Making Estimates in Research and Elsewhere

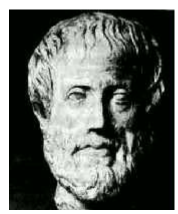
Lance Cooper, Nadya Mason

Q: How many cows can you fit in a barn?

A: *assume* spherical cows of radius ~R
&
assume rectangular barn w/ volume ~ LxW

$N \sim (L \times W)/\pi R^2$

Making Estimates in Research: Why?



It is the mark of an instructed mind to rest assured with that degree of precision that the nature of the subject admits, and not to seek exactness when only an approximation of the truth is possible.

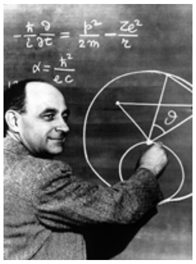
- Aristotle

Why make estimates in science?

- To provide a rough check of more exact calculations
- To provide a rough check of research results or hypotheses
- To obtain estimates of quantities when other resources aren't available
- To obtain estimates of quantities that are difficult to measure precisely
- To obtain estimates of quantities for which no firm theoretical prediction exists
  - ⇒ particularly important in interdisciplinary sciences, soft matter, astrophysics
- To provide bounds for possible design alternatives

The ability to estimate – to within an order of magnitude or so – the size or probability of various quantities is useful in science as well as in many other endeavors.

Making Estimates in Research: How?



Enrico Fermi

How do you estimate the answer to a question that appears impossible to determine at all, or at least without access to an encyclopedia, internet connection, or omniscient being?

e.g., how many grains of sand are there on earth's beaches?
how many piano tuners are there in Chicago?
how many atoms are in your body?

These problems are sometimes referred to as Fermi problems, after the physicist Enrico Fermi, who was famous for (among other things) posing and solving such problems.

Getting started:

(1). Don't panic when you see the problem

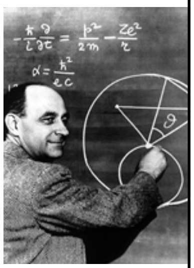
(2). Write down any fact you *do* know related to the question

(3). Outline one or more possible procedures for determining the answer

(4). List the things you'll need to know to answer the question

(5). Keep track of your assumptions

Making Estimates in Research: How?



Other general guidelines for making order-of-magnitude estimates:

Make everything as simple as possible!

(1). Don't worry about specific values: round numbers to "convenient values"
e.g., $\pi \approx 3$; 8.4 $\approx$10; etc.

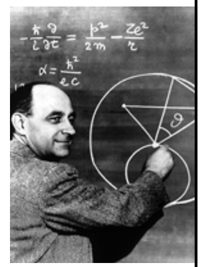
(2). Choose convenient geometries when modeling
e.g., a spherical cow, a cubic grain of sand, etc.

(3). Make "educated" guesses or even upper and lower bounds of quantities you don't know.
try to make good guesses, and keep track of these guesses, as they will set bounds on the fidelity of your estimate

(4). Use ratios when possible – by comparing the value of one quantity (e.g., force, energy, etc.) in comparison to a related quantity – in order to eliminate unknown parameters and get a dimensionless parameter

(5). If possible, exploit plausible scaling behavior of some quantity, i.e., estimate an unknown quantity by assuming it scales linearly - from known values – with some parameter

Making Estimates in Research: How?



More guidelines for making order-of-magnitude estimations:

Checking your estimates:

(1). Make sure that your estimates and calculations are dimensionally correct! ⇒ This is a very powerful tool!

(2). Check the plausibility of your estimate, if possible
e.g., if your answer exceeds the speed of light or the size of the universe, you've got a problem!

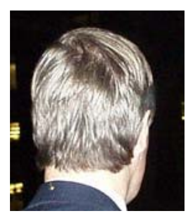
(3). Check the plausibility of your estimate using an alternate calculation method
do the two methods agree to within an order of magnitude?

(4). Perform a "reality check" on your estimate based on the number and size of the approximations you made

(5). More quantitatively - place "bounds" on your estimate:
To obtain an "upper bound" – in equations, put largest estimated values of quantities in the numerator and the smallest estimated values in the denominator

To obtain a "lower bound" – in equations, put smallest estimated values of quantities in the numerator and the largest estimated values in the denominator

Making Estimates: Hairs on a Human Head



How many hairs on a human head?

(1). What do we need to know? Size of a typical scalp, and approximate number of hairs per square inch

Hair diameter: 0.1 - 0.2mm → 20 – 40 per inch (guess or measure) ⇒ 400 – 1600 per inch^2

(2). Do we need a model? We don't want a precise, specific answer, so assume "average" head that is a hemisphere (keep it simple!)

Radius of "typical" head: ~ 5 inch (guess or measure)
⇒ area of scalp ~ $\frac{1}{2}(4\pi r^2)$ ~ 150 inch^2
(you can take π ~ 3! Excellent!)

(3). Make your estimate!

Note dimensional consistency!

\# of hairs ~ area of scalp * hairs/unit area ~ 60,000 – 240,000 hairs

OR N ~ 10^5 hairs

Rounding up is OK, even encouraged! ⇒

(4). Now, check: How good were our approximations?

Chemistry Estimates: Atoms in a Grain

How many atoms in a grain of sand?

(1). What do we need to know? Size of a grain of sand, size of an atom

Dimension of a grain of sand: assume L_{sand} ~ 1 mm = 10^{-3} m × 10^{10} Å/m = 10^7 Å

Dimension of an atom: assume L_{atom} ~ 1 Å

(2). Do we need a model? Don't worry about geometry…assume a cubic grain of sand!

Volume of grain of sand: V_{sand} ~ $(10^7)^3$ Å^3 ~ 10^{21} Å^3

Volume of an atom: V_{atom} ~ $(1)^3$ Å^3 ~ 1 Å^3

(3). Make your estimate!

# of atoms in 1 grain ~ (volume of 1 grain)/(volume of 1 atom) ~ 10^{21} atoms per grain

(4). Now, check: How good were our approximations?

Close to N_A… so this is reasonable

"Musical" Estimates: Piano Tuners in CU

How many piano tuners are there in Champaign?

(Similar to an original Fermi problem!)

"Let me through - I'm a piano tuner!"

(1). What do we need to know? How many families are there in Champaign? How many families own a piano? How often are pianos tuned? How many tuners are needed?

# of people in CU: Estimate 150,000

# of families in CU: Estimate 150,000/4 ~ 38,000

# of families owning a piano: Estimate 1 in 10 ⇒ ~ 4,000 pianos in Champaign

rate at which pianos are tuned: Estimate 1 time each year ⇒ ~ 4,000 tunings/year

rate at which piano tuners can perform a tuning: Estimate ~ 4 tunings/day, working ~ 200 days/year (exclude weekends and holidays) ⇒ ~ 800 tunings/year per tuner

(2). Make your estimate!

# of tuners ~ (rate at which piano tunings are needed)/(rate at which each tuner performs tunings)

⇒ (4000 tunings/year)/(800 tunings/year*tuner) ~ 5 tuners (± 3 tuners)

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PICTURE FRAMES–DEALERS

From C/U Yellow Pages…

Estimate: 2-8
Actual: 7

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Culinary Estimates: Supersize Me

How many McDonalds franchises are there in the US?

(1). What do we know? # of McDonalds in Champaign, population of Champaign, population of US

# of McDonalds in Champaign: estimate $N_{champaign} \sim 7$

population of Champaign: ~ 150,000 ~ 1.5exp5

population of US: ~ 300,000,000 ~ 3exp8

⇒ Reasonably assume that # of McDonalds franchises scales with population!

(2). Make your estimate!

Assume simple scaling relationship

N_{USA} ~ (population of US/population of Champaign)*(# of McDonalds in Champaign)

$N_{USA} \sim 14{,}000$

Actual: 14,000

Technological Estimates: Storage Capacity of CDs

Introduction: CDs store information as a high density of "pits" or bumps in an aluminum layer

(a) How many "pits" in a typical CD, and what is their spacing?

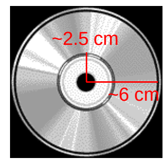
(1). What do we need to know for (a)? Active area of CD, storage capacity of CD

Consider 700 Mbyte CD $\Rightarrow$ (8 bits = 1 byte) 700 Mbytes = 5.6×10^9 bits! $\Leftarrow$ # of pits!

To get the pit spacing, we need to estimate the active area on the CD:

$A_{CD} = \pi(0.06)^2 - \pi(0.025)^2$ m^2 (guess or measure)

$A_{CD} \sim 9 \times 10^{-3}$ m^2



(2). Make your estimate!

Note dimensional consistency!

Area associated w/ each bit, $A_{bit} = A_{CD}/(\text{\# of bits}) \sim 1.6 \times 10^{-12}$ m^2/bit

$\Rightarrow$ Separation between each bit, $d \sim (A_{bit})^{1/2} \sim 1.3 \times 10^{-6}$ m = 1.3 microns!

λ=0.65 microns = 650 nm

Blu-ray Disc™

The benefit of using a blue-violet laser (405nm) is that it has a shorter wavelength than a red laser (650nm), which makes it possible to focus the laser spot with even greater precision. This allows data to be packed more tightly and stored in less space, so it's possible to fit more data on the disc even though it's the same size as a CD/DVD. This together with the change of numerical aperture to 0.85 is what enables Blu-ray Discs to hold 25GB/50GB.

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Other resources on making estimates

A View From the Back of the Envelope
http://www.vendian.org/envelope/

University of Maryland Fermi Problems Site
http://www.physics.umd.edu/perg/fermi/fermi.htm

Old Dominion University Fermi Problems Site
http://www.physics.odu.edu/~weinstei/wag.html

Order of Magnitude Astrophysics
http://www.astronomy.ohio-state.edu/~dhw/Oom/questions.html

Back-of-the-Envelope Physics, Clifford Swartz (Baltimore, Johns Hopkins University Press, 2003).

The Back of the Envelope, E.M. Purcell, monthly column in the *American Journal of Physics,* July 1984 – Jan. 1993.

Consider a Spherical Cow : A Course in Environmental Problem Solving, John Harte (Berkeley, University Science Books, 1988).

Powers of Ten : About the Relative Size of Things in the Universe, and the Effect of Adding Another Zero, Philip Morrison and Phylis Morrison (Scientific American Library, 1982, 1994).

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In-Class Activity Questions*

(1). How much money did American consumers pay to fill up their cars with gas last year?

(2). How many nails are needed to make sleeping on a bed of nails safe (but still crazy!)?

(3). How much kinetic energy does the earth have due to its rotation about the sun? How much rotational kinetic energy does the earth have?

(4). Legend has it that water in Southern Hemisphere washbasins drains in the opposite sense to that of water draining in Northern Hemisphere washbasins, i.e., that the Coriolis force, $|F_c| \sim 2m(\omega \times v)$ (where ω is the rotational frequency of the earth, v is the radial speed of the water, and m is the mass of the water), governs the behavior of draining water. Is this legend reasonable? (Hint: Compare your estimate of $|F_c|$ with another force in the problem, i.e., the gravitational force $F_{grav} = mg$.)

(5). How fast does human hair grows (on average) in mph?

(6). How much sand is there on all the beaches of the earth?

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In-Class Activity Questions*

(7). Estimate the surface area and volume of a typical human.

(8). Estimate the number of cells in the human body.

(9). Estimate the weight of earth's atmosphere. [Hint: Atmospheric pressure at sea level is ~ 14.7 psi]

(10). In a letter to the Royal Society in 1774, Ben Franklin reported that the equivalent of 0.1 cm^3 of oil dropped on a lake spread to a maximum area of 40 m^2. What is the thickness of the oil and does this make sense?

(11). Estimate the number of leaves on a tree.

(12). Estimate the weight of Mt. Everest [height ~ 29,000 ft].

*Example solutions are on the following slides

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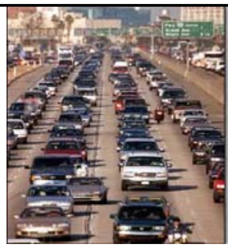
Environmental Estimates: Gas Guzzlers



How many gallons of gas are consumed by automobiles in the US each year?

(1). What do we know? Roughly how many gallons a typical automobile consumes

Gas capacity of 'typical' car: ~ 12 gallons/tank

Gas consumption rate of 'typical' car: ~ 1 filling per week ⇒ 12 gallons/week

Yearly gas consumption of 'typical' car: (12 gallons/wk)*(52 weeks) ~ 600 gallons/year

Now, we need to estimate the # of automobiles in the US:

Population of the US: ~ 250,000,000

How many cars? Assume 1 in 3 people own a car in US ⇒ ~ 80,000,000 cars in US

(2). Make your estimate of total yearly gas consumption!

Yearly total gas consumption in US: ~ (8×10^7 cars)*(6×10^2 gallons/car) ~ 5×10^{10} gallons

Assume gas ~ $2.50/gallon ⇒ Total price paid for gas in US last year ~ $100 billion

Note, that there are roughly 40 gallons of gas per barrel of oil, so this amounts to ~ 10^9, or 1 billion barrels of oil/year from cars alone (about 1/7 of total oil consumption in the US)!

Orthopedic Estimates: Bed of nails?

Introduction: Some people like their mattresses very stiff!

How many nails are needed to make sleeping on a bed of nails safe (but still crazy!)?

(1). What do we need to know? 'Typical' mass of a person, amount of force for each nail that can be comfortably sustained.

Typical mass ~ 80 kg (~ 175 lbs)   Typical weight $\Rightarrow$ 800 N

Now, how much force from a nail can you comfortably bear? You can try to measure this with a force meter…

As another estimate, your finger can certainly support the weight of a nail (maybe 10 grams or so) without discomfort, so choose this as your pain threshold $\Rightarrow$ Force/nail ~ 0.01 kg/nail * 10 m/s^2 ~ 0.1 N/nail

(2). Make your estimate!

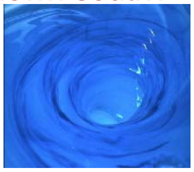
Note dimensional consistency!

# of nails needed ~ typical weight/(Force/nail) ~ 8000 nails needed

How about the spacing? Estimate 'typical' contact area: A_{con} ~ 200 cm x 30 cm ~ 6000 cm^2

Area/nail = A_{con}/(# of nails) = 6000 cm^2/8000 nails ~ 1 cm^2/nail

Physics Estimates: How does water drain down south?



Introduction: Legend has it that water in southern hemisphere washbasins drains in the opposite sense to water draining in northern hemisphere washbasins

Is this legend reasonable?

(1). What do we need to know? Coriolis force acting on draining water, rotational frequency of the earth

Magnitude of Coriolis force: $|F_c| \sim 2\,m\,(\omega \times v)$, where ω is the earth's rotational frequency, m is the mass of the water, and v is the radial speed of the water

(2). Estimate parameters: $\omega = 2\pi$ radians/(24 hours x 3600 sec/hr) ~ 7 x 10^{-5} rad/sec

v ~ 1 m/sec (just guess!)

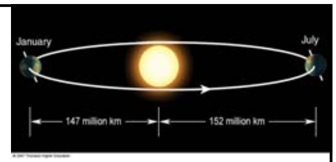
(3). Make your estimate!

Rather than estimate the mass of the water, it is better to compare the Coriolis force on the water to the gravitational force acting on the water:

$|F_c|/mg = 2\omega v/g \sim 2(7 \times 10^{-5}\ \text{rad})(1\ \text{m/sec})/(10\text{m/sec}^2) \sim 10^{-5}$

$\Rightarrow$ The Coriolis force is weaker than the gravitational force by a factor of roughly 10^5! The legend is wrong…the Coriolis force doesn't determine sense of rotation!

Physics Estimates: Earth's Kinetic Energy



What is the earth's rotational kinetic energy? What is the earth's kinetic energy due to its rotational around the sun?

Let's start with the earth's rotational kinetic energy:

What do we need to know? Rotational kinetic energy = $\frac{1}{2}I\omega^2$.

$\omega = 2\pi f = 2\pi$*(1rev/day)*(1day/24 hrs)*(1hr/3600 s) = 7.27×10^{-5} rad/s

$I_{sphere} = (2/5)M_{earth} R^2$, where R = 6371 km. To get M_{earth}, notice that $F = GM_{earth}*m/R^2$ and $F = mg$. So $GM_{earth}/R^2 = g$ and $M_{earth} = gR^2/G = 9.8 * (6371 \times 10^3)^2/(6.67 \times 10^{-11}) = 6 \times 10^{24}$ kg.

Rotational kinetic energy = $\frac{1}{2}I\omega^2 = (1/5)M_{earth}R^2\omega^2 = (1/5)(6 \times 10^{24})(6.37 \times 10^6)^2(7.3 \times 10^{-5})^2 = 2.6 \times 10^{29}$ J

How about the earth's orbital kinetic energy:

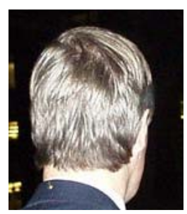
What do we need to know? Orbital kinetic energy = $\frac{1}{2}I\omega^2$, where $I = M_{earth}r^2$

$I = M_{earth}r^2 = (6 \times 10^{24})(1.5 \times 10^{11})^2 = 1.35 \times 10^{47}$ kg-m^2

$\omega = 2\pi f = 2\pi$*(1rev/year)*(1year/365days)*(1day/24hrs)*(1hr/3600s) = 2×10^{-7} rad/s

Orbital kinetic energy = $\frac{1}{2}I\omega^2 = \frac{1}{2}(1.35 \times 10^{47})*(2 \times 10^{-7})^2 = 2.7 \times 10^{33}$ J

Estimates for Barbers



How fast does human hair grow in mph?

What do we need to know? Rate of hair growth…to estimate this, use yourself as an example:

If hair is cut every n months (usually $n<2$) and the average amount cut off is x inches, then the hair growth rate is x/n inches per month.

Now, convert this to mph: x/n * 1/(5280*12 inches/mile)*1/(30*24 hours/month) mph ~ 10^{-8} (x/n) mph. If n=2 and x=1 then hair growth is approximately 10^{-8} mph.

Recreational Estimates: Sand on the Earth

How many grains of sand on the earth's beaches?

(1). What do we need to know? Volume of a grain of sand, total volume of beaches on earth (!)

Volume of grain of sand: $V_{grain} \sim (10^7)^3$ Å^3 $\sim 10^{21}$ Å^3 * $(10^{-10}$ m/Å$)^3 \sim 10^{-9}$ m^3

Dimensions of a typical beach: Just guess! width ~ 100 m; depth ~ 10 m

OK, that was pretty easy, but – to get a total volume of beaches – we now need to estimate the total length of beaches on earth!

Take a look at a map…there is a lot of coastline! To estimate the total length of coastline, it is useful to compare the coastlines with another length scale you're more familiar with, i.e., the circumference of the earth!

Now the issue is a bit easier to visualize: how many times would all the coastlines wrap around the earth? Certainly more than once, but not dozens of times. Let's just assume for simplicity that they would wrap around the earth ~ 10 times: $\Rightarrow L_{beach} \sim 10*(2\pi R_{earth}) \sim 10*6*(6 \times 10^6)$ m $\sim 4 \times 10^8$ m

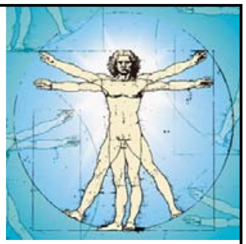
So, the volume of all beaches on earth: $V_{beach} \sim (100\text{ m})*(10\text{ m})*(4\times10^8\text{ m}) \sim 10^{11}$ m^3

(2). Estimate! How many grains of sand in this volume?

On order of # of atoms/grain!

# of grains on earth ~ (volume of beaches)/(volume of 1 grain) ~ 10^{20} grains on earth

Anatomical Estimates



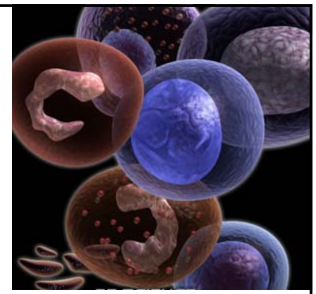
Estimate the surface area and volume of the human body.

What do we need to know? Typical dimensions of humans. Let's make this easy on ourselves by approximating humans as cylinders of radius r and height h:

If r ~ 0.5 ft and h~6 ft, then the volume $V=\pi*r^2*h \sim 4.5$ ft^3 and the surface area is $S=2*\pi*r*h \sim 20$ ft^2. Since 1 ft ~ 0.3 m, V ~ 0.1 m^3 and S ~ 2 m^2.

Another approach: humans have a density close to water, or 1 gm/cm^3. So one kg occupies 1000 cm^3 or 1 liter. A person weighing 170 lbs (77 kg) has a volume of 77 liters or ~0.08 m^3.

Anatomical Estimates

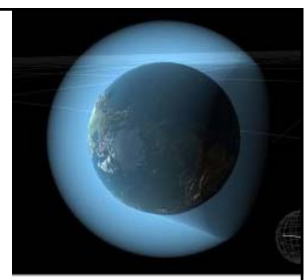


Estimate the number of cells in the human body

What do we need to know? Typical volume of human body and typical volume of cells.

Assume average cell diameter of 10 microns or 10^{-5} m, and use result from previous estimate ($V_{body} \sim 0.1\ m^3$), to get $N \sim V_{body}/V_{cell} \sim 0.1\ m^3/(10^{-5})^3 \sim 10^{14}$ cells in the human body.

Earth Science Estimates

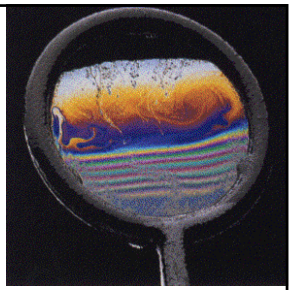


Estimate the weight of the earth's atmosphere

What do we need to know? Atmospheric pressure at sea level [~ 14.7 lbs/in^2] and surface area of earth.

Note that atmospheric pressure at sea level is ~ 14.7 lb/in^2. So, all we need to do is calculate the surface area of the earth in square inches, because each square inch supports a column of air weighing 14.8 lbs. Assume a smooth sphere of radius 3960 miles. $W = 4*\pi*(3960*5280*12)^2 *14.7 \sim 1.2 \times 10^{19}$ lbs.

More environmental estimates:



In a letter to the Royal Society in 1774, Ben Franklin reported that the equivalent of ~ 0.1 cm^3 of oil dropped on a lake spread to a maximum area of 40 m^2. Estimate the thickness of oil this would make. Does this estimate make sense?

What do we need to know? If d is the thickness of the layer in meters, then $V_{drop} = V_{slick}$. So, $40*d = 10^{-7}\ m^3$. So, $d = 25 \times 10^{-10}$ m or 25 angstroms. This is the thickness of several monolayers of oil molecules, so it makes sense. Notice the dark region in the picture above arises from destructive interference of the light between front and back surfaces of the oil film, indicating the thickness t in this region is substantially smaller than the wavelength of light, $t << \lambda$.

Horticultural Estimates

Estimate the number of leaves in the canopy of a tree.

What do we need to know? We need a simple model of a tree's canopy. Let's treat it as a perfect sphere. Then, if r is the typical radius of the tree's canopy, the surface area of the canopy is $4\pi r^2$. Then, if d is the leaf dimension (treat the leaf as a square), then the number of leaves is $N \sim 4\pi r^2/d^2$.

For a small tree, r ~ 4 ft and d ~ 1 in, so $N \sim 4\pi(48\text{ in})^2/(1\text{ in})^2 \sim$ 30,000 leaves

Geological Estimates

Estimate the weight of Mt. Everest [height ~ 29,000 ft)

What do we need to know? We need a simple model of a mountain. First, let's approximate it as a right circular cone [$V = (\pi/3)r^2h$]. Next, let's assume Everest is composed completely of granite [$\rho \sim 2.7$ gm/cm^3]. Finally, let's assume Everest base has a diameter approximately equal to its height, i.e., $r \sim h/2$.

Approximate volume of Everest ~ $(1/3)*\pi*(14500)^2*29000$ ft^3 ~ 6.4×10^{12} ft^3.

2.7 gm/cm^3 * (2.54 cm)^3/(1 in)^3 * (12 in)^3/(1 ft)^3 * 10^{-3} kg/gm = 76.5 kg/ft^3. So, Everest has an estimated mass of (6.4×10^{12} ft^3)*(77 kg/ft^3) ~ 5×10^{14} kg.

Now, 1 kg = 0.069 slugs, so 1 kg has a weight of 0.069*32ft/s^2 = 2.2 lbs. So, our estimated weight of Everest is (5×10^{14} kg) * 2.2 = 11×10^{14} lbs.