1-Body Motion $\vec{f} = \dot{\vec{p}} = m\ddot{\vec{r}}$ $\quad \vec{p} = m\vec{v} = m\dot{\vec{r}}$

$\vec{l} \equiv \vec{r} \times \vec{p}$ $\quad \vec{\tau} \equiv \vec{r} \times \vec{f} = \dot{\vec{l}}$ $\quad \vec{f}_{\text{grav}} = -(GMm/r^2)\hat{r}$

$\vec{f}_{\text{air}} = -(bv + cv^2)\hat{v} = \vec{f}_{\text{lin}} + \vec{f}_{\text{quad}}$ $\quad$ Reynolds $R \equiv Dv\rho/\eta$

sphere, diameter D: $b = \beta D$ $\quad c = \gamma D^2$ $\quad R = 48\, f_{\text{quad}} / f_{\text{lin}}$

Rocket Motion $m\dot{v} = -\dot{m}v^{\text{ex}} + F^{\text{EXT}}$

Miscellaneous 1 m/s ≈ 2.2 mph

$f_{\text{friction}} \begin{cases} = \mu_{\text{kin}} N \\ \le \mu_{\text{static}} N \end{cases}$ $\quad \vec{f}_{\text{EM}} = q(\vec{E} + \vec{v} \times \vec{B})$

Collective Motion * assuming Newton's 3rd Law ➔ F^{INT} cancel

Notation for collective properties

- unsubscripted capital letter ➔ "TOTAL", except for …
- unsubscripted capital position, velocity, accel ➔ "OF THE CM"

◇ subscript ≠ coordinate index ➔ "OF"

◆ no superscript ➔ "RELATIVE TO ORIGIN"
◆ superscript () ➔ "RELATIVE TO (POINT)"
◆ superscript prime′ ➔ "RELATIVE TO THE CM"

CM: $M\vec{R} \equiv \sum_i m_i \vec{r}_i$ $\quad \vec{P} = M\dot{\vec{R}}$ $\quad \vec{L}_{CM} = \vec{R} \times \vec{P}$

Rotating Body: for any BODY-FIXED vector $\vec{B}$, $\dot{\vec{B}} = \vec{\omega} \times \vec{B}$

Moment of Inertia: for any BODY-FIXED point B,

$I_{\hat{\omega}}^{(B)} \equiv \sum m_i \left| \vec{r}_i^{\,(B)} \times \hat{\omega} \right|^2$ $\quad L_{\omega}^{(B)} = I_{\hat{\omega}}^{(B)} \omega$ $\quad T^{(B)} = \tfrac{1}{2} I_{\hat{\omega}}^{(B)} \omega^2$

EOM in Inertial Frames

$\vec{F}^{\text{EXT}} = \dot{\vec{P}}$

$\vec{\tau}^{\text{EXT},(A)} = \dot{\vec{L}}^{(A)}$ if reference point A

- is the CM, or
- is not accelerating, or
- $\ddot{\vec{r}}_A \parallel (\vec{R} - \vec{r}_A)$

$T + U^{\text{EXT}} + U^{\text{INT}}$ = conserved for conservative forces

Decompositions

$\vec{P} = \vec{P}_{CM}$ $\quad I_{\hat{\omega}}^{(B)} = I_{\text{CM},\hat{\omega}}^{(B)} + I'_{\hat{\omega}}$

$\vec{L} = \vec{L}_{\text{CM}} + \vec{L}'$ $\quad \vec{\tau}_{CM}^{\text{EXT}} = \dot{\vec{L}}_{CM}$

$T = T_{\text{CM}} + T'$ $\quad T = T^{(\text{stationary point})}$

Uniform Gravity: If $\vec{f}^{\text{EXT}} = m\vec{g}$ only → $\vec{F}^{\text{EXT}} = M\vec{g}$ $\quad \vec{\tau}^{\text{EXT}} = \vec{R} \times M\vec{g}$ $\quad \vec{\tau}'^{\,\text{EXT}} = 0$ $\quad U^{\text{EXT}} = MgH$

Variational Calc / Mech * Gen. coord q_i must be indep

$S \equiv \int_{t_1,\vec{q}_1}^{t_1,\vec{q}_1} dt\, L(q_i, \dot{q}_i, t)$ $\quad \delta S = 0 \to \dfrac{\partial L}{\partial q_i} = \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{q}_i}\right)$ for each q_i

$H \equiv \sum_i \dot{q}_i \dfrac{\partial L}{\partial \dot{q}_i} - L$ $\quad \dfrac{dH}{dt} = -\dfrac{\partial L}{\partial t}$

Mechanics Principle of Least Action :

$L = T - U \;\to\; \delta S = 0$ @ true $\{q_i(t)\}$

Gen. force $Q_i \equiv \dfrac{\partial L}{\partial q_i}$, momentum $p_i \equiv \dfrac{\partial L}{\partial \dot{q}_i}$

H equals T+U when ∃ no t-dep constraints

Accel Frames $\vec{f} \equiv \vec{F}/m \;\to\; \vec{f}_{\text{lin}}^{\,*} = -\vec{A}_0$ $\quad \vec{f}_{\text{cf}}^{\,*} = \left(\vec{\Omega} \times \vec{r}^{\,*}\right) \times \vec{\Omega} = \Omega^2 s^* \hat{s}^*$ $\quad \vec{f}_{\text{Cor}}^{\,*} = 2\vec{v}^{\,*} \times \vec{\Omega}$ $\quad \vec{f}_{\text{azim}}^{\,*} = \vec{r}^{\,*} \times \dot{\vec{\Omega}}$

For $\vec{B}$ constant in S* frame: $\left.\dfrac{d\vec{B}}{dt}\right|_{\substack{\text{S}\\\text{frame}}} = \vec{\Omega} \times \vec{B}$

For general vector $\vec{b}$: $\left.\dfrac{d\vec{b}}{dt}\right|_{\substack{\text{S}\\\text{frame}}} = \vec{\Omega} \times \vec{b} + \left.\dfrac{d\vec{b}}{dt}\right|_{\substack{\text{S*}\\\text{frame}}}$

Damped / Driven Linear Oscillators

EOM : $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f(t)$ where $\vec{F}_{\text{restore}} = -k\vec{x} = -m\omega_0^{\,2}\vec{x}$, $\quad \vec{F}_{\text{damp}} = -b\vec{v} = -m(2\beta)\vec{v}$, $\quad \vec{F}_{\text{drive}} = m f(t)$

Resonance: Quality factor $Q = \dfrac{\omega_{\text{peak}}}{\Delta\omega_{\text{FWHM}}}$ where $A^2\left(\omega_{\text{peak}} \pm \dfrac{\Delta\omega_{\text{FWHM}}}{2}\right) = \dfrac{1}{2} A^2(\omega_{\text{peak}})$ and $A \equiv$ amplitude of $x(t)$

$Q = 2\pi \dfrac{E_{\text{stored}}}{E_{\text{dissipated / cycle}}}$ where $E_{\text{stored}} = T + U$ for mechanical oscillator

For weak damping $\beta \ll \omega_0$: $\quad \omega_{\text{peak}} \approx \omega_0$ $\quad A_{\text{peak}} \approx \dfrac{f_0}{2\beta\omega_0}$ $\quad Q \approx \dfrac{\omega_0}{2\beta}$