Physics 487 – Homework #10

due Friday Nov 6 (ideal) – Wed Nov 11 @ midnight (cutoff)

The first two principles we discussed concerning the solution of time-dependent Hamiltonians were the **sudden approximation** and the **adiabatic approximation**. These are simple to apply, but we have yet to do any problems with them, and they are important.

Please do a 2 minute review first as we covered these techniques a while ago: They were presented in Lecture 7B, right before we embarked on time-dependent perturbation theory. For these simplest-of-all $H(t)$ techniques we did not develop any formulae, just concepts[1].

Problem 1 : A 3D Infinite Well ™ of Changing Size *Qual Problem*

A particle of mass M bounces elastically between two infinite plane walls separated by a distance D. The particle is in its lowest possible energy state.

(a) What is the energy of this state?

(b) The separation between the walls is slowly (i.e., adiabatically) increased to 2D.

(i) How does the expectation value of the energy change?

(ii) Compare this energy change with the result obtained classically from the mean force exerted on a wall by the bouncing ball?

(c) Now assume that the separation between the walls is increased rapidly, with one wall moving at a speed >> $(E/M)^{1/2}$. Classically, there is no change in the particle's energy since the wall is moving faster than the particle and cannot be struck by the particle while the wall is moving.

(i) What happens to the expectation value of the energy quantum-mechanically?

(ii) Compute the probability that the particle is left in its lowest possible energy state.

Problem 2 : A 1D SHO with a Suddenly Applied Electric Field *Qual Problem*

A particle of mass m experiences a simple-harmonic potential in one dimension, so the particle's Hamiltonian is

$$H_0 = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} .$$

(a) You are told that the form of the ground state wavefunction is $\psi_0(x) = Ne^{-\alpha^2 x^2/2}$.
Calculate the constants N and α **WITHOUT using the 1D SHO reference section of our formula sheets.** (A little bit of a review/memory-refresher on basic things.)

(b) What is the energy of the ground state? Again please derive the result **WITHOUT using the 1D SHO reference section of our formula sheets.**

(c) At time $t = 0$, a constant, uniform electric field is switched on, adding this new term to the Hamiltonian:

$$H' = eEx$$

[1] Of course one can *extend* both approximations by developing formulae for higher-order *corrections* to them, but that is beyond the scope of what we covered.

where E is a constant (the magnitude of the electric field). Despite the notation "H_0" and "H'" from perturbation theory, **you may NOT assume that the perturbing electric potential is small compared to the harmonic-oscillator potential!** Calculate the exact ground state energy of the new hamiltonian $H_0 + H'$.
▶ HINT: Complete the square.

(d) Assuming that the field is switched on instantaneously, what is the probability that the particle stays in the ground state?

(e) Obviously one cannot turn on anything "instantaneously", that is just code for "fast enough that we can use the sudden approximation". Well, how fast is fast? Fill in the following sentence with an order-of-magnitude quantity (i.e. we don't care about factors of 2 or 5 or whatever) :

> "In order to use the sudden approximation in part (d), the electric field must be switched on over a time interval that is much shorter than _________".

▶ HINT: Lecture 7B.