Homework 6

PHYS 485: Fall 2017
Due Date: 11/8/2017

To recieve fully credit please write legibly and clearly, show all required steps and calculations and answer any explicit questions in full sentences.

I. TRANSMISSION THROUGH A RECTANGULAR BARRIER [8]

An electron having total energy 2 eV is incident upon a rectangular barrier of height 4 eV. What is the transmission coefficient for this electron if the barrier has thickness

(a) 10^{-10} m [2]

(b) 9×10^{-9} m [2]

(c) 10^{-9}m [2]

(d) Comment on the difference between T values for (a), (b) and (c). Is the smallest value consistent with common quantum physics expectations? [2]

II. TWO-DIMENSIONAL INFINITE SQUARE WELL [13]

Consider a particle of mass m in a two-dimensional infinite square well potential having sides of length a and b given by

$$V(x) = \begin{cases} 0 & 0 < x < a \text{ and } 0 < \text{y} < \text{b} \\ \infty & \text{otherwise.} \end{cases}$$

(a) Assuming that $a = b$, write down the normalized wavefunctions and the corresponding energies. Note that these wavefunctions and energies can be parametrized by two quantum numbers, say, n_x and n_y. [2]

(b) Give the quantum numbers (n_x, n_y) for the first eight states that have the lowest energies. Do all of these states correspond to different energy values? Clearly state the degeneracy (how many states have the same energy) for each energy value. [8]

(c) Suppose that five electrons are trapped in this two-dimensional "box". Keeping in mind that no two electrons can occupy the same state i.e. be described by the same set of quantum numbers, what is the highest energy level that will be occupied in this case, while maintaining the lowest total energy of the electrons? *Disregard* any effects of electronic spin. [2]

(d) How would your answers to (b) and (c) change if a and b are *not* the same, though still of comprable value? Explain. [1]

III. ANGULAR MOMENTUM STATES [6]

Consider an angular wavefunction expressed in terms of spherical harmonics (for instance, see p. 183 in the text):

$$\psi(\theta, \phi) = \frac{1}{2} Y_{1,1} + \frac{i}{\sqrt{2}} Y_{1,0} - \frac{1}{2} Y_{1,-1}$$

(a) What is the probability that a measurement of the square of the total orbital angular momentum L^2 will yield $2\hbar^2$? [1]

(b) What is the probability that a measurement of the z-component of angular momentum L_z will yield $-\hbar$? [1]

(c) The operator corresponding to the z-component of angular momentum can be written as, in spherical polar coordinates,

$$L_z = -i\hbar \frac{\partial}{\partial \phi}.$$

Explicitly prove that $Y_{1,1}$ and $Y_{1,0}$ are eigenstates of this operator i.e using the explicit form of $Y_{1,1}$ and $Y_{1,0}$, in terms of θ and ϕ, show that $L_z Y_{1,1} = \lambda_1 Y_{1,1}$ and $L_z Y_{1,0} = \lambda_0 Y_{1,0}$ and determine the eigenvalues λ_1 and λ_0. [2]

(d) Sketch $|Y_{1,1}|^2$ and $|Y_{1,0}|^2$. *Hint: See Fig. 6.3 in the text.* [2]

IV. HYDROGEN WAVEFUNCTIONS [8]

Suppose an electron is in the $n = 2$, $\ell = 1$, $m_\ell = 1$ state of the hydrogen atom.

(a) Sketch the radial wavefunction and a polar plot of the angular dependence. [2] *Hint: See Fig. 6.3 in the text.*

(b) Find the value of $r = r_{\max}$ at which the radial probability density is a maximum. [2]

(c) If the electron makes a transition from the state given above to the $n = 1$, $\ell = 0$, $m_\ell = 0$ state it has to emit a photon. What is the energy of this photon? Would this photon be more or less energetic if the transition was from the $n = 2$, $\ell = 1$, $m_\ell = -1$ state to the $n = 1$, $\ell = 0$, $m_\ell = 0$ state instead? Explain. [2]

(d) Give the electronic configuration of sodium (Na), treating it as a hydrogen-like atom i.e. distribute its 11 electrons into states labelled by (n, l, m) – these are the $1s$, $2s$, $2p$ etc. orbitals. [2]
Note: The spin of an electron is an additional quantum number so that, for instance, (n,l,m,up) and (n,l,m,down) describe two distinct states.