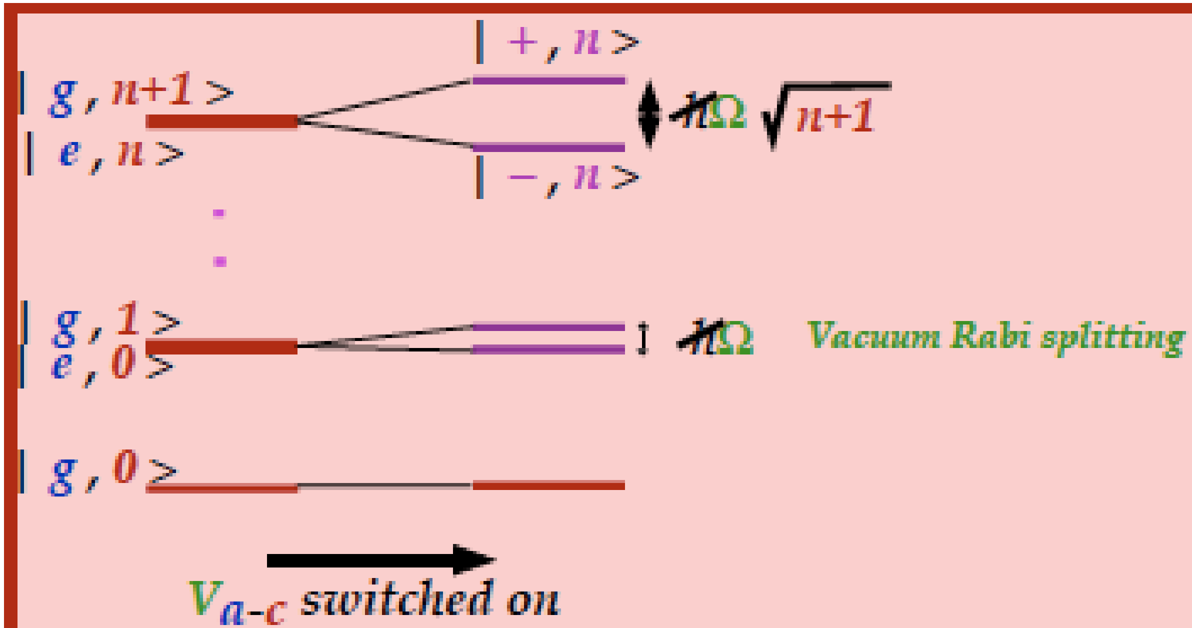
Hamiltonian: $H = H_a + H_c + V_{a-c}$

$$H_a = \frac{\hbar \omega_a}{2} (| e \rangle\langle e | - | g \rangle\langle g |)$$

$$H_c = \frac{\hbar \omega_c}{2} (a^\dagger a + a a^\dagger) = \hbar\omega_c (a^\dagger a + 1/2)$$

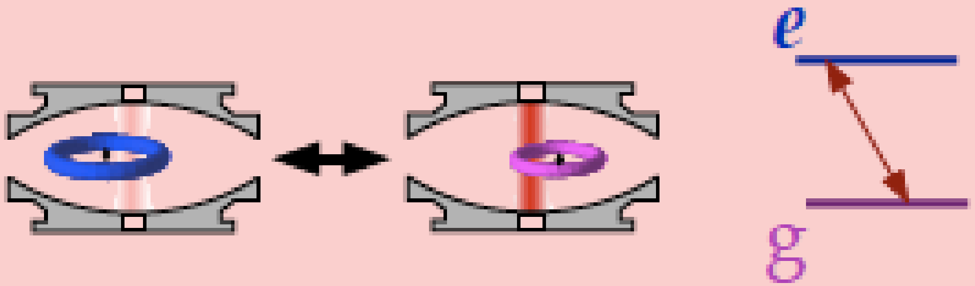
$$V_{a-c} = - \vec{D}_a . \vec{E}(r)$$



Eigenvalues $E_{\pm,n} = \pm \frac{\hbar\Omega}{2}\sqrt{n+1}$

Eigenstates $| \pm , n \rangle = \frac{1}{\sqrt{2}} \{ | e , n \rangle \mp i | g , n+1 \rangle \}$

Quantum Rabi Oscillation



$$|e,0\rangle \leftrightarrow |g,1\rangle$$

***Reversible** photon emission and absorption*

More generally :

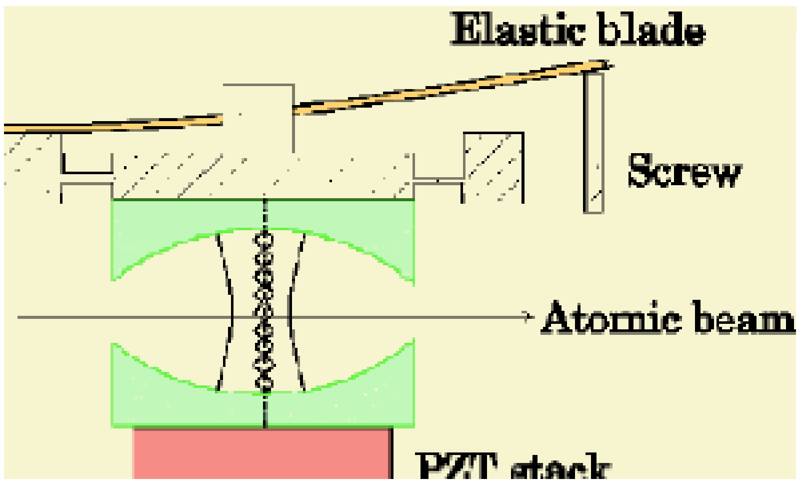
$$|e,n\rangle \leftrightarrow |g,n+1\rangle$$

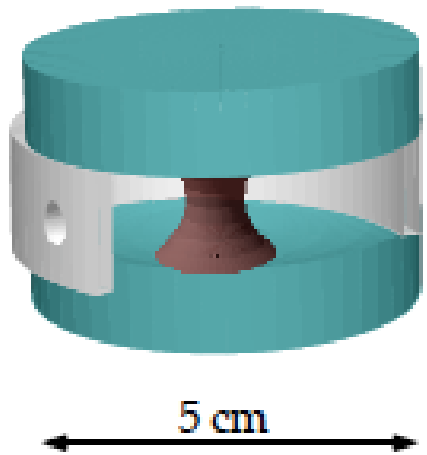
***Temporal evolution, starting from** $|e,n\rangle$ **at** $t=0$:*

$$|\psi(t)\rangle = \cos\left(\frac{\Omega\sqrt{n+1}\,t}{2}\right)|e,n\rangle - \sin\left(\frac{\Omega\sqrt{n+1}\,t}{2}\right)|g,n+1\rangle$$

Atom and cavity field generally entangled!

The superconducting Fabry-Perot Cavity





Gaussian field mode with 6mm waist

Large field per photon (1.5 mV/m)

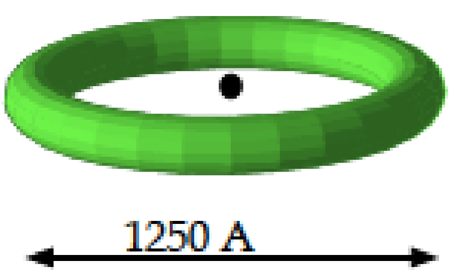
Long photon life-time (1ms) improved by ring around mirrors

Easy tunability

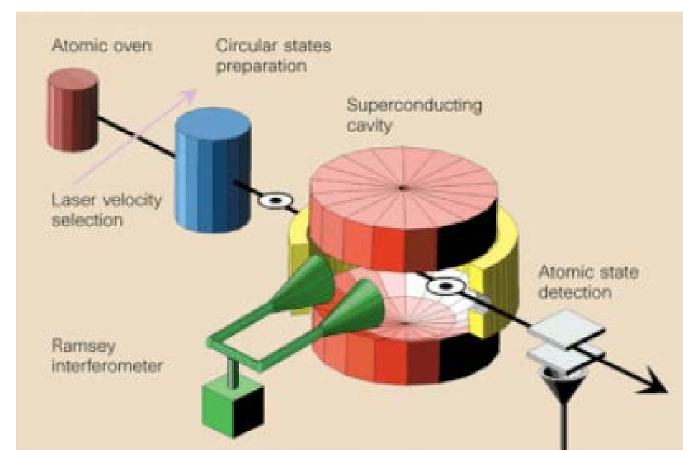
Very small thermal photon background

Circular Rydberg atoms

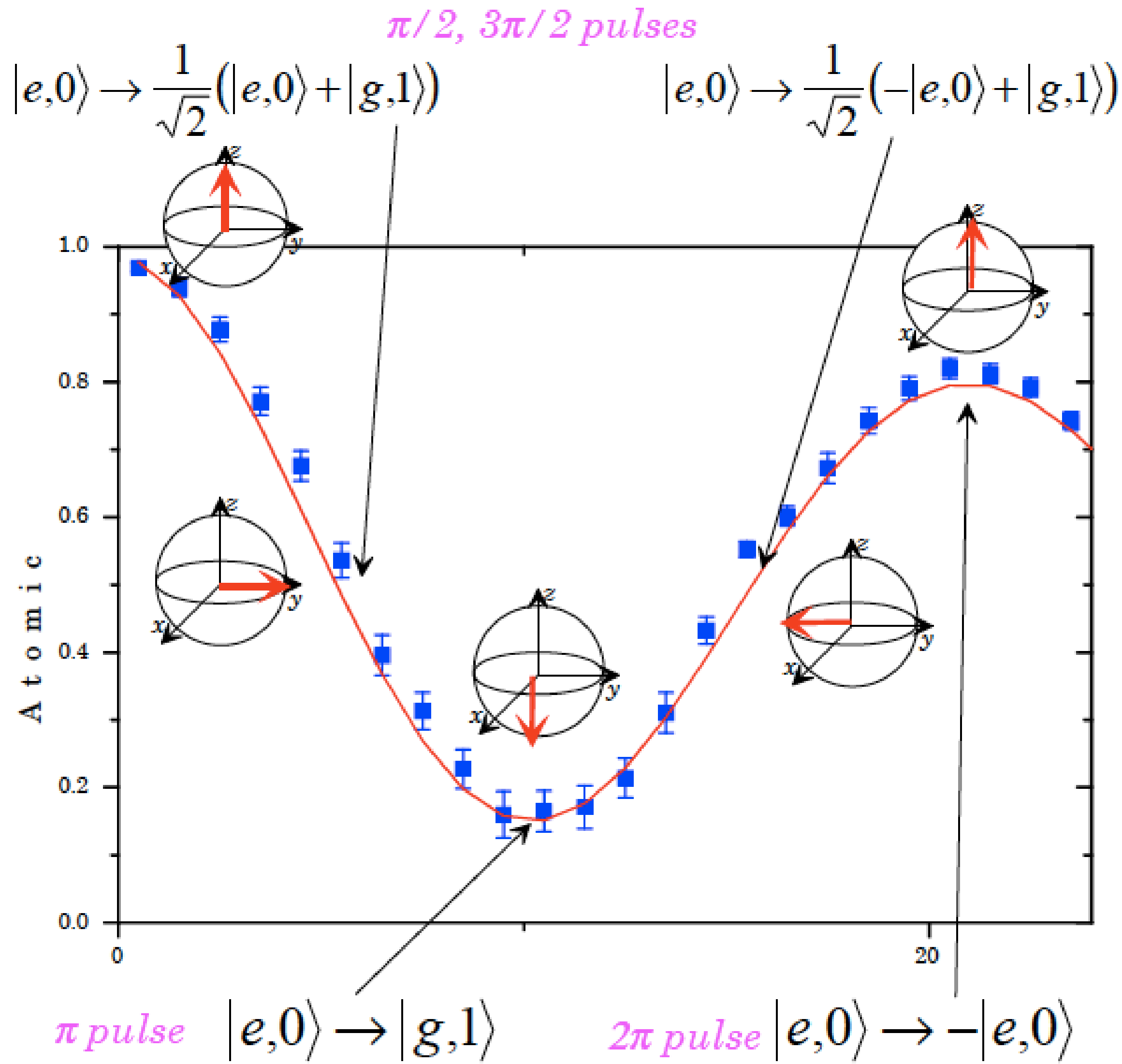
$n = 51$



Large circular orbit
Strong coupling to microwaves
Long radiative life time (30 ms)
level tunability by Stark effect
Easy state selective detection



$$|e,0\rangle \rightarrow \cos\frac{\Omega_0 t}{2}|e,0\rangle + \sin\frac{\Omega_0 t}{2}|g,1\rangle \qquad |g,1\rangle \rightarrow -\sin\frac{\Omega_0 t}{2}|e,0\rangle + \cos\frac{\Omega_0 t}{2}|g,1\rangle$$



Is |45>

= |H> + |V> an entangled state?

Is $(|1>_a + |1>_b >)$/sqrt 2?

Entangled state

send in 1st atom:

$$|e,0\rangle \xrightarrow[\pi/2\ pulse]{} \frac{|e,0\rangle + |g,1\rangle}{\sqrt{2}} \quad \text{(atom-photon entanglement)}$$

send in 2nd atom:

$$|G\rangle \frac{|e,0\rangle + |g,1\rangle}{\sqrt{2}} \xrightarrow[\pi\ pulse]{} \frac{|G\rangle|e,0\rangle - |E\rangle|g,0\rangle}{\sqrt{2}} = \frac{|G\rangle|e\rangle - |E\rangle|g\rangle}{\sqrt{2}} \quad \text{(atom-atom "singlet")}$$

measure in arbitrary basis: apply π/2-pulse to 2nd atom, shifted by ϕ w.r.t. pulse on 1st atom

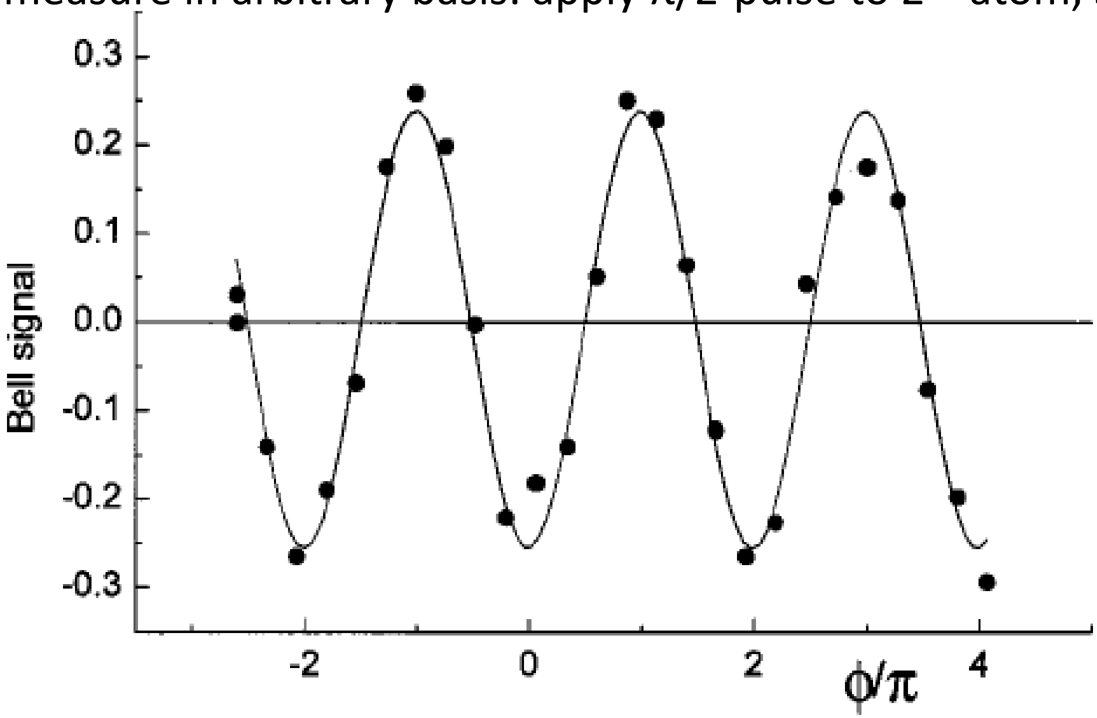
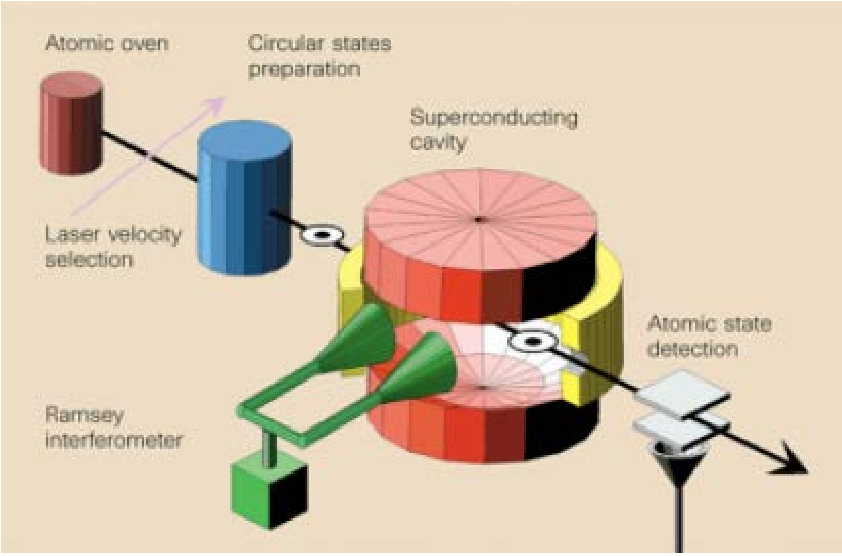
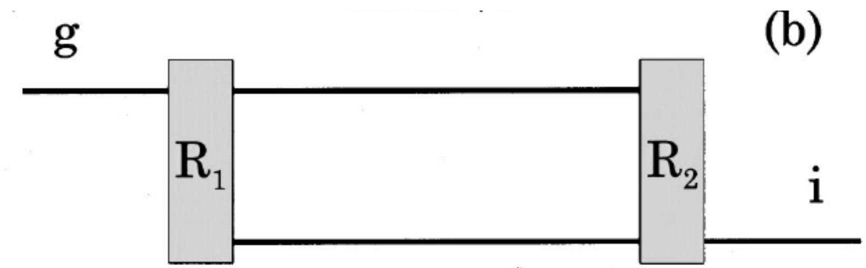


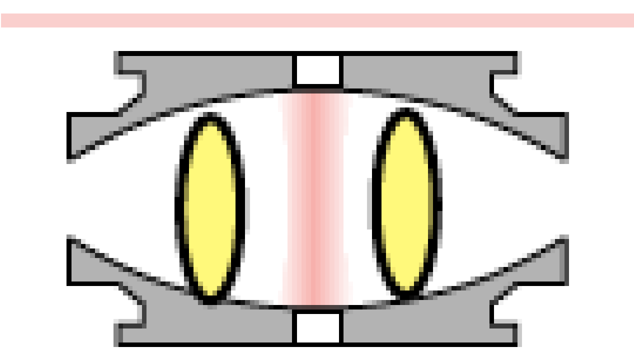
FIG. 7. "Bell signal" plotted versus the relative phase ϕ (in units of π) of pulses R_1^{eg} and R_2^{eg}. The line is a sine fit.

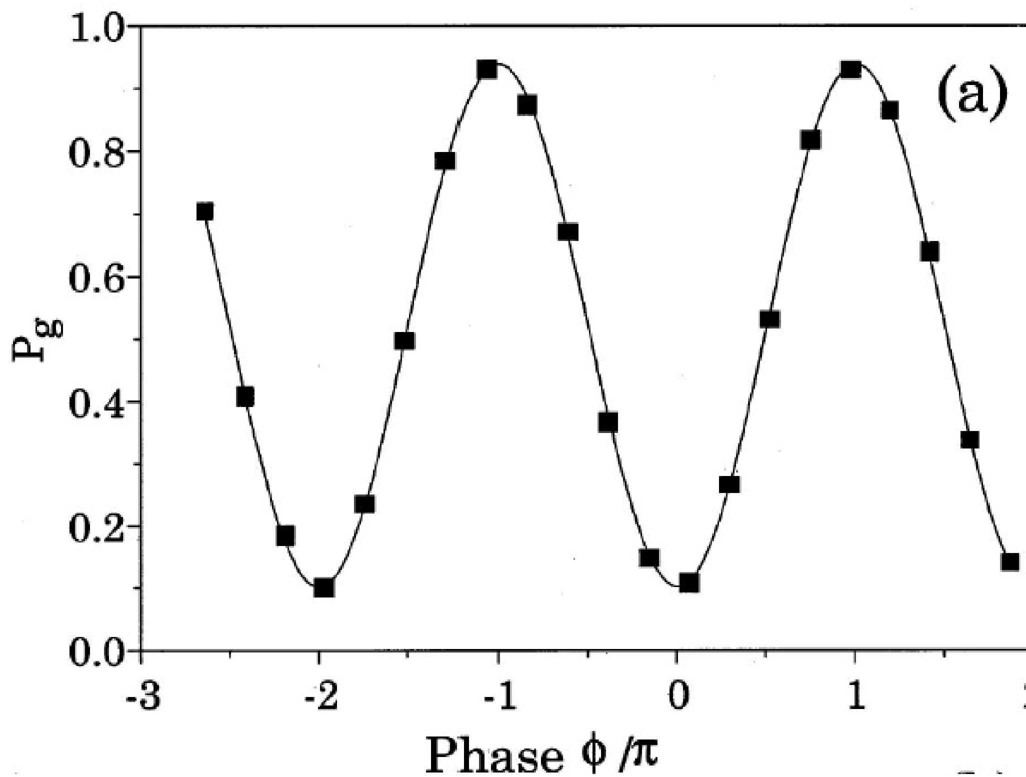
Correlations insufficient to violate Bell inequality (V = 45%)
- imperfect Ramsey interference
- residual blackbody photons
- 2-atom events

Ramsey fringes

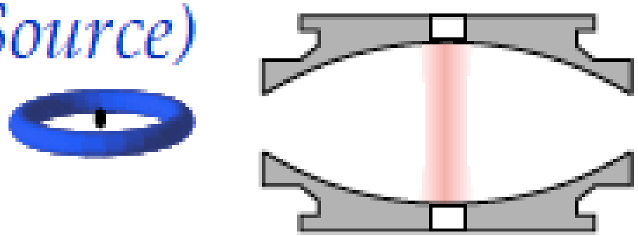








Atom #1
(Source)

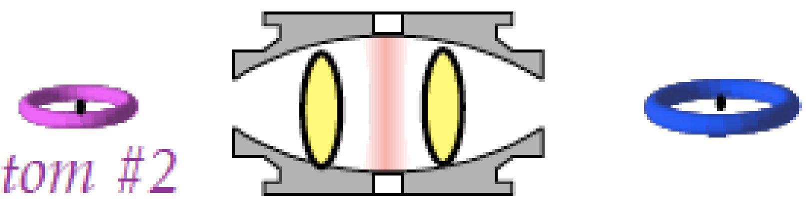


π/2 *Rabi pulse on atom #1*

$$(1/\sqrt{2})\,\{\, | e_1, 0 > \; + \; | g_1, 1 > \}$$

First "source atom" emits with 50% probability a single photon in C. Detecting it reduces field to 0 or 1 photon

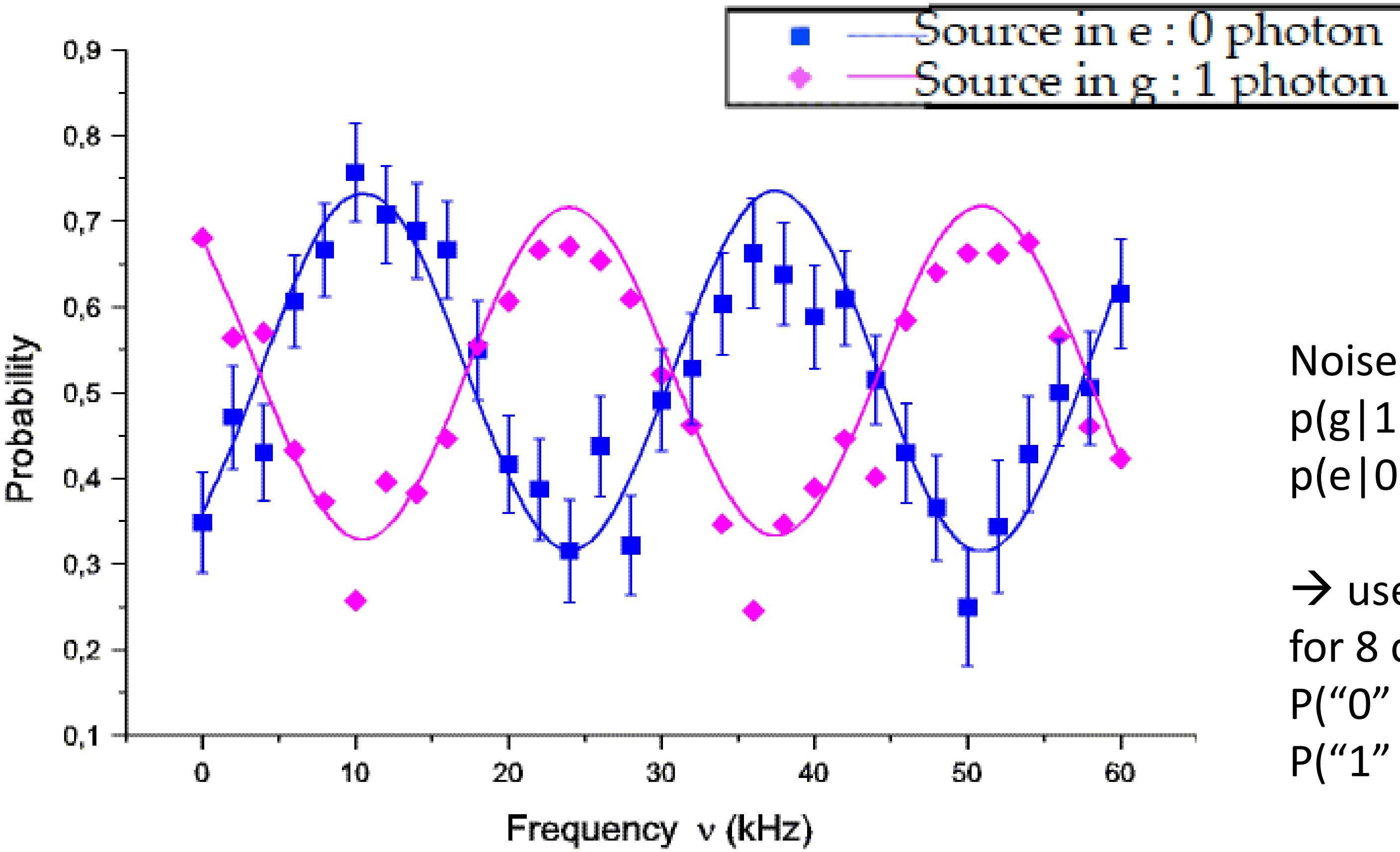
Second "meter" atom reads out the photon number by Ramsey interferometry



Atom #2
(QND

2π *Rabi pulse on atom #2*

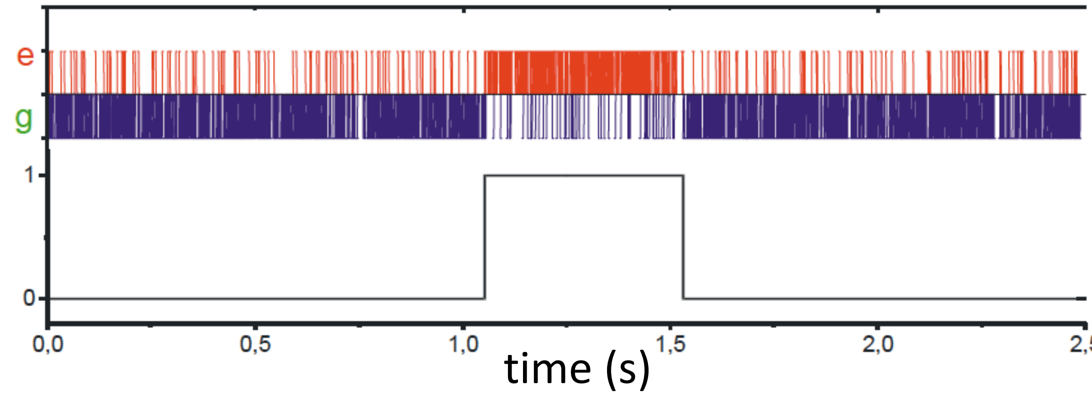
QND measurement



Noise probabilities:
p(g|1) = 13%
p(e|0) = 9%

→ use majority vote for 8 consecutive atoms
P("0"|1) < 0.0014
P("1"|0) < 0.00025)

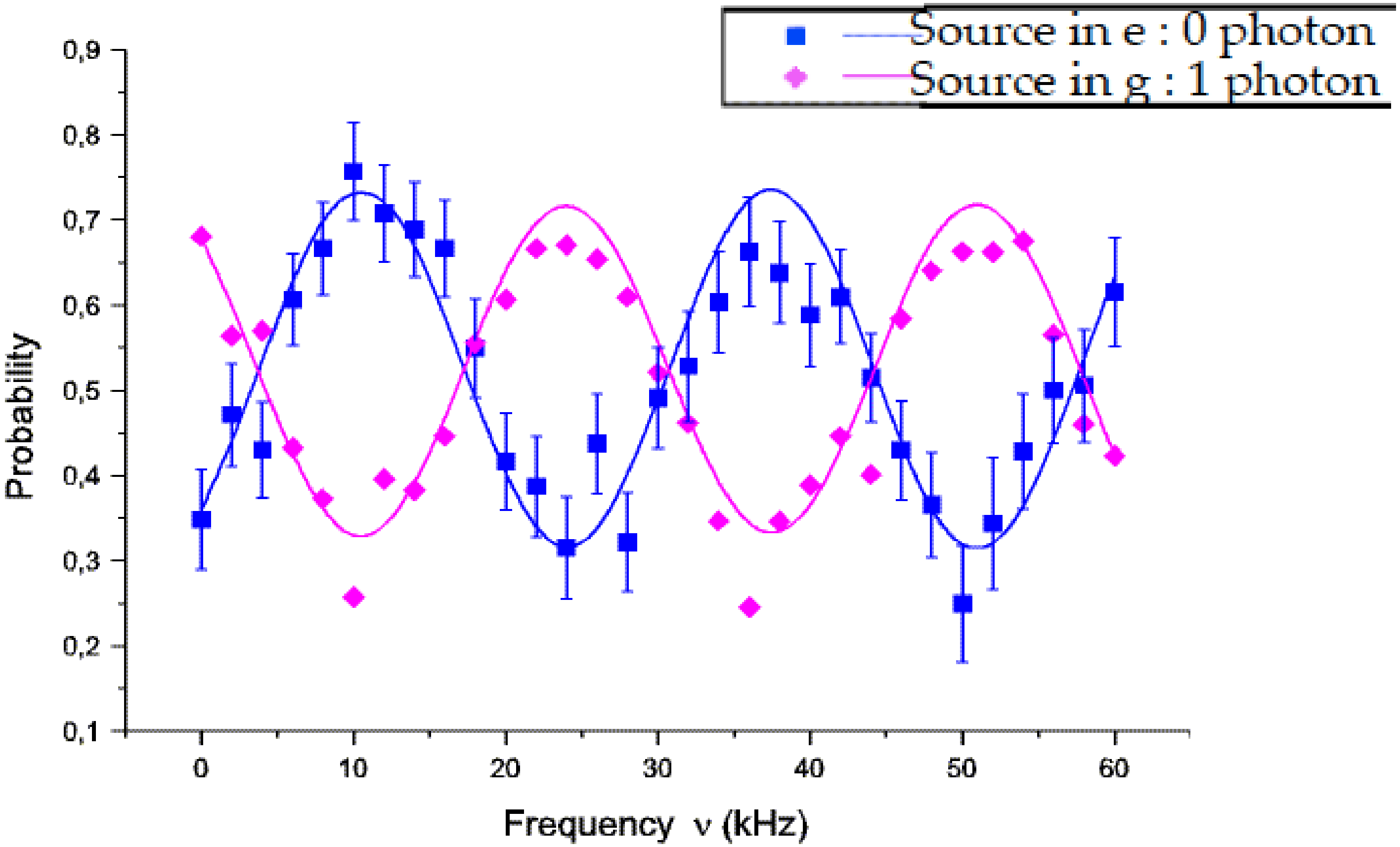
Quantum jumps of light recording the birth and death of a photon in a cavity, Gleyes et al., Nat. **446**, 297 (2007)



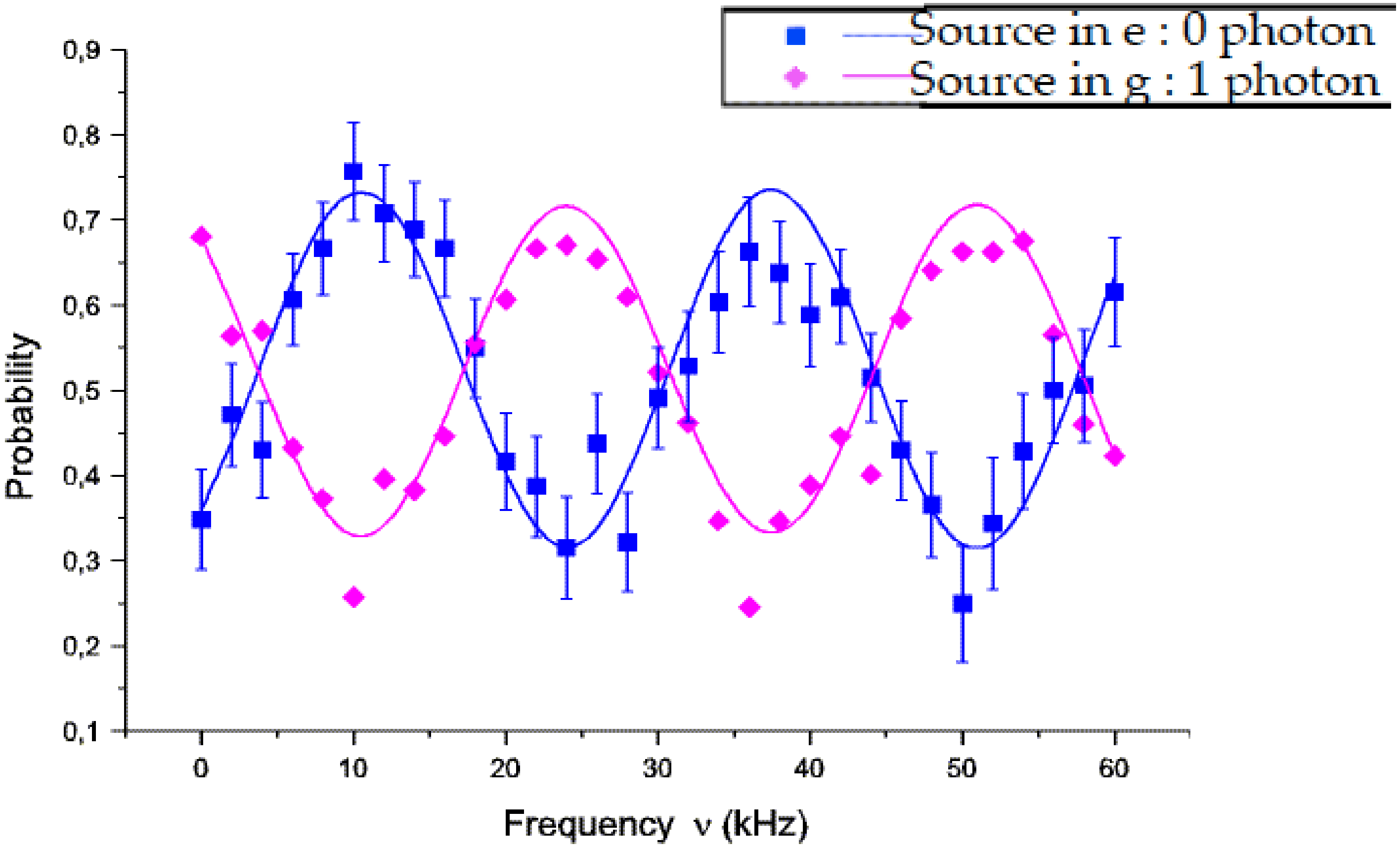
This QND distinguished between 0 and 1 photon.

Question: Can we use it to distinguish between 0, 1 and 2 photons?

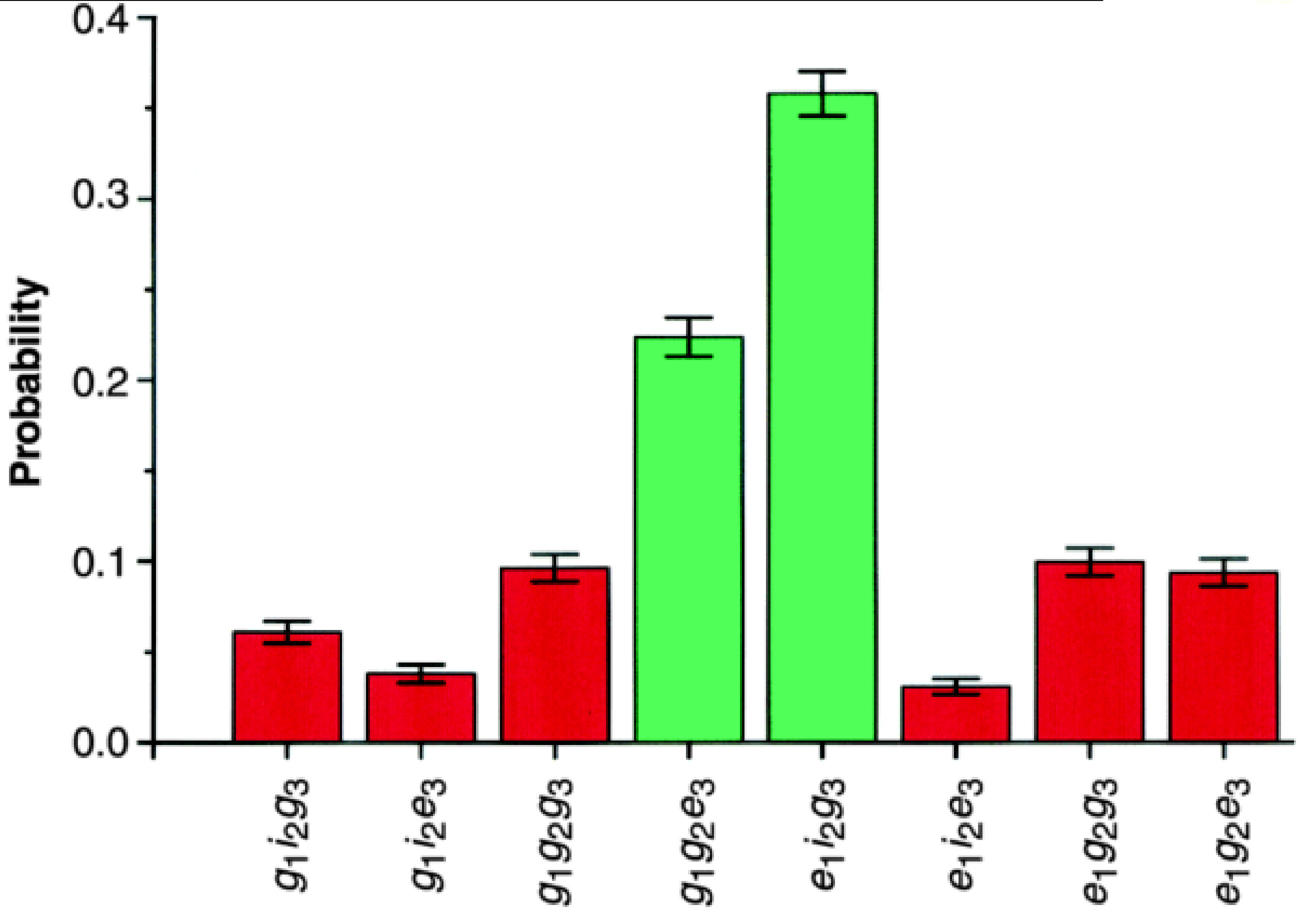
QND measurement

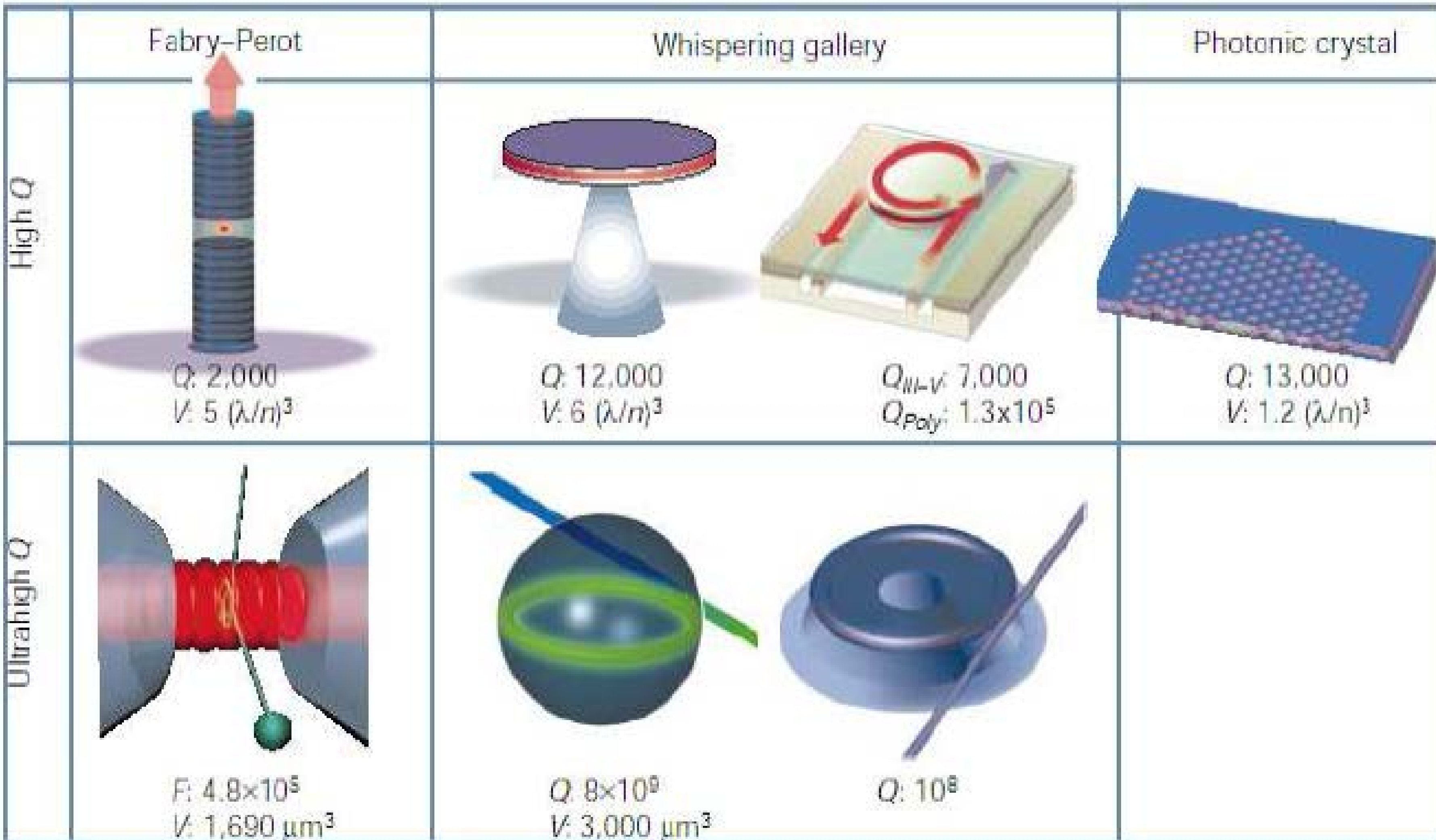


Quantum Phase Gate



GHZ Three-atom entanglement

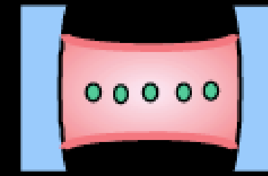


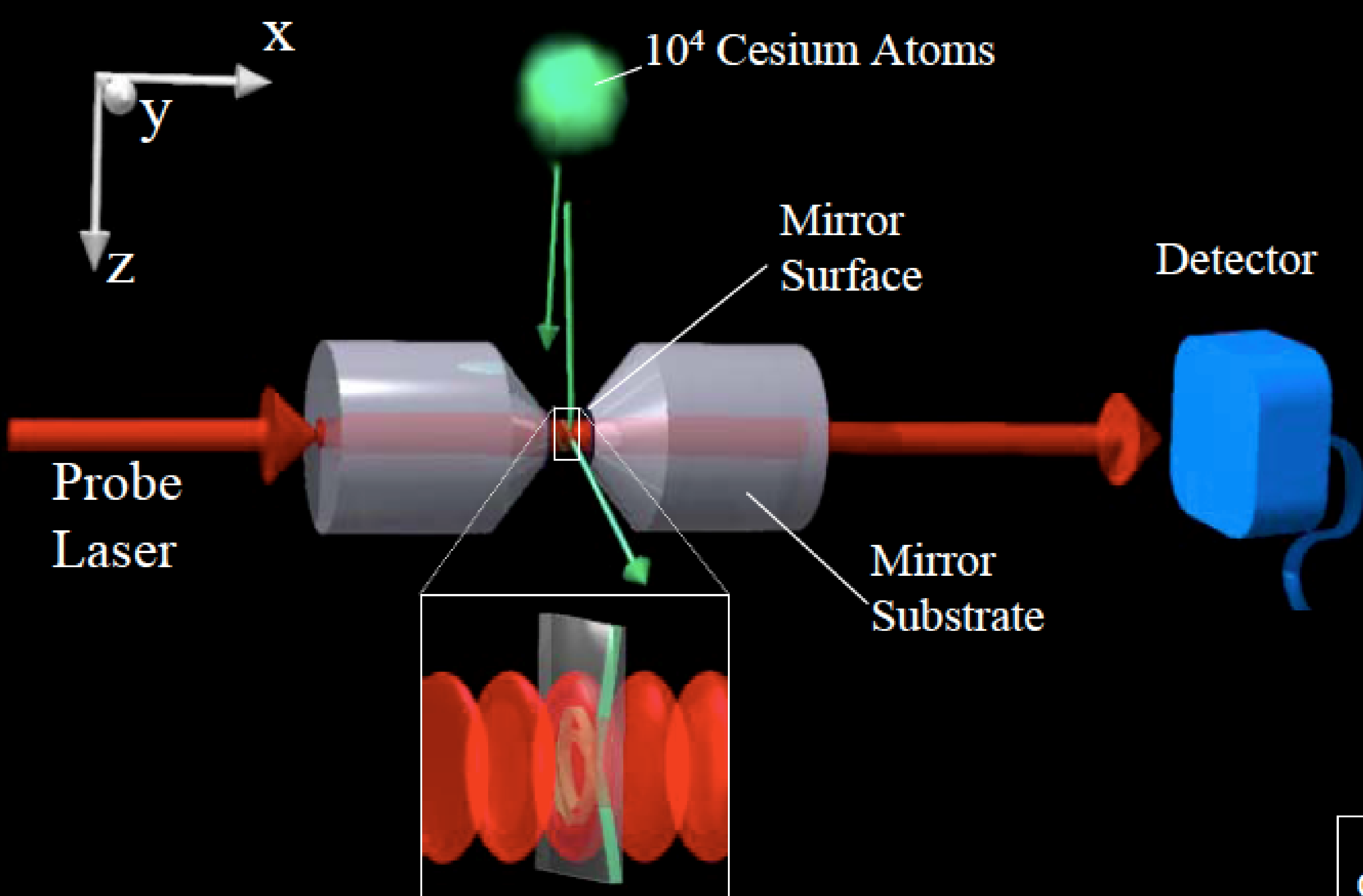


Fabry–Perot
Whispering gallery
Photonic crystal
High Q
Ultrahigh Q
Q: 2,000
V: 5 $(\lambda/n)^3$
Q: 12,000
V: 6 $(\lambda/n)^3$
Q_{III-V}: 7,000
Q_{Poly}: 1.3×10^5
Q: 13,000
V: 1.2 $(\lambda/n)^3$
F: 4.8×10^5
V: 1,690 μm^3
Q: 8×10^9
V: 3,000 μm^3
Q: 10^8

Cavity QED with Cold Atoms

- Goal: localized atoms in a regime of strong coupling





Caltech
Quantum
Optics

VOLUME 92, NUMBER 12 PHYSICAL REVIEW LETTERS week ending 26 MARCH 2004

Scalable Photonic Quantum Computation through Cavity-Assisted Interactions

L.-M. Duan[1,2] and H. J. Kimble[3]

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[2]*Laboratory of Quantum Information, USTC, Hefei, Anhui 230026, China*
[3]*Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125, USA*
(Received 25 September 2003; published 25 March 2004)

We propose a scheme for scalable photonic quantum computation based on cavity-assisted interaction between single-photon pulses. The prototypical quantum controlled phase-flip gate between the single-photon pulses is achieved by successively reflecting them from an optical cavity with a single-trapped atom. Our proposed protocol is shown to be robust to practical noise and experimental imperfections in current cavity-QED setups.

Nondestructive Detection of an Optical Photon

Andreas Reiserer, Stephan Ritter,* Gerhard Rempe

All optical detectors to date annihilate photons upon detection, thus excluding repeated measurements. Here, we demonstrate a robust photon detection scheme that does not rely on absorption. Instead, an incoming photon is reflected from an optical resonator containing a single atom prepared in a superposition of two states. The reflection toggles the superposition phase, which is then measured to trace the photon. Characterizing the device with faint laser pulses, a single-photon detection efficiency of 74% and a survival probability of 66% are achieved. The efficiency can be further increased by observing the photon repeatedly. The large single-photon nonlinearity of the experiment should enable the development of photonic quantum gates and the preparation of exotic quantum states of light.

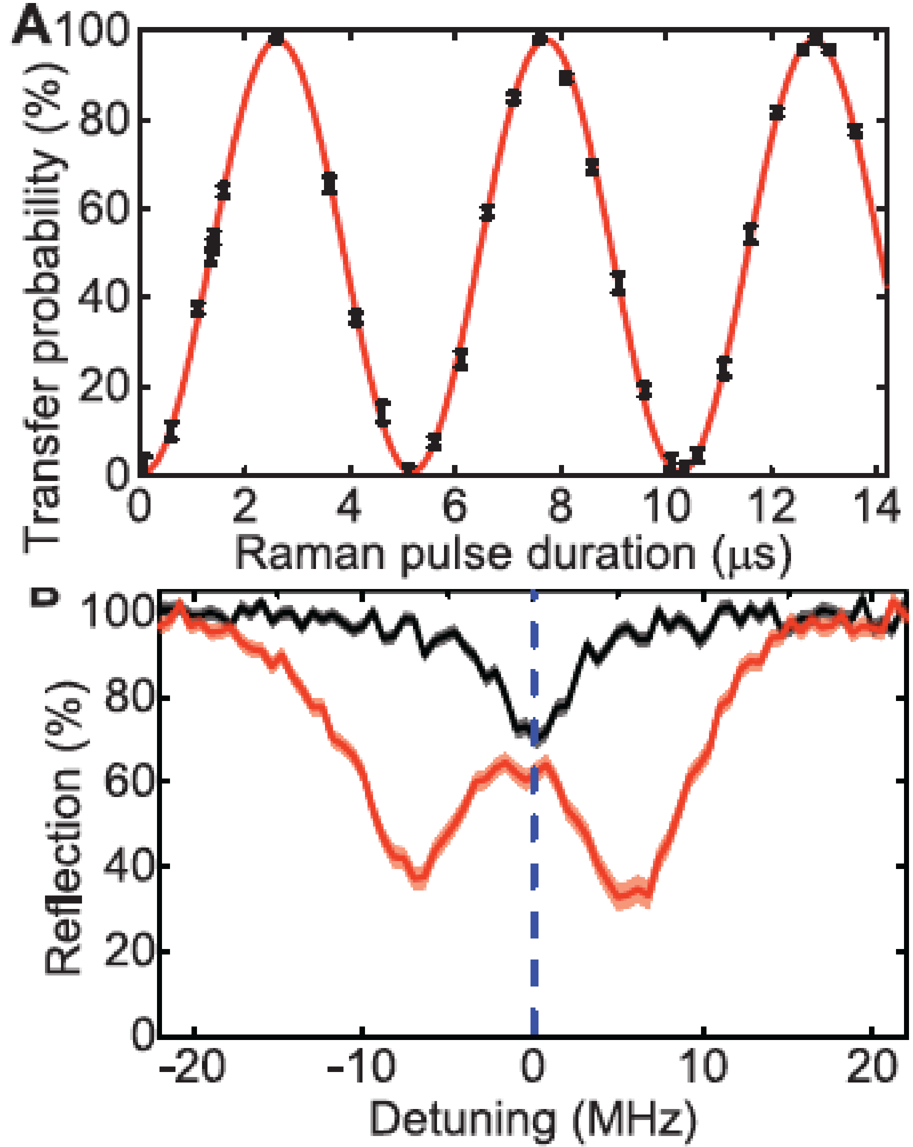
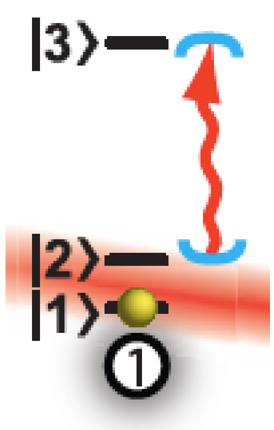


Fig. 2. Atomic state manipulation and cavity reflection spectrum. (**A**) Rabi oscillations of the atomic population when the atom is prepared in $|2\rangle_a$, and two Raman laser beams are applied for a variable duration. The red fit curve gives a visibility of 97%. (**B**) Reflection off the atom-cavity system as a function of probe laser frequency, with the atom in the strongly coupled state $|2\rangle_a$ (red) or in the uncoupled state $|1\rangle_a$ (black). The statistical SE is given by the thickness of the lines.



field decay rate is $\kappa = 2\pi \times 2.5$ MHz, the atomic dipole decay rate is $\gamma = 2\pi \times 3$ MHz, and the measured atom-cavity coupling constant on the $|2\rangle_a \leftrightarrow |3\rangle_a$ transition is $g = 2\pi \times 6.7$ MHz (*16*). Thus, the system operates in the strong-coupling regime of cavity quantum electrodynamics.

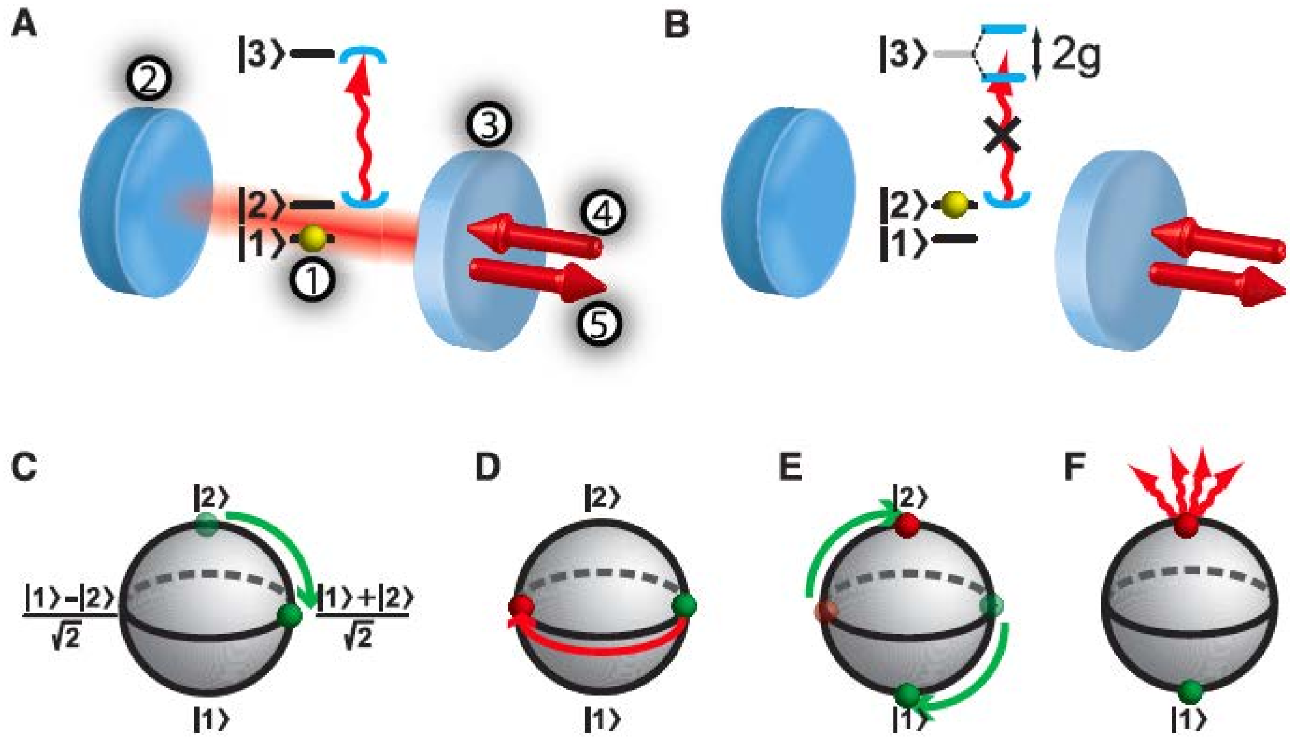
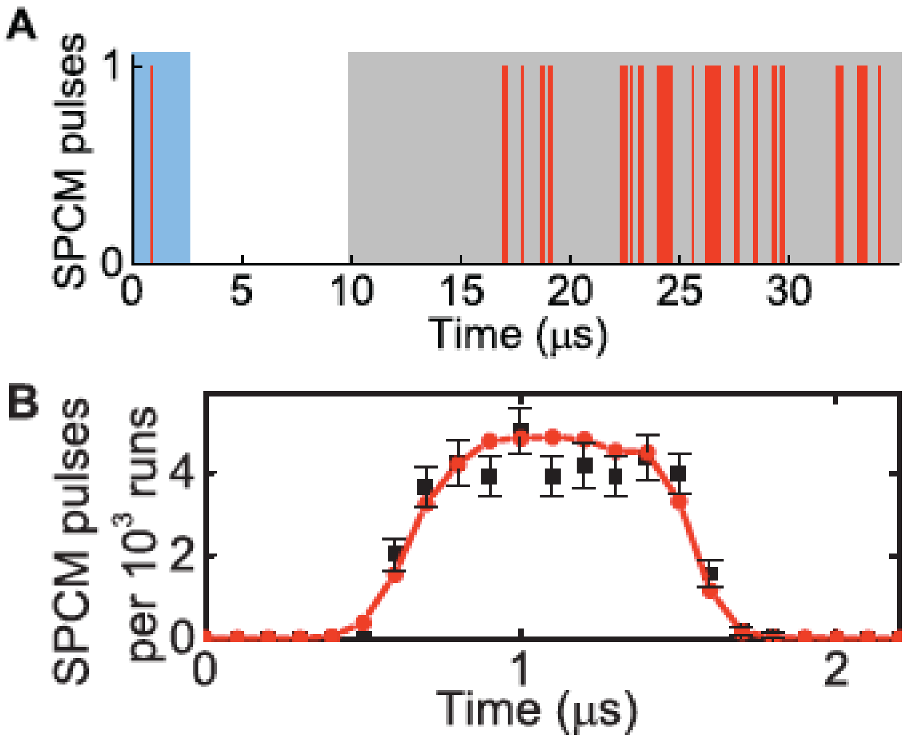


Fig. 1. Nondestructive photon detection. (**A** and **B**) Sketch of the setup and atomic level scheme. A single atom, (1), is trapped in an optical cavity that consists of a high-reflector, (2), and a coupling mirror, (3). A resonant photon is impinging on, (4), and reflected off, (5), the cavity. (A) If the atom is in state $|1\rangle_a$, the photon (red wavy arrow) enters the cavity (blue semicircles) before being reflected. In this process, the combined atom-photon state acquires a phase shift of π. (B) If the atom is in $|2\rangle_a$, the strong coupling on the $|2\rangle_a \leftrightarrow |3\rangle_a$ transition leads to a normal-mode splitting of $2g$, so that the photon cannot enter the cavity and is directly reflected without a phase shift. (**C** to **F**) Procedure to measure whether a photon has been reflected. (C) The atomic state, visualized on the Bloch sphere, is prepared in the superposition state $\frac{1}{\sqrt{2}}(|1\rangle_a + |2\rangle_a)$. (D) If a photon impinges, the atomic state is flipped to $\frac{1}{\sqrt{2}}(|1\rangle_a - |2\rangle_a)$. (E) The atomic state is rotated by $\frac{\pi}{2}$. (F) Fluorescence detection is used to discriminate between the states $|1\rangle_a$ and $|2\rangle_a$.



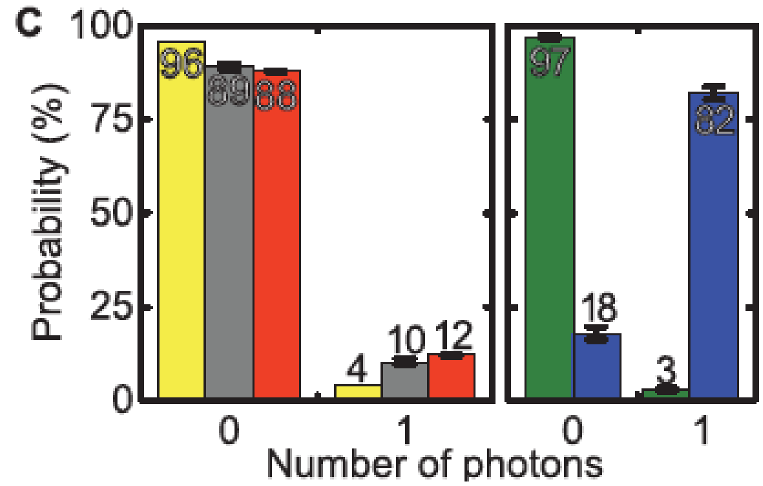


Fig. 3. Experimental results. (**A**) Typical trace of an experimental run. A photon (red bar) impinging in the trigger interval (blue area) leads to the emission of many photons in the readout interval (gray area). When the input pulse is blocked, no photons are detected in both intervals. (**B**) Temporal envelope of the reflected photon pulse when an atom is present (black squares) compared with a reference run without atom (red points). Within the errors, no deviation in the pulse shape is observable, except for a small amplitude change stemming from the slightly different reflectivities (Fig. 2B). (**C**) Nondestructive detection of a single photon. The probability of detecting zero or one photon is plotted. Yellow, result of the SPCM detection; gray, calculated input pulse, taking into account the SPCM detection efficiency; red, result of the atomic state readout; green, atomic state readout without impinging light; blue, atomic state, conditioned on the SPCM detection of a reflected photon in the trigger interval.

Generating Arbitrary Photon States

John Martinis
UC Santa Barbara

IARPA

- Quantum Integrated Circuits
  - Quantum currents & voltages
  - Microfabricated "atoms"
- Digital to Analog Converter

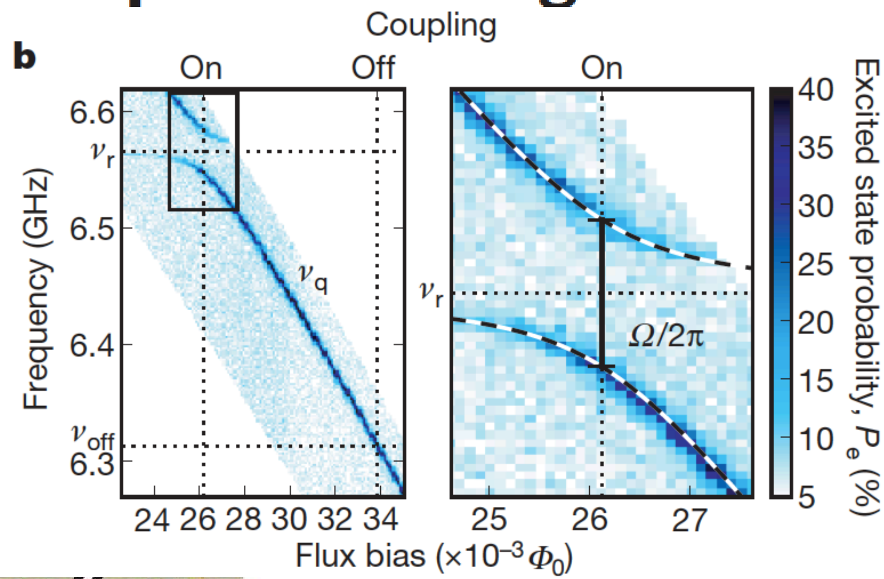
Classical: V = a[0] 1 + a[1] 2 + a[2] 4 + …a[9] 512

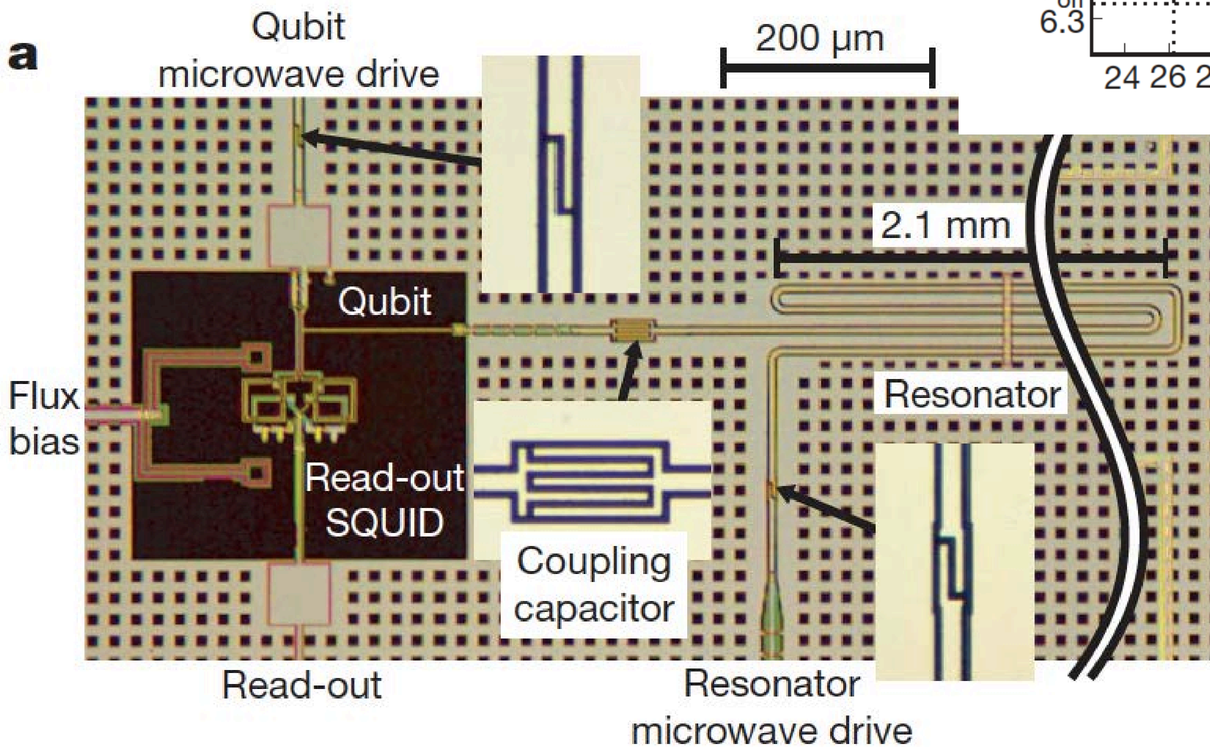
Quantum: $\Psi = \alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle + \ldots \alpha_9 |9$

nature Vol 454|17 July 2008|doi:10.1038/nature07136

Generation of Fock states in a superconducting quantum circuit

Max Hofheinz[1], E. M. Weig[1]†, M. Ansmann[1], Radoslaw C. Bia
H. Wang[1], John M. Martinis[1] & A. N. Cleland[1]



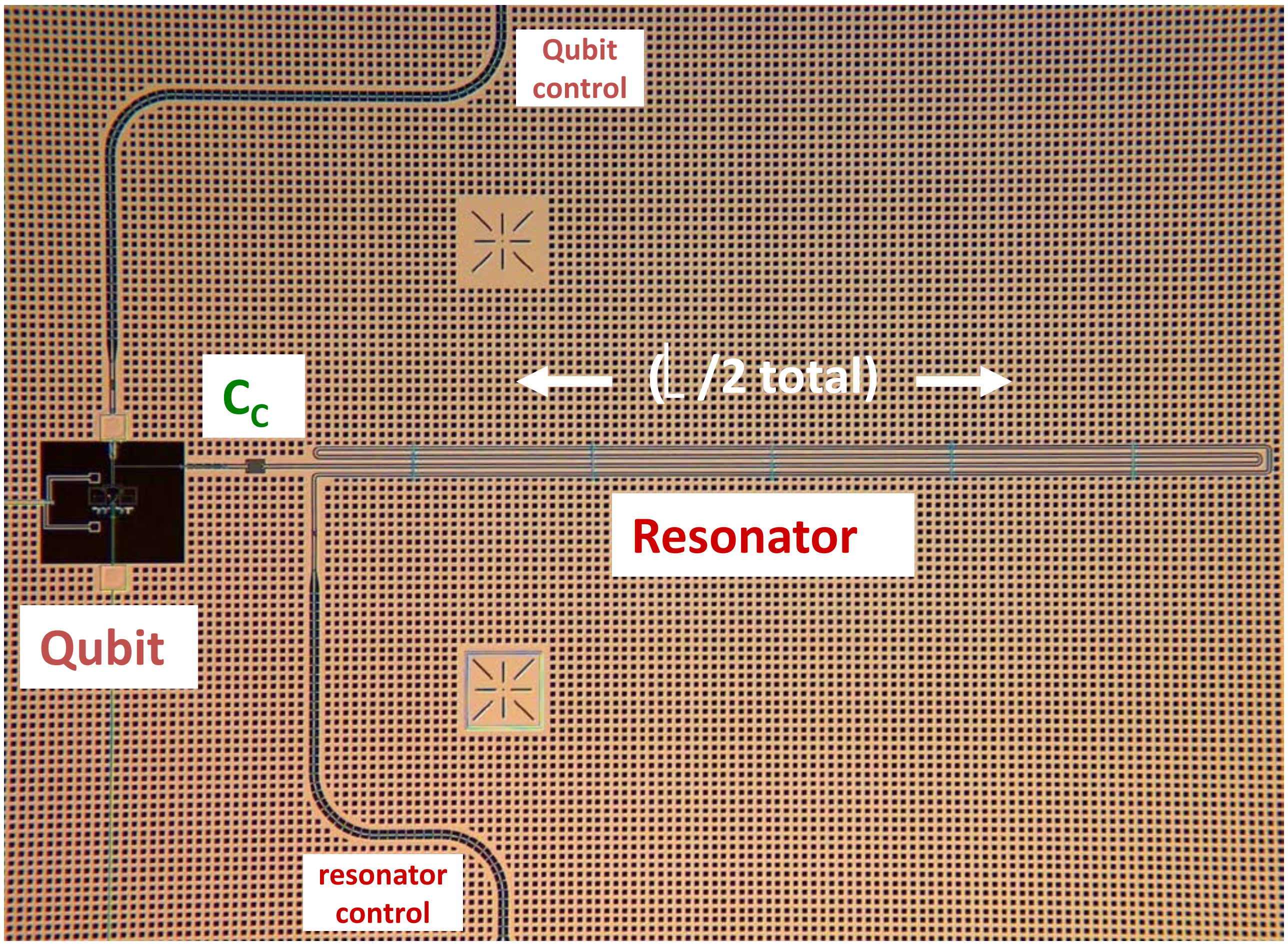


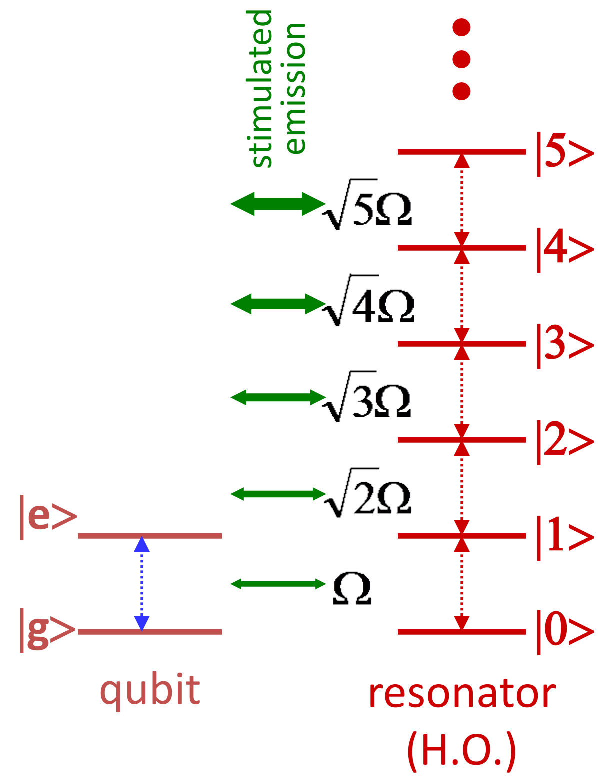
Qubit Coupled to Photons (Harmonic Oscillator)

Qubit-resonator first demonstrated in Saclay (1987),
GHz coupling easy in phase qubits

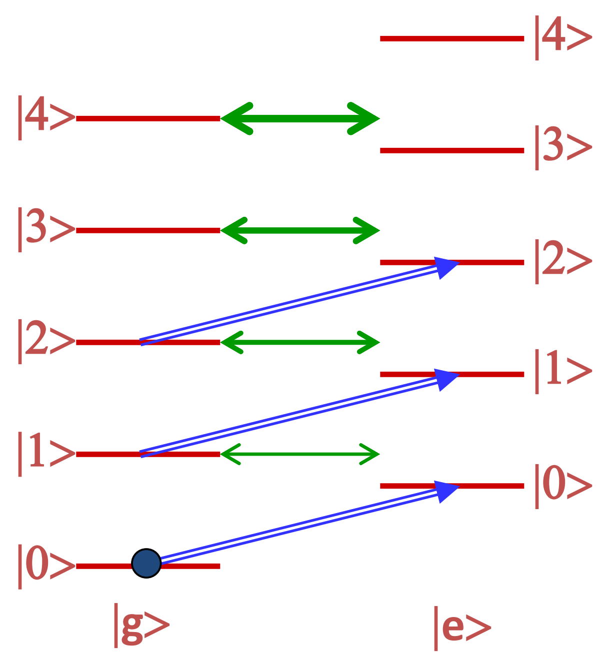
Design follows cQED (Yale)

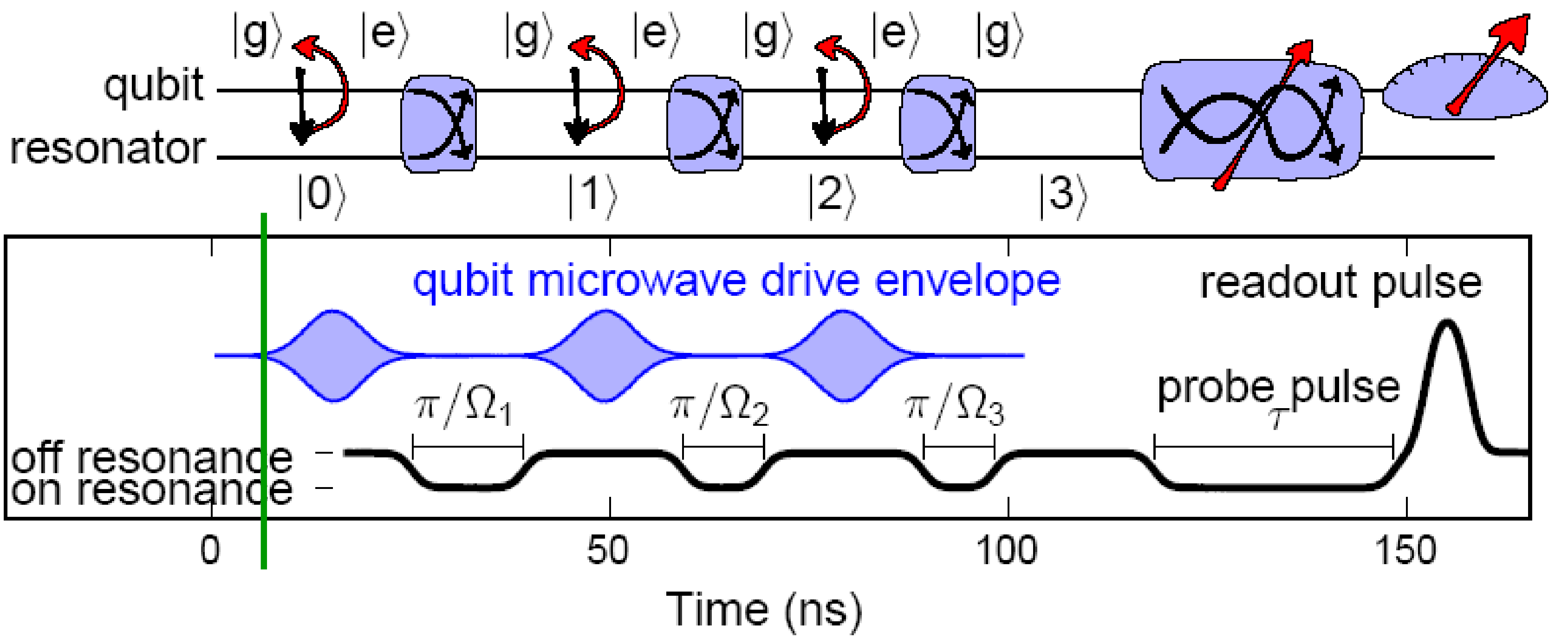
Readout through qubit (Saclay87, NIST), high fidelity
Similar to ion traps (NIST), Rydberg (ENS)





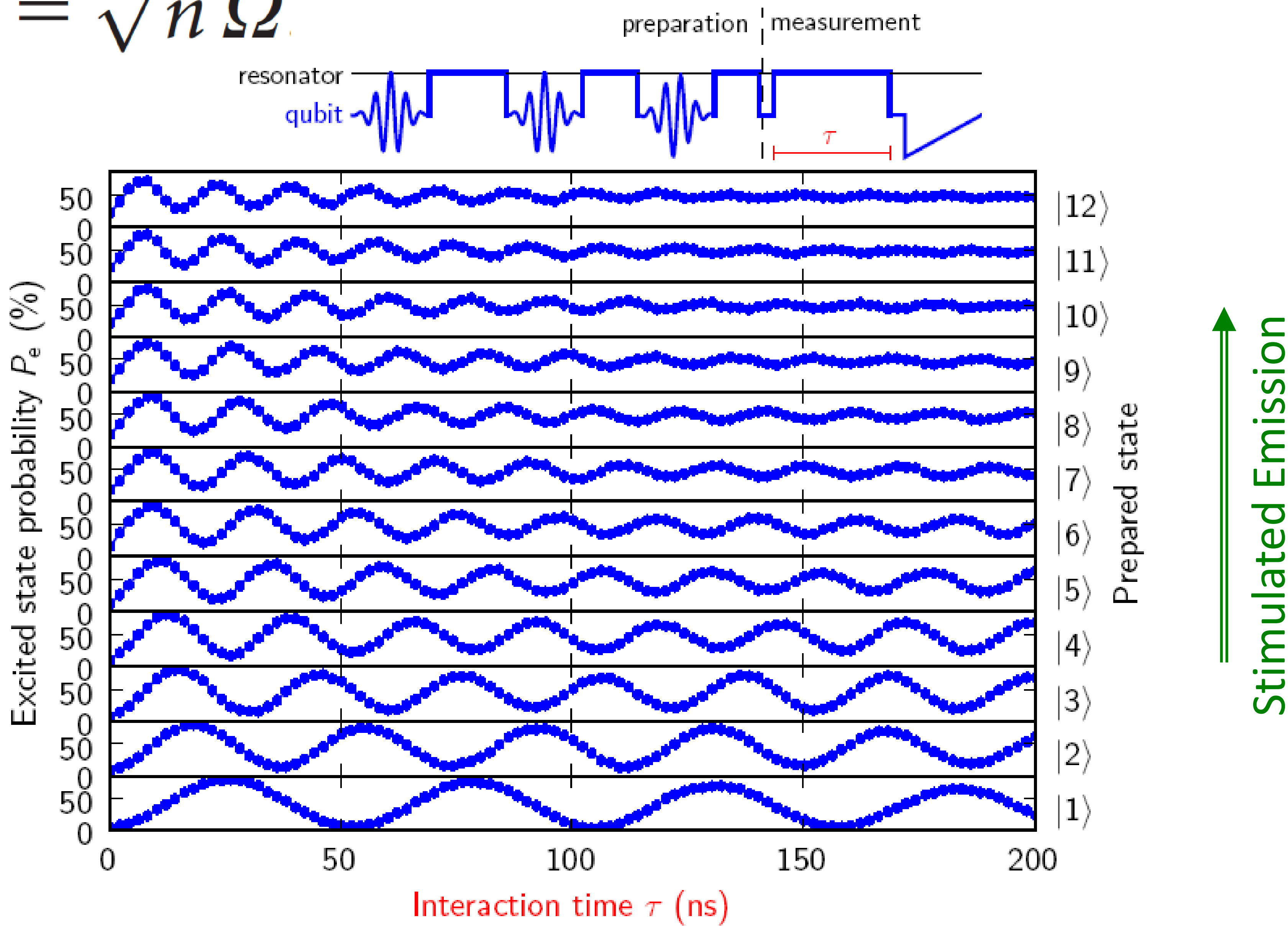
Generating Fock States: Pumping Photons One by One



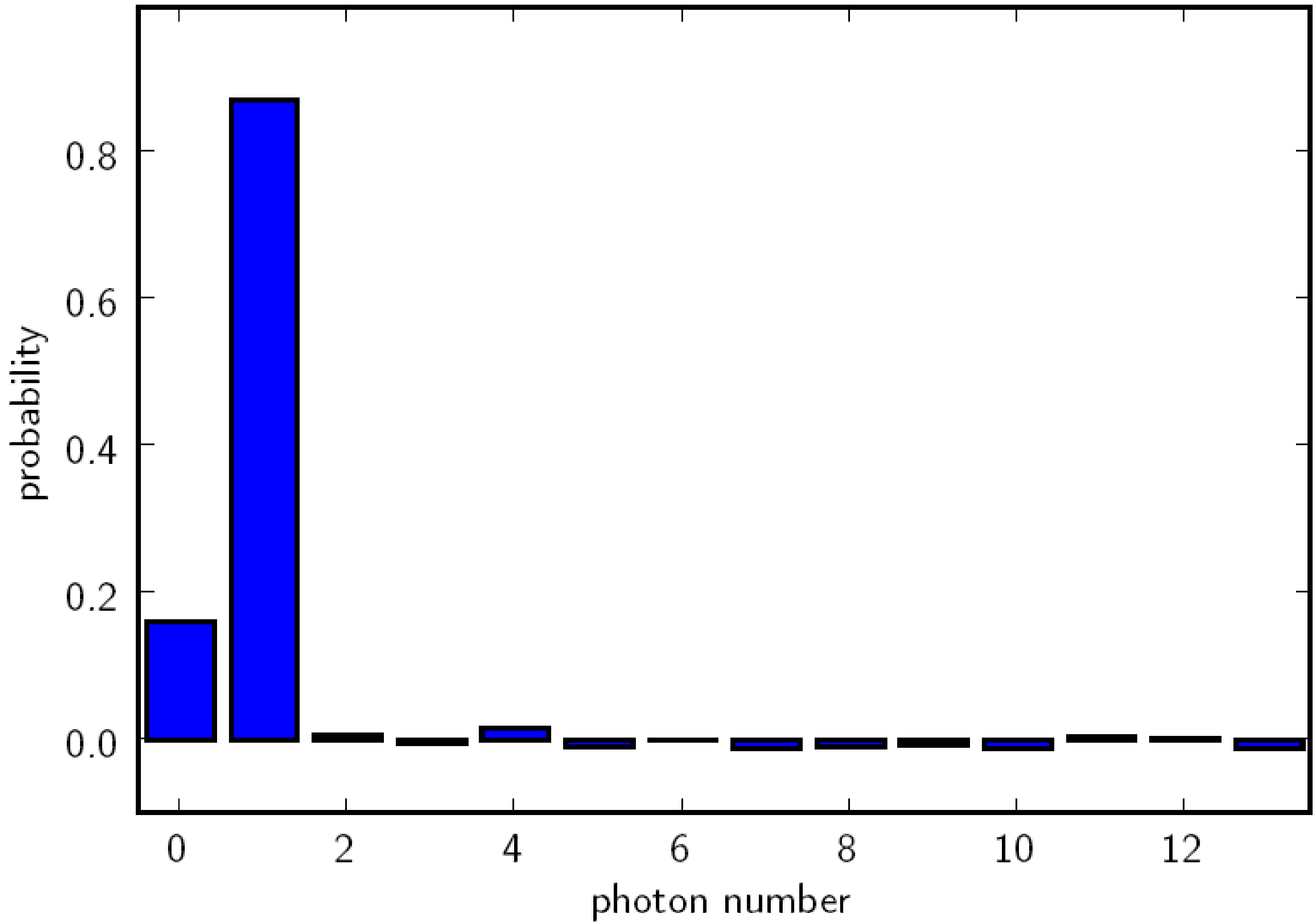


Fock States: Swap oscillations depend on |n⟩

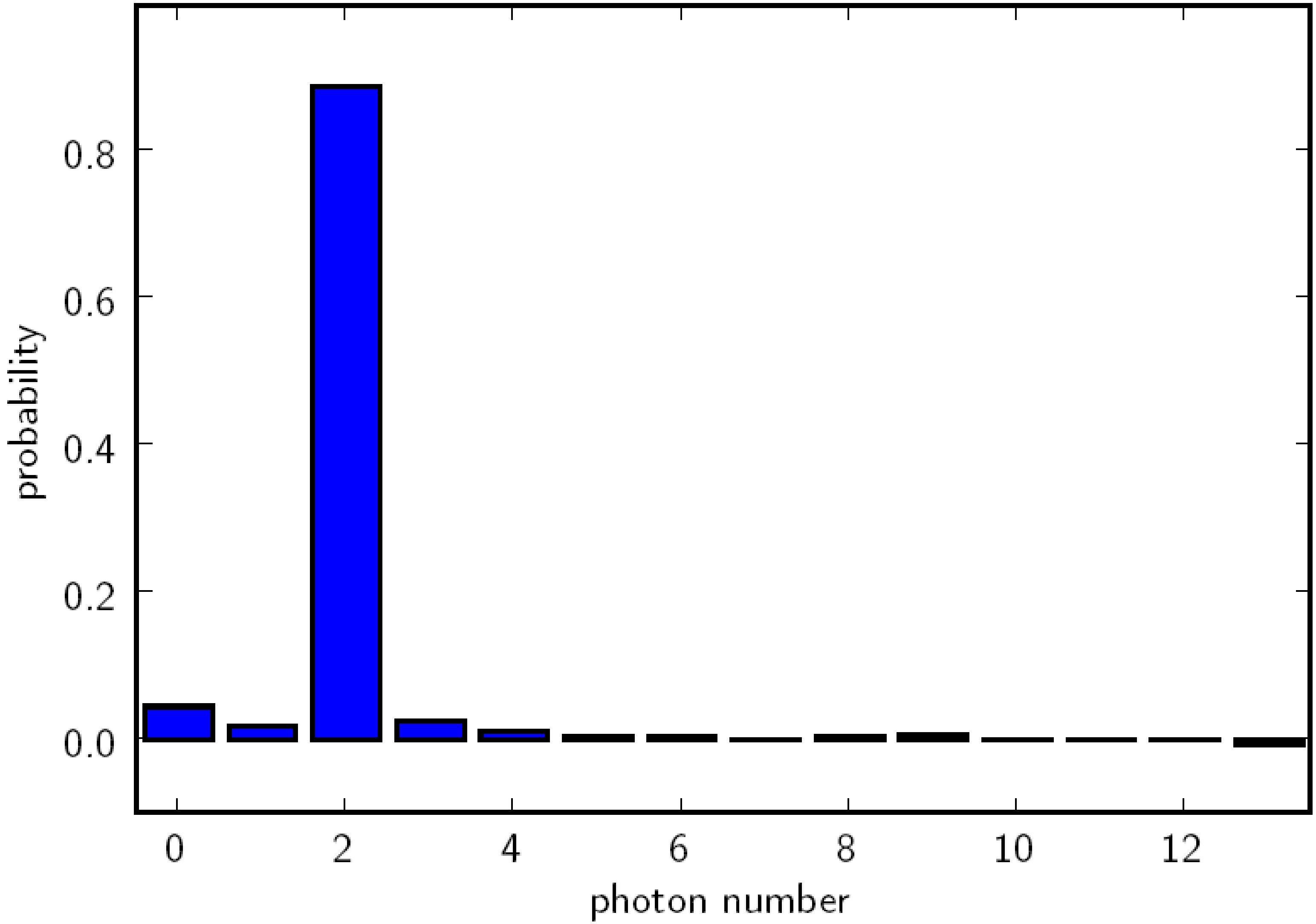
$$\Omega_n = \sqrt{n}\,\Omega$$



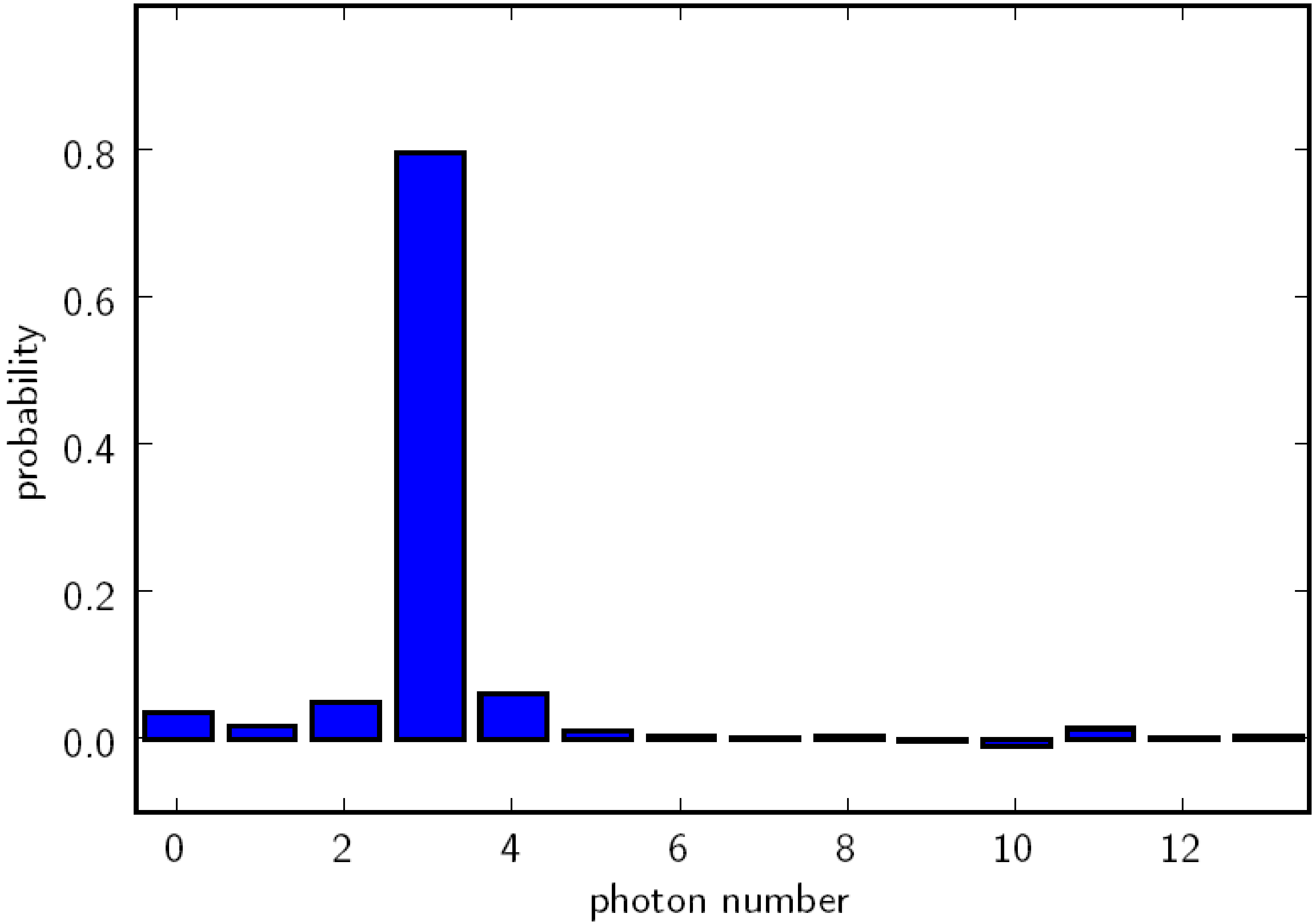
Fock States: Photon number distribution for $|1\rangle$



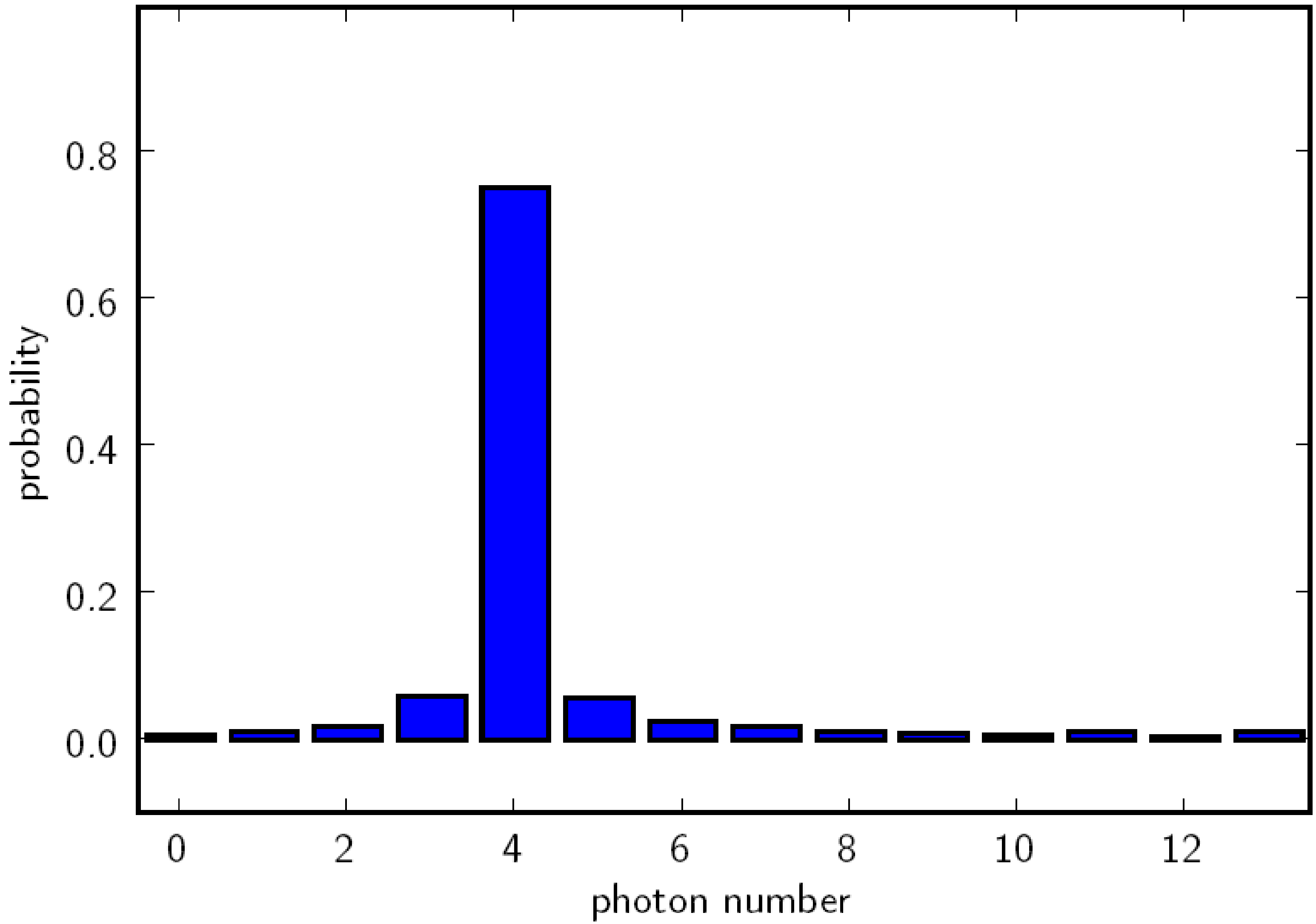
Fock States: Photon number distribution for $|2\rangle$



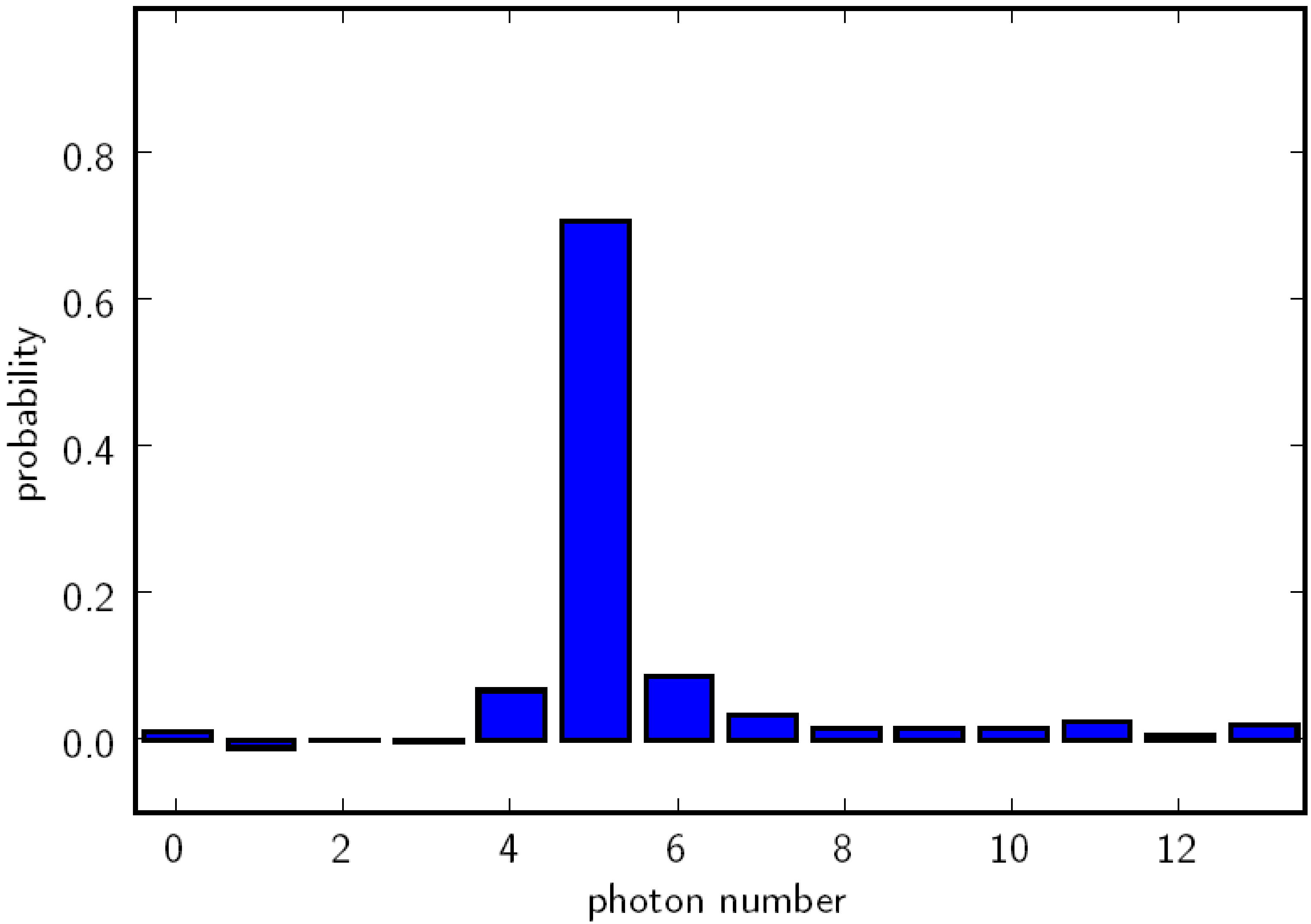
Fock States: Photon number distribution for $|3\rangle$



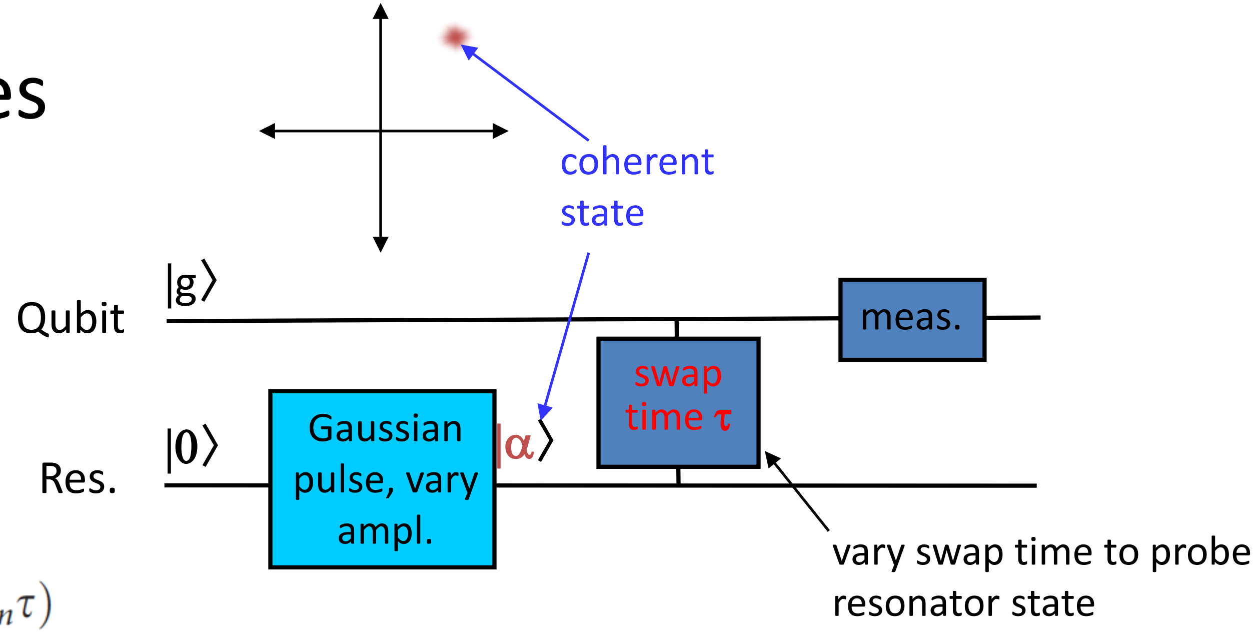
Fock States: Photon number distribution for $|4\rangle$



Fock States: Photon number distribution for $|5\rangle$

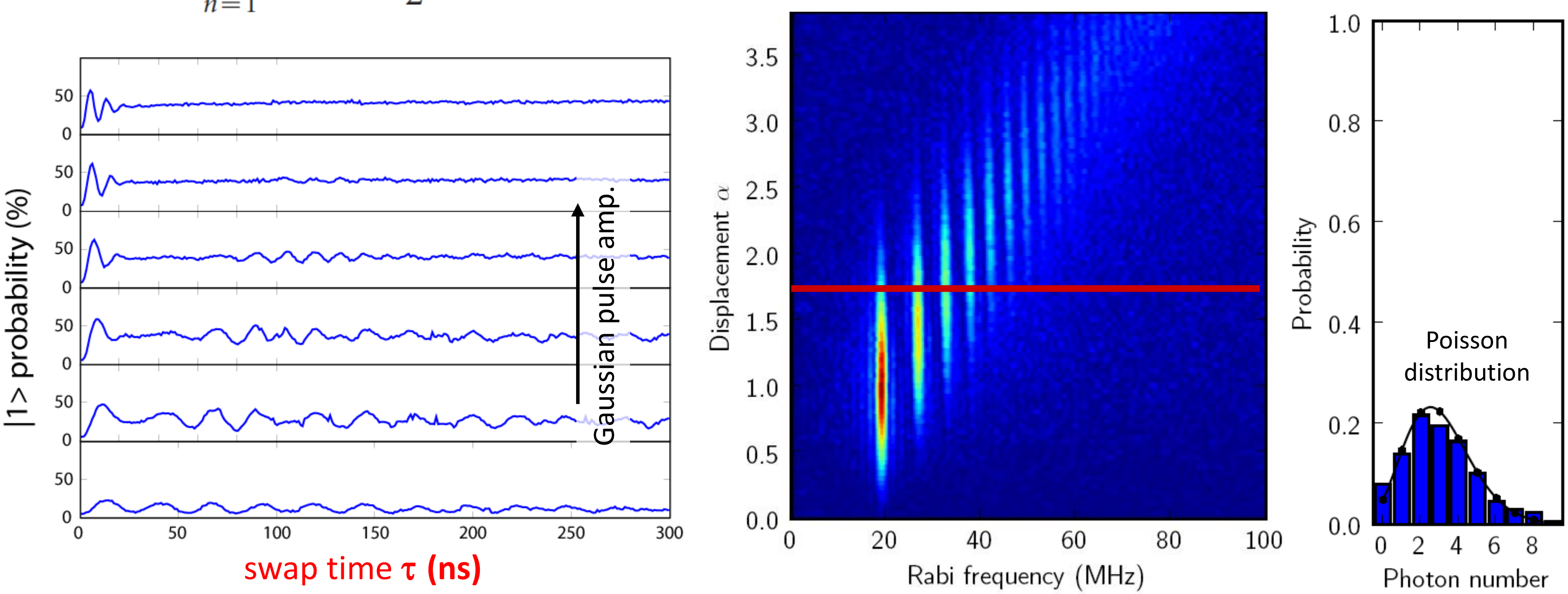


Coherent states



State preparation:

$$P_e(\tau) = \sum_{n=1}^{\infty} P_n \frac{1-\cos(\Omega_n \tau)}{2}$$

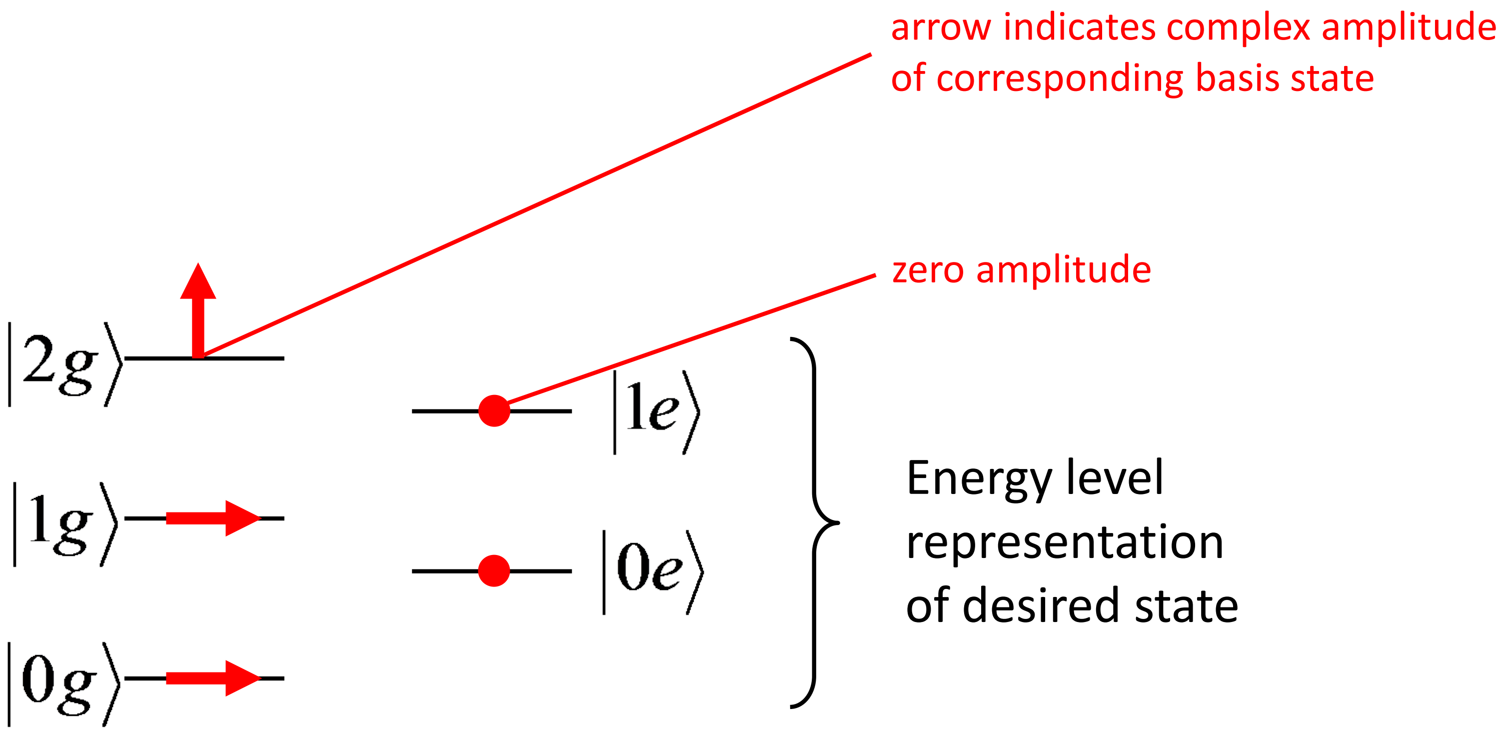


Generating Arbitrary States

Desired final state ➡ $|\psi\rangle = 0.577\left(|0\rangle + |1\rangle + i|2\rangle\right) \otimes |g\rangle$

Law and Eberly, PRL (1996)

Reverse-engineer final state by building pulse sequence backwards

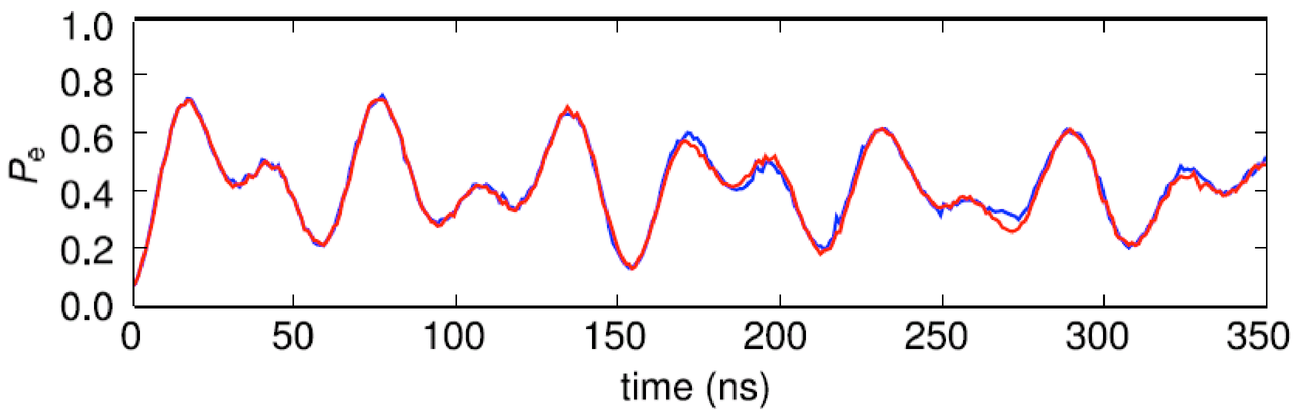


Generating Arbitrary States:
Quantum Digital to Analog Converter

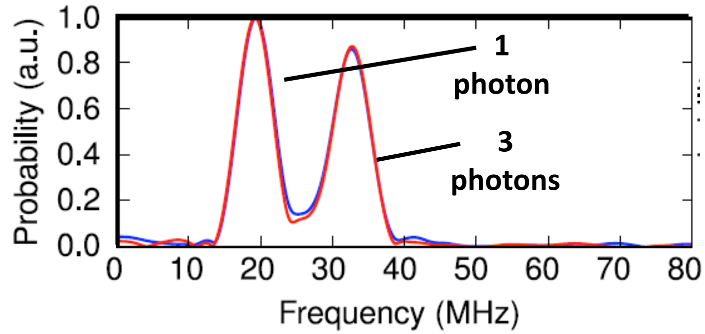
$$|\psi_a\rangle = |1\rangle + |3\rangle$$

Now try: $|\psi_b\rangle = |1\rangle + \boldsymbol{i}\,|3\rangle$

Measure using qubit and watching evolution (usual method):



Fourier transform yields number composition:



Cannot distinguish based on this measure!

Synthesizing arbitrary quantum states in a superconducting resonator

Max Hofheinz[1], H. Wang[1], M. Ansmann[1], Radoslaw C. Bialczak[1], Erik Lucero[1], M. Neeley[1], A. D. O'Connell[1], D. Sank[1], J. Wenner[1], John M. Martinis[1] & A. N. Cleland[1]

Vol 459 | 28 May 2009 | doi:10.1038/nature08005

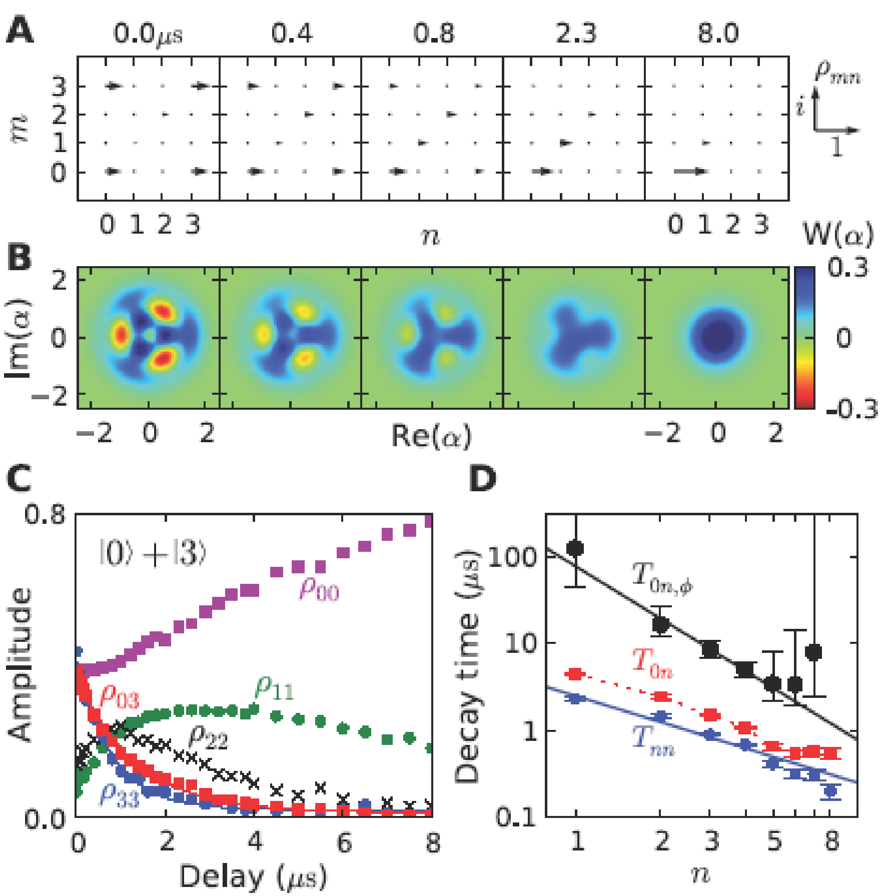
PRL 103, 200404 (2009) PHYSICAL REVIEW LETTERS week ending 13 NOVEMBER 2009

Decoherence Dynamics of Complex Photon States in a Superconducting Circuit

H. Wang, M. Hofheinz, M. Ansmann, R. C. Bialczak, Erik Lucero, M. Neeley, A. D. O'Connell, D. Sank, M. Weides, J. Wenner, A. N. Cleland,* and John M. Martinis†

Off-Diagonal Decay of Density Matrix

$|0\rangle + |N\rangle$ (far-element decay)



$|2\rangle + |3\rangle$ (decay cascade)