M. Stone, 2024

Syllabus for Physics 508: Mathematical Physics I

Course Title: CRN 52340; Physics 508: Mathematical Physics I

Instructor F2024, Professor Michael Stone

Course description: A graduate level course on the basic mathematics used in physics research. The course covers four related areas:

1) Calculus of Variations. Equations of mathematical physics as variational problems, conservation laws, Lagrange multipliers, origin of eigenproblems, variational approximation schemes.
2) Ordinary differential equations. Linear equations: Solution space, linear independence, Wronskians, normal forms. Eigenvalue problems: importance of boundary conditions, formal and true self-adjointness, completeness of eigenfunctions, Fourier series, continuous spectra and Fourier integrals. Green Functions: Range-nullspace theorem, Fredholm alternative, constructing Green functions via jump conditions.
3) Partial Differential equations. Classification of PDE's. Hyperbolic equations: wave equation, method of characteristics, shocks and weak solutions. Heat equation: solution by integral transforms. Elliptic equations: Dirichlet and Neumann problems, Poisson's equation, Legendre functions, spherical harmonics, Bessel and spherical Bessel functions, examples from electrostatics.
4) Integral Equations. Type I and type II Fredholm and Volterra equations, solution via Fourier and Laplace transforms, Abel's equation. Separable Kernels: compact and Hilbert-Schmidt operators, Fredholm alternative again. Perturbation methods: Neumann and Fredholm series.

Prerequisites and Co-requisites: MATH 285.

Credit Hours: N/A

Current course website: https://courses.physics.illinois.edu/phys508/fa2024/
This site includes Includes all homeworks, example exams, and links to other sites of interest.

Grade assignment: Grades will be determined from your total scores weighted as follows: Homework 50%, Midterm exam 20%, Final Exam 30%.

Texts and Supplies: Recommended but not required: Michael Stone and Paul Goldbart *Mathematics for Physics: A guided tour for graduate students*, (Cambridge University Press 2009). (A draft version of the book is linked to on the course website.)

Weekly Schedule: This guide is *approximate* because the actual progress through the course material will depend on questions and discussions that arise during the class-time.

Week1: Introduction to the calculus of variations: what the calculus of variations is good for; calculus of variations with one dependent and one independent variable; variations with fixed end-points; Euler's equation; readily integrable systems; conservation laws.

Week 2: Variations at boundaries: natural boundary conditions, example of flexible rod. Constraints: Lagrange multipliers; connection with eigenproblems;

Week 3: Hilbert and Banach spaces. Norms and Completeness. Fourier series and integrals.

Week 4: Ordinary Differential Equations: linear equations; dimension of solution space; linear independence; Wronskians; normal forms.

Week 5: Eigenvalue Problems: importance of boundary conditions; adjoint equation and self- adjointness; completeness of eigenfunctions for self-adjoint problems; Fourier series.

Week 6: The continuous spectrum: density of states; normalization of continuum eigenfunctions, completeness again, Fourier integral transforms. Linear equations: range-nullspace theorem and Fredholm alternative for matrices.

Week 7: Green functions: Fredholm alternative for balanced boundary value problems; null solutions of adjoint operator and constraints on source terms; Green functions via jump conditions; modified Green functions; eigenfunction expansion of Green function and resolvent.

Week 8: Linear and non-linear partial differential equations: elliptic, hyperbolic and parabolic equations; method of characteristics; the wave equation, shocks and weak solutions of PDE's.

Week 9: The Heat equation: solution by Green function; solution by integral transforms. Elliptic boundary value problems: Laplace's equation; Dirichlet and Neumann bound- ary conditions. Poisson's equation.

Week 10: Separation of variables: Legendre polynomials; spherical harmonics; examples from electrostatics. Bessel functions and spherical Bessel functions.

Week 11: Bessel functions: Orthogonality and recurrence relations,

Week 12: Integral equations: Type I and II Fredholm and Voltera equations. Solution via Fourier and Laplace transformations. Abel's equation.

Week 13: Separable Kernels. Compact and Hilbert-Schmidt operators. Fredholm Alternative.

Week 14: Perturbative methods: Neumann and Fredholm series.

Learning Objectives and Outcomes: For each topic in the list above you will master the basic concepts to the level that they can use them to solve physics problems. You will acquire the necessary skills though working illustrative examples in homework assignments. You will demonstrate competence by your performance in the midterm and final examinations.

Disability Access: The Department of Physics is committed to being an open and welcoming environment for all of our students. We are committed to helping all of our students succeed in our courses. To obtain disability-related academic adjustments and/or auxiliary aids, students with disabilities must contact the course instructor and the Disability Resources and Educational Services (DRES) as soon as possible. To contact DRES, you may visit 1207 S. Oak St., Champaign, call 333-4603, e-mail disability@illinois.edu, or go to the DRES website. If you are concerned you have a disability-related condition that is impacting your academic progress, there are academic screening appointments available on campus that can help diagnosis a previously undiagnosed disability by visiting the DRES website and selecting "Sign-Up for an Academic Screening" at the bottom of the page.

Academic Integrity: All activities in this course are subject to the Academic Integrity rules as described in *Article 1, Part 4, Academic Integrity*, of the Student Code.

Infractions include, but are not limited to:

- Cheating, plagiarism, fabrication
- facilitating infractions of academic integrity.
- academic interference
- computer-related infractions
- unauthorized use of university resources
- sale of class materials or notes

Violations of any of these rules will be prosecuted and reported to the student's home college in compliance with the Student Code: *Article 1, Part 4, Academic Integrity*, of the Student Code.