Handout

Calculate α for semiclassical fields:

Let $I_1(t) =$ intensity of optical disturbance going to left

$I(t) =$ intensity of optical disturbance going to right

$I_r(t) =$ intensity going to detector r; $I_t(t) \rightarrow$ detector t

$\overline{I(t)} \equiv$ time average

$$N_1 \propto \eta_1 \overline{I_1}$$

$$N_{1r} \propto \eta_1 \eta_r \overline{I_1 I_r} = \eta_1 \eta_r \frac{\overline{I_1 I}}{2} \qquad \text{(1/2 from beamsplitter)}$$

$$N_{1t} \propto \eta_1 \eta_t \overline{I_1 I_t} = \eta_1 \eta_t \frac{\overline{I_1 I}}{2}$$

$$N_{1rt} \propto \eta_1 \eta_r \eta_t \overline{I_1 I_r I_t} = \eta_1 \eta_r \eta_t \overline{\frac{1}{4} I_1 I^2}$$

$$\therefore \qquad \alpha = \frac{\left(\frac{1}{4}\eta_1 \eta_r \eta_t \overline{I_1 I^2}\right)\eta_1 \overline{I_1}}{\left(\frac{1}{2}\eta_1 \eta_r \overline{I_1 I}\right)\left(\frac{1}{2}\eta_1 \eta_t \overline{I_1 I}\right)} = \frac{\overline{I_1 I^2}\,\overline{I_1}}{\left(\overline{I_1 I}\right)^2}$$

Let $f \equiv \sqrt{I_1} I, \quad g \equiv \sqrt{I_1} \quad \Rightarrow \alpha = \dfrac{\overline{f^2}\,\overline{g^2}}{\left(\overline{fg}\right)^2}$

But $\left(\overline{f^2}\right)\left(\overline{g^2}\right) \geq \left(\overline{fg}\right)^2$ (Cauchy-Schwartz Inequality)

$$\therefore \alpha_{semi-classical} \geq 1 \quad (= 1 \text{ for coherent state})$$

Proof: $\overline{fg} = \int dr P(r) f(r) g(r) = \int dr \left[A \equiv \sqrt{P(r)} f(r)\right]\left[B \equiv \sqrt{P(r)} g(r)\right] = (A,B)$

$\overline{f^2} = \int dr \sqrt{P(r)} f(r) \sqrt{P(r)} f(r) = (A,A) \qquad \overline{g^2} = (B,B)$

$$(\mathrm{A},\mathrm{A})(B,B) \quad \geq \quad (\mathrm{A},\mathrm{B})^2$$