Physics 487 – Homework #10 **due Monday, Nov 11 @ 12:29 pm**

The first two principles we discussed concerning the solution of time-dependent Hamiltonians were the **sudden approximation** and the **adiabatic approximation**. These are simple to apply, but we have yet to do any problems with them, so let's get to it now! :-)

Before you do anything else, please do a 2 minute review as we covered these techniques a while ago. They were presented in Lecture 9B, starting with blackboard 9B-3. Board 9B-4 describes the sudden approximation, which is all you need for this homework, and 9B-5 describes the adiabatic approximation. These simplest-of-all $H(t)$ techniques require only one blackboard each as there are no formulae, just concepts[1]. (The sudden and adiabatic approximations appeared again at 10A-2 and 10A-3, to construct an example.)

Problem 1 : A 1D Infinite Well ™ of Changing Size

A particle of mass m is contained in a 1D Infinite Well ™ that extends from $x = -L/2$ to $x = +L/2$. At times $t \le 0$, the particle is in its ground state.

(a) Write down the eigenfunctions of the ground state and the first excited state.

(b) At time $t = 0$, the walls of the box are moved outward instantaneously to form a well that extends from $x = -L$ to $x = +L$. Calculate the probability that the particle will still be in the ground state at time $t = 0+$.

NOTE: This is equivalent to asking "what is the probability that the particle will be in the ground state at times $t > 0$", i.e if the particle stays in the ground state, it will remain in the ground state after the well expands. If it is not clear to you why that is, ask!

(c) Calculate the probability that the particle will instead be in the first excited state at $t = 0+$.

Problem 2 : A 1D SHO with a Suddenly Applied Electric Field

A particle of mass m experiences a simple-harmonic potential in one dimension, so the particle's Hamiltonian is

$$H_0 = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} \,.$$

(a) You are told that the form of the ground state wavefunction is $\psi_0(x) = N\, e^{-\alpha^2 x^2/2}$.
Calculate the constants N and α **WITHOUT using the 1D SHO reference section of our formula sheets.** (A bit of a refresher on how to do basic things! :-))

(b) What is the energy of the ground state? Again please derive the result **WITHOUT using the 1D SHO reference section of our formula sheets.**

(c) At time $t = 0$, a constant, uniform electric field is switched on, adding this new term to the Hamiltonian:

$$H' = eEx$$

where E is a constant (the magnitude of the electric field). Despite the notation "H_0" and "H'" from perturbation theory, **you may NOT assume that the perturbing electric potential is small compared to the harmonic-oscillator potential!** Calculate the exact ground state energy of the new hamiltonian $H_0 + H'$.
▶ HINT: Complete the square.

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[1] Of course one can *extend* both approximations by developing formulae for higher-order *corrections* to them, but that is beyond the scope of what we are covering at the moment.

(d) Assuming that the field is switched on instantaneously, what is the probability that the particle stays in the ground state?

(e) Obviously one cannot turn on anything "instantaneously", that is just code for "fast enough that we can use the sudden approximation". Well, how fast is fast? Fill in the following sentence with an order-of-magnitude quantity (i.e. we don't care about factors of 2 or 5 or whatever) :

> "In order to use the sudden approximation in part (d), the electric field must be switched on over a time interval that is much shorter than _________".

▶ HINT: Lecture 9B-3.