transition were from $n = 17$ to $n = 18$? Explain. [3]

(c) Now consider two tennis balls attached by a spring of the same spring constant and having a reduced mass of 6 grams.

(i) If the system is vibrating with an energy of about 0.1 Joules, what quantum number does this correspond to for the state of harmonic vibration? (An order of magnitude estimate is fine.) [2]

(ii) Would quantum effects be noticeable in this case? Explain. [1]

II. FINITE SQUARE WELL [7]

In this problem, you will compare the wavefunctions of a finite and an infinite square well, shown in Fig. 1 (a) and (b).

(a) Assuming all state are bound i.e. confined to the vicinity of the wells, draw the energy eigenfunctions for the three lowest energy states in each well shown. [3]

(b) Compare and contrast the wavefunctions you have drawn. Are they different? How? Why? [2]

(c) Consider the first excited state of the finite square well and the first excited state of the infinite square well. Which will have a larger energy? Explain why. [2]

III. STEP POTENTIAL [10]

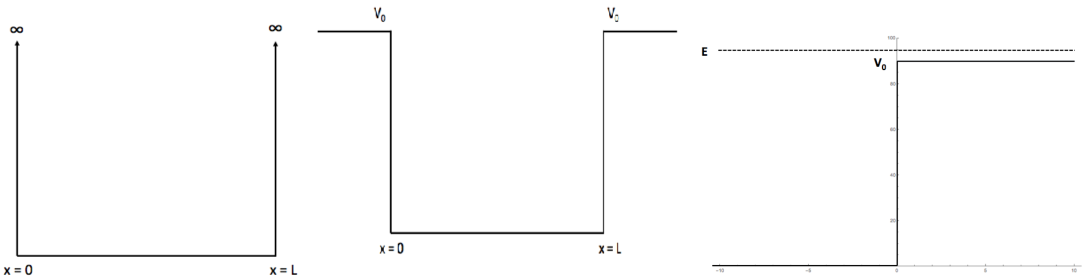
In this problem we examine an energy state above a "step" potential given by

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & x > 0. \end{cases}$$

More precisely, we take $E > V_0$ as shown in Fig. 1 (c).

(a) What is the most general form of the wavefunction corresponding to E, $\psi_E(x)$, for $x < 0$? This form should include two undetermined constants, A and B. [2]

(b) What is the most general form of the wavefunction corresponding to E, $\psi_E(x)$, for $x > 0$? This form should include two undetermined constants, C and D. Assume that there is no probability current propagating to the left in this region so that one of the constants, say D, is *identically zero*. [3]



(a) (b) (c)

FIG. 1: Potentials relevant for Problems II and III.

(c) Write down a set of equations that have to be true in order for $\psi_E(x)$ and its derivative to be continuous. [2]

(d) Use these equations to express B and C in terms of A. What is the physical significance of A, B and C? *Hint: See Townsend Sec. 4.6.* [3]

IV. SUPERHERO TUNNELING [13]

Superman has been locked up in a cube of length 3m and walls of thickness 10 cm. Having kept up with modern physics – as all superheroes should – he is considering quantum tunneling as a means of escape. Assume Superman's mass is 90 kg, the maximum speed he can acquire is 2000 m/s (faster than any bullet or even sound waves), and that his kinetic energy reaches only 9/10 of the potential barrier, V_0, energy provided by each wall. He models each wall as a rectangular barrier and his plan is to keep bouncing back and forth linearly against two sets of walls, tunneling out a little each time (which is absurd for classical physics).

(a) Sketch the situation described as a simple one-dimensional potential having two rectangular barriers. Clearly mark the height V_0 and draw a line for Superman's energy E (equal to $mv^2/2$ when the potential energy is zero). [2]

(b) Calculate the probability that he will tunnel through one of the walls upon approaching it once. It may be helpful to recall that

$$\log(T) \approx -2 \int \sqrt{\frac{2m(V(x)-E)}{\hbar^2}}\,dx$$

(see Townsend Eq. 4.137). Express your answer in the form of an exponential of a power of 10 (e^{-a10^x} for some appropriate x and a.) [3]

(c) Now, to estimate the time for all of him to tunnel out, because the probability T that this happens in a single attempt is small he has to repeat this step some n number of times until $nT = 1$ and he has certainly tunneled out. Given that he is moving at the speed v (given above) and has to traverse the distance of L from the barrier he has reflected from to the the barrier on the other side of the box, what is the total time that is necessary to complete these n tunneling events? Give an estimate in the form of an exponential of a power of 10 (e^{b10^y} for some appropriate y and b.) How does this time compare with the age of the universe? [3]

(d) Now suppose that Ant-Man is trapped instead and he can shrink to the nanoscale and that his mass is equal to the mass of an electron. His captors have appropriately imprisoned him in a cube of length 100 nm having walls of thickness 1 nm. How long would it take the shrunken Ant-Man to tunnel out of this nano-box? How does this escape time compare to Superman's? Assume that he can attain the same speed and that his kinetic energy is 9/10 of the barrier height again: $KE = m_e v^2/2 = 0.9V_0$ where m_e is the mass of the electron. Discuss. [6]