Discussion 14

Problem 1

1. In this exercise we recast the scattering problem in terms of the flux

$$\vec{j} = \frac{-i\hbar}{2m}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right)$$

which we have used in cases where non-normalizable plane wave solutions are convenient.

With an incident plane wave and a scattered spherical wave, we have written the full solution as

$$\psi_{tot} = e^{i\vec{k}\cdot\vec{r}} + f(\theta)\frac{e^{ikr}}{r}$$

where the first term is the incident wave and the second is the scattered wave.

a) Show that the cross-terms between the incident and scattered wave functions in $\vec{j}$ all contain a term $e^{\pm ikr(1-\cos\theta)}$. As $r \to \infty$ (where our detector is located), these terms all average to zero and can be ignored.

b) After dropping the exponential terms in a), show that, up to terms of order $1/r^2$,

$$\vec{j} = \frac{\hbar\vec{k}}{m} + \frac{\hbar k}{m}\frac{|f(\theta)|^2}{r^2}\hat{r}$$

where the first term is the incident flux and the second term is the scattered flux and $\hat{r}$ points towards the detector.

c) The number scattered into a small detector of area $d\vec{A}$ per unit time is

$$\vec{j}_{sc}\cdot d\vec{A} = \vec{j}_{sc}\cdot\hat{r}r^2 d\Omega$$

Where $\vec{j}_{sc}$ is the scattered flux.

The differential cross section $d\sigma/d\Omega$ is the ratio of the number incident per unit time per unit solid angle to the number scattered per unit time per unit solid angle. Show that

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$