Phys 487 Discussion 12 — Higher-Order Selection Rules

Problem 1 : E2 Radiation

In writing the electric field $\vec{E}$ that an atom is exposed to as (Griffiths Equation 9.31)

$$\vec{E}(t) = E_0 \cos\omega t\ \hat{k}$$

it is assumed that the atom is so small (in comparison to the wavelength of light) that spatial variations in the field can be safely ignored. The *true* electric field would be

$$\vec{E}(\vec{r},t) = \vec{E}_0 \cos\left(\vec{k}\bullet\vec{r} - \omega t\right)$$

If the atom is centered at the origin, then $\vec{k}\bullet\vec{r} \ll 1$ over the relevant volume ($\left|\vec{k}\right| = 2\pi/\lambda$, so $\vec{k}\bullet\vec{r} \sim r/\lambda \ll 1$), and that's why we could afford to drop the term. Suppose, however, that we keep the first-order correction:

$$\vec{E}(\vec{r},t) = \vec{E}_0\left[\cos\omega t + \left(\vec{k}\bullet\vec{r}\right)\sin\omega t\right]$$

The first term gives rise to the *allowed* (*electric dipole*) transitions we considered in the text; the second leads to the so-called *forbidden* (*magnetic dipole* and *electric quadrupole*) transitions (higher powers of $\vec{k}\bullet\vec{r}$ lead to even *more* "forbidden" transitions associated with high multipole moments).

(a) Show that the spontaneous emission rate for forbidden transitions (don't bother to average over polarization and propogation directions, though this should really should be done to complete the calculation) is given by

$$R_{a\to b} = \frac{q^2\omega^5}{\pi\varepsilon_0\hbar c^5}\left|\langle a|\left(\hat{n}\bullet\vec{r}\right)\left(\hat{k}\bullet\vec{r}\right)|b\rangle\right|^2$$

(b) Show that for a one-dimensional oscillator, the forbidden transitions go from level n to level $n-2$ and that the transition rate (suitably averaged over $\hat{n}$ and $\hat{k}$) is

$$R = \frac{\hbar q^2\omega^3 n(n-1)}{15\pi\varepsilon_0 m^2 c^5}$$

[*Note*: Here ω is the frequency of the *photon*, not the oscillator.]

Find the *ratio* of the "forbidden" to the "allowed" rate and comment on the terminology.

(c) Show that the $2S \to 1S$ transition in hydrogen is not possible even by a "forbidden" transition.
[*Aside*: As it turns out, this is true for all the higher multipoles as well; the dominant decay is in fact by *two-photon* emission, and the lifetime is about a tenth of a second.]