PHYS514 **Homework 4**
Fall 2013 **due 10/15**

1. von Neumann entropy

The von Neumann entropy S is defined as $S = -tr(\rho \ln \rho)$. If ρ is written in terms of its eigenvectors $\{|\lambda_i\rangle\}$, then $S = \sum_i \lambda_i \ln \lambda_i$, where $\{\lambda_i\}$ are the eigenvalues of ρ.

a. What is S for a two-level system in the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$?

b. What is S for a two-level system in the mixed state $\rho = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$?

c. Consider the entangled state of two two-level systems $|\psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle_a|-\rangle_b - |-\rangle_a|+\rangle_b)$. Compute ρ and the reduced density matrices ρ_a and ρ_b. What is S for ρ, ρ_a, and ρ_b?

d. Consider the non-entangled state of two two-level systems $\frac{1}{\sqrt{2}}(|+\rangle_a + |-\rangle_a) \otimes \frac{1}{\sqrt{2}}(|+\rangle_b - |-\rangle_b)$. Compute ρ and the reduced density matrices ρ_a and ρ_b. What is S for ρ, ρ_a, and ρ_b?

What general properties can you infer about the von Neumann entropy? How might it be useful for characterizing entanglement?

2. Foot 1.8…*classical radiative lifetime*

3. Foot 7.5…*saturation*

4. Foot 7.6…*transitions in hydrogen*