Photon-antibunching and sub-Poissonian photon statistics

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It is shown by example that sub-Poissonian photon-counting statistics need not imply photon antibunching, but can be accompanied by photon bunching, i.e., by the tendency of two photons to be close together more frequently than further apart. Some comments on the relation between antibunching and sub-Poissonian statistics are made.

It is well known that the phenomena of photon antibunching and sub-Poissonian photon-counting statistics are manifestations of certain nonclassical states of lights, i.e., states having no description in terms of electromagnetic waves. Both these effects were first observed in the process of resonance fluorescence from an atom,[1,2] but they have since been observed in other ways also. Perhaps because the effects often tend to occur together, there has been a widespread tendency in the literature to mix them up or even to regard them as one and the same. It is the purpose of this paper to emphasize that the two effects are distinct and need not occur together.

Let us first make clear what we mean. Photon bunching is the tendency of photons (or other particles) to distribute themselves preferentially in bunches rather than at random, so that when a light beam falls on a photodetector more photon pairs are detected close together in time than further apart. Antibunching is the opposite effect, in which fewer photon pairs are detected close together than further apart. As was shown by Glauber,[3] when light falls on a photodetector, the joint probability density $P_2(t,t+\tau)$ for detecting one photon at time t and another one at later time $t+\tau$ is given by

$$P_2(t,t+\tau)=K\langle \Upsilon{:}\hat{I}(t)\hat{I}(t+\tau){:}\rangle \ , \tag{1}$$

where K is a constant characteristic of the detector, $\hat{I}(t)$ is the light intensity operator, and the operator product is written in normal order and in time order. By introducing the normalized correlation function

$$\lambda(t,t+\tau)\equiv\frac{\langle T{:}\hat{I}(t)\hat{I}(t+\tau){:}\rangle}{\langle\hat{I}(t)\rangle\langle\hat{I}(t+\tau)\rangle}-1 \ , \tag{2}$$

we can rewrite Eq. (1) for a stationary light field in the form

$$P_2(t,t+\tau)=K\langle\hat{I}\rangle^2[1+\lambda(\tau)] \ , \tag{3}$$

because λ depends only on the time difference τ and $\langle\hat{I}(t)\rangle$ is time independent. It is then apparent that bunching, which implies that $P(t,t+\tau)$ falls with increasing τ from $\tau=0$, occurs when $\lambda(0)>\lambda(\tau)$, whereas antibunching, which implies that $P_2(t,t+\tau)$ rises with increasing τ from $\tau=0$, occurs when $\lambda(0)<\lambda(\tau)$. The last condition violates the Schwarz inequality for a field obeying classical statistics, and therefore one can occur only in a quantum field.

By contrast, some authors have regarded negative values of $\lambda(0)$ as the signature of photon antibunching, irrespective of the question whether $\lambda(\tau)$ increases with τ from $\tau=0$ or not. This leads to the somewhat strange conclusion that $\lambda(\tau)=-$const represents antibunching, even though the photodetection probability density $P_2(t,t+\tau)$ would then be independent of τ, and would not favor either short or long time intervals τ between photons. Nevertheless, this definition can be found in articles reviewing the subject,[4] although some authors have given both definitions.[5,6]

On the other hand, from the probability[7,8] $p(N,t,t+T)$ that the detector registers N photodetections in the time interval from t to $t+T$, it may readily be shown that for a stationary field

$$\langle(\Delta N)^2\rangle-\langle N\rangle=\langle N\rangle^2\frac{1}{T^2}\int_{-T}^{T}d\tau(T-|\tau|)\lambda(\tau) \ . \tag{4}$$

It follows that the sign of $\lambda(\tau)$ is crucial in determining whether $\langle(\Delta N)^2\rangle$ exceeds $\langle N\rangle$ or $\langle N\rangle$ exceeds $\langle(\Delta N)^2\rangle$, i.e., whether the photon-counting statistics are super- or sub-Poissonian. A negative correlation function $\lambda(\tau)\leq 0$ for all τ always implies sub-Poissonian statistics, and this is also a signature of a nonclassical state.

It has been shown by Surendra Singh[9] that in the process of resonance fluorescence the photon-counting statistics can be either super- or sub-Poissonian, even though the photons always exhibit antibunching. In the following we draw attention to the opposite phenomenon, that the counting statistics can be sub-Poissonian when the photons exhibit bunching in time.

Let us consider a plane, polarized electromagnetic field in the Fock state $|\{n\}\rangle$, in which the occupation number n_{ks} for the plane-wave mode $\mathbf{k},s$ is zero unless $\mathbf{k}$ points in some given direction and $s=s_0$. Hence the nonvacuum modes can be labeled by the frequency suffix ω. It is not difficult to show from the definition, that if $\hat{I}(t)$ is the photon density defined by

$$\hat{I}(t)=\frac{1}{L^3}\sum_{\mathbf{k},s}\sum_{\mathbf{k}',s'}\hat{a}^{\dagger}_{ks}\hat{a}_{k's'}e^{i(\omega-\omega')t} \ , \tag{5}$$

then

$$\lambda(\tau)=\left[\left|\sum_{\omega}n_{\omega}e^{-i\omega\tau}\right|^2-\sum_{\omega}n_{\omega}^2-\sum_{\omega}n_{\omega}\right]\Bigg/\left[\sum_{\omega}n_{\omega}\right]^2 . \tag{6}$$

It follows that for a single occupied mode,

$$\lambda(\tau) = -1/n_{\omega} , \tag{7}$$

so that the photon-counting statistics are necessarily sub-Poissonian. However, there is no antibunching in time because $\lambda(\tau)$ and $P_2(t, t+\tau)$ do not depend on τ and long intervals are not preferred over short intervals. On the other hand, if two modes of frequency ω_1 and ω_2 are occupied with $n_{\omega_1} = n_{\omega_2} = \frac{1}{2}n$, then we find from Eq. (6),

$$\lambda(\tau) = \tfrac{1}{2}\cos(\omega_1 - \omega_2)\tau - 1/n , \tag{8}$$

and from Eqs. (4) and (8),

$$\langle (\Delta N)^2 \rangle - \langle N \rangle = \langle N \rangle^2 \left[\frac{1}{2} \left[\frac{\sin(\omega_1 - \omega_2)T/2}{(\omega_1 - \omega_2)T/2} \right]^2 - \frac{1}{n} \right] . \tag{9}$$

It follows from this that for certain counting intervals, such as $T = 2\pi/|\omega_1 - \omega_2|$, the photon-counting statistics are evidently sub-Poissonian. Yet $\lambda(\tau)$ falls with increasing τ from $\tau = 0$, so that the photons are more likely to be close together; they therefore exhibit short time bunching rather than antibunching.

These considerations, although contrived, show that sub-Poisson counting statistics need not be associated with antibunching but can be accompanied by bunching. Therefore, sub-Poisson statistics and antibunching are distinct effects, and it is important that the definitions of these phenomena not be confused.

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