Units

Note Title 1/15/2005

We'll mix up Gaussian/cgs, mks/SI, and atomic units (a.u.)

— this is just an unfortunate reality in AMO physics (many of us raised with red edition of Jackson, and theorists push a.u.)

see Bransden and Joachain, Appendix 14 for a.u.:

$m_e = e = \hbar = a_0 = 1$, $c = 137.036$ a.u. (people are inconsistent about this)

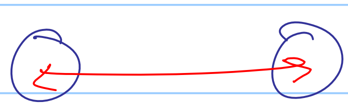
Table A14.2 Atomic units.

Quantity	*Unit*	*Physical significance*	*Value*
Mass	m or m_e	Electron mass	$9.109\,38 \times 10^{-31}$ kg
Charge	e	Absolute value of electron charge	$1.602\,18 \times 10^{-19}$ C
Angular momentum	$\hbar$	Planck's constant divided by (2π)	$1.054\,57 \times 10^{-34}$ J s
Length	a_0	Bohr radius for atomic hydrogen (with infinite nuclear mass)	$5.291\,77 \times 10^{-11}$ m
Velocity	$v_0 = \alpha c$	Magnitude of electron velocity in first Bohr orbit	$2.187\,69 \times 10^{6}$ m s^{-1}
Momentum	$p_0 = mv_0$	Magnitude of electron momentum in first Bohr orbit	$1.992\,85 \times 10^{-24}$ kg m s^{-1}
Time	$\dfrac{a_0}{v_0}$	Time required for electron in first Bohr orbit to travel one Bohr radius	$2.418\,88 \times 10^{-17}$ s

Frequency	$\frac{v_0}{2\pi a_0}$	Angular frequency of electron in first Bohr orbit (v_0/a_0) divided by (2π)	$6.579\,69 \times 10^{15}\ s^{-1}$
Energy ("Hartree"–H)	$\frac{e^2}{4\pi\varepsilon_0 a_0} = \alpha^2 mc^2$	Twice the ionisation potential of atomic hydrogen (with infinite nuclear mass)	$4.359\,74 \times 10^{-18}$ J $= 27.2114$ eV
Wave number	$\frac{\alpha}{2\pi a_0} = 2\tilde{R}(\infty)$	Twice the Rydberg constant, i.e. twice the wave number corresponding to the ionisation potential of atomic hydrogen (with infinite nuclear mass)	$2.194\,75 \times 10^{7}\ m^{-1}$
Electric field strength	$\frac{e}{(4\pi\varepsilon_0)a_0^2}$	Strength of the Coulomb field experienced by an electron in the first Bohr orbit of atomic hydrogen (with infinite nuclear mass)	$5.142\,21 \times 10^{11}\ V\,m^{-1}$
Magnetic field strength	$\frac{\hbar}{e a_0^2}$		$2.350\,52 \times 10^{5}$ T

<u>Let's try an example:</u>

interaction between 2 atoms:

^{87}Rb, 5s  ^{87}Rb, 5s

R – internuclear distance

$$V = \frac{C_6}{R^6} + \ldots$$ interaction energy

^{87}Rb, 5s atoms – $C_6 \simeq 4700$ a.u.

What's V in joules when $R = 100$ nm?

in a.u., $R = \frac{100\,\text{nm}}{a_0} \simeq \frac{100\,\text{nm}}{0.05\,\text{nm}} \simeq 2000$ a.u.

$$V = \frac{4700}{2000^6} = 7 \times 10^{-17}\ H$$

note that using a.u. does not serve a real purpose here — but C_6 reported this way in literature!

$$= 7 \times 10^{-17}\, H \cdot \frac{4.4 \times 10^{-18}\ J}{H} = 3.2 \times 10^{-34}\ J$$

For cgs, see "red" Jackson 619–620:

Quantity	Gaussian	mks
Velocity of light	c	$(\mu_0\epsilon_0)^{-1/2}$
Electric field (potential, voltage)	$\mathbf{E}(\Phi, V)$	$\sqrt{4\pi\epsilon_0}\,\mathbf{E}(\Phi, V)$
Displacement	$\mathbf{D}$	$\sqrt{\frac{4\pi}{\epsilon_0}}\,\mathbf{D}$
Charge density (charge, current density, current, polarization)	$\rho(q, \mathbf{J}, I, \mathbf{P})$	$\frac{1}{\sqrt{4\pi\epsilon_0}}\rho(q, \mathbf{J}, I, \mathbf{P})$
Magnetic induction	$\mathbf{B}$	$\sqrt{\frac{4\pi}{\mu_0}}\,\mathbf{B}$
Magnetic field	$\mathbf{H}$	$\sqrt{4\pi\mu_0}\,\mathbf{H}$
Magnetization	$\mathbf{M}$	$\sqrt{\frac{\mu_0}{4\pi}}\,\mathbf{M}$
Conductivity	σ	$\frac{\sigma}{4\pi\epsilon_0}$
Dielectric constant	ϵ	$\frac{\epsilon}{\epsilon_0}$
Permeability	μ	$\frac{\mu}{\mu_0}$
Resistance (impedance)	$R(Z)$	$4\pi\epsilon_0 R(Z)$
Inductance	L	$4\pi\epsilon_0 L$
Capacitance	C	$\frac{1}{4\pi\epsilon_0}C$

Physical Quantity	Symbol	Rationalized mks	Gaussian	
Length	l	1 meter (m)	10^2	centimeters (cm)
Mass	m	1 kilogram (kg)	10^3	grams (gm)
Time	t	1 second (sec)	1	second (sec)
Force	F	1 newton	10^5	dynes
Work Energy	W U	1 joule	10^7	ergs
Power	P	1 watt	10^7	ergs sec^{-1}
Charge	q	1 coulomb (coul)	3×10^9	statcoulombs
Charge density	ρ	1 coul m^{-3}	3×10^3	statcoul cm^{-3}
Current	I	1 ampere (coul sec^{-1})	3×10^9	statamperes
Current density	J	1 amp m^{-2}	3×10^5	statamp cm^{-2}
Electric field	E	1 volt m^{-1}	$\frac{1}{3} \times 10^{-4}$	statvolt cm^{-1}
Potential	Φ, V	1 volt	$\frac{1}{300}$	statvolt
Polarization	P	1 coul m^{-2}	3×10^5	dipole moment cm^{-3}
Displacement	D	1 coul m^{-2}	$12\pi \times 10^5$	statvolt cm^{-1} (statcoul cm^{-2})
Conductivity	σ	1 mho m^{-1}	9×10^9	sec^{-1}
Resistance	R	1 ohm	$\frac{1}{9} \times 10^{-11}$	sec cm^{-1}
Capacitance	C	1 farad	9×10^{11}	cm
Magnetic flux	ϕ, F	1 weber	10^8	gauss cm^2 or maxwells
Magnetic induction	B	1 weber m^{-2}	10^4	gauss
Magnetic field	H	1 ampere-turn m^{-1}	$4\pi \times 10^{-3}$	oersted
Magnetization	M	1 ampere m^{-1}	10^{-3}	magnetic moment cm^{-3}
*Inductance	L	1 henry	$\frac{1}{9} \times 10^{-11}$	