PH 514

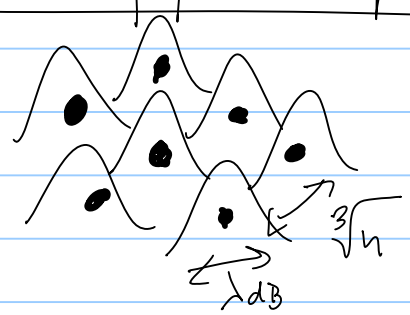
BEC and Fermi gases

Note Title 4/6/2008

BEC: ^{87}Rb and ^{23}Na first in 1995
then: ^{7}Li, ^{85}Rb, ^{133}Cs, H, ^{39}K, ^{41}K, $^{4}He^{*}$, ^{174}Yb, ^{52}Cr, Sr, ...

Fermi gases: ^{40}K first in 1999
then: ^{6}Li

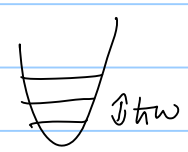
Usual picture of quantum degeneracy: $n\lambda_{dB}^3 = 1$



in this regime quantum statistics dominate → must write down a properly symmetrized many-body wavefunction

To be more exact:

- bosons, specialized to a 3D harmonic potential



  - phase transition to BEC when the number of atoms in the ground state becomes "macroscopic"
  - or, when excited states are just enough to hold all atoms N, and cooling to lower T will result in an atom entering the ground state
  - BEC = macroscopic occupation of a single quantum state

Statistical mechanics (ignoring interactions):

$$f(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/kT} - 1}$$ distribution function

$P(\varepsilon) = g(\varepsilon) f(\varepsilon)$ probability to find a particle with energy ε, where $g(\varepsilon)$ = density of states

- for a 3D harmonic oscillator, $g(\varepsilon) = \frac{\varepsilon^2}{2(\hbar\omega)^3}$

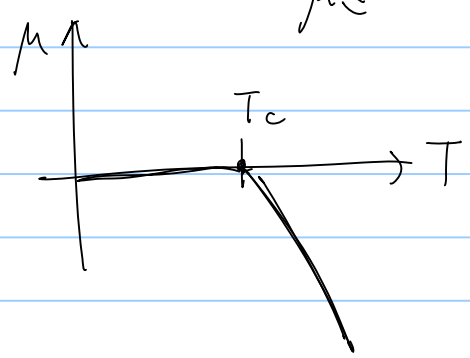
- chemical potential μ determined by $N = \int_0^\infty g(\varepsilon) f(\varepsilon)\, d\varepsilon$
- usually we use the microcanonical ensemble, so that temperature is determined by $U = \int \varepsilon\, g(\varepsilon) f(\varepsilon)\, d\varepsilon$, where U is the total energy
- i.e., μ is the Lagrange multiplier that conserves particle number, and T is the Lagrange multiplier that conserves energy

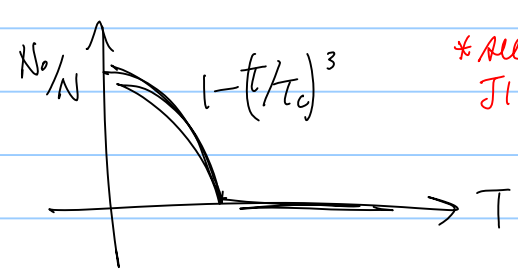
phase transition determined by:

$N = N_{ex} = \int_0^\infty g(\varepsilon) f(\varepsilon)\, d\varepsilon$ (since $g(0)$ mistakenly equals 0)

and $N_0 = N - N_{ex}$ is the number in the condensate

$\mu \leq 0$ always for bosons:





*see original JILA data in slides

$T_c = 0.94\, \hbar\omega N^{1/3}$ (equivalent to $n_{ph} \lambda_{dB}^3 = 2.6$)

Data primarily from time-of-flight imaging — see original JILA data in the slides

- "thermal component" N_{ex} (or "N_t") has rms velocity roughly determined by $\frac{1}{2}mv^2 = \frac{1}{2}kT$ along any direction (there is a correction to this for $T \lesssim T_c$), and the velocity distribution is approximately Gaussian (again, not exactly for $T \lesssim T_c$)
- momentum distribution of the thermal component must be isotropic, so we observe something spherical at long expansion times
- but, for the BEC, the momentum distribution is not isotropic for a trap with $\omega_r \neq \omega_z$
  - think about the ground state of a harmonic oscillator

$$\sqrt{\langle v^2 \rangle} = \sqrt{\frac{\hbar\omega}{m}}$$

  - so, if interactions are weak, then we get anisotropic expansion and the aspect ratio reverses at long times

<u>What about interactions?</u>

- we usually treat this problem using mean field theory and at $T=0$

<u>mean field theory</u>: $\hat{\psi} = \phi + \hat{\psi}'$, where $\hat{\psi}$ and $\hat{\psi}'$ are field operators, i.e., $\hat{\psi} = \sum_\alpha \psi_\alpha(\vec{x}) a_\alpha$, and ϕ is just a number — the order parameter!

• the condensate wavefunction is described very well by the Gross-Pitaevskii equation

It all starts with:

$$H = \sum_i \left[-\frac{\hbar^2}{2m} \nabla_i^2 + V(\vec{r}_i) \right] + \sum_{i<j} \frac{4\pi\hbar^2}{m} a\, \delta^3(\vec{r}_i - \vec{r}_j)$$

for a BEC: $\psi = \prod_i \phi_0(\vec{r}_i)$, where we need to find $\phi_0(\vec{r})$

$$\langle H \rangle = E = \int d^3\vec{r} \left[\frac{\hbar^2}{2m} |\vec{\nabla}\phi_0|^2 + V(r)|\phi_0|^2 + \frac{N}{2} u_0 |\phi_0|^4 \right]$$

factor of ½ to avoid double counting, $|\phi_0|^4$ just like our calculation for 2 particles in a box

where $u_0 = \frac{4\pi\hbar^2 a}{m}$

• if we minimize δE with respect to $\delta\psi$ and $\delta\psi^*$, then we find $\psi = \frac{1}{\sqrt{N}}\phi_0$ (sorry, switching notation) has to satisfy

GP equation: $$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + \frac{4\pi\hbar^2 a}{m}|\psi|^2 \right]\psi = E\psi = \mu\psi$$

• where we've introduced a "chemical potential" μ

- we can also write down a time-dependent GP equation, with $\psi(\vec{r},t) = e^{-i\mu t/\hbar}\psi(\vec{r})$
- all of this is OK in the dilute limit $na^3 \ll 1$ (we can ignore changes in wavefunction in a region of size a)

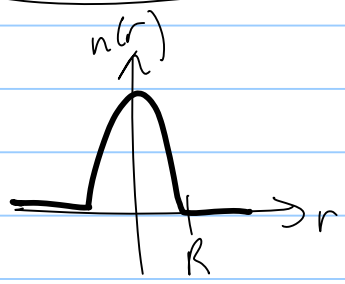
What does the ground state look like?

• in the "Thomas Fermi" approximation, we ignore the kinetic energy

$$\left[V(r)+\frac{4\pi\hbar^2 a}{m}|\psi(r)|^2\right]\psi(r)=\mu\,\psi(r)$$

so, $V(r)+\frac{4\pi\hbar^2 a}{m}|\psi(r)|^2=\mu$, or $n(r)=|\psi(r)|^2=\frac{\mu-V(r)}{U_0}$

<u>physical interpretation:</u> energy to add a particle must be the same everywhere, or $V(r)+n(r)U_0=\text{constant}$



- in a harmonic trap, we get a parabola

$$\mu=\frac{15^{2/5}}{2}\left(\frac{Na}{a_{ho}}\right)^{2/5}\hbar\omega \qquad a_{ho}=\sqrt{\hbar/m\omega}$$

$$R\simeq 1.7\left(\frac{Na}{a_{ho}}\right)^{1/5}a_{ho}$$

this grows as a and N grow; collapse occurs beyond critical N for $a<0$

- this approximation fails near the edges of the cloud, where the kinetic energy per particle becomes bigger than the interaction energy per particle

*see original Han data in the slides

- one last thing: the shortest distance over which ψ can change is the healing length ξ (typically few hundred nm)

$$\underbrace{\frac{\hbar^2}{2m\xi^2}}_{\text{kinetic energy}}=\underbrace{nU_0}_{\text{interaction energy (or chemical potential)}}$$

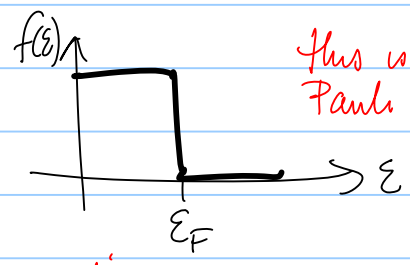
- if the kinetic energy is greater than μ, then atoms (statistically) leave the BEC

Fermi gases

- Of course, fermions do something rather different

$$f(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/kT}+1}$$

at zero temperature:



this is just the Pauli exclusion principle at work

* see thermodynamic plots in the slides

the Fermi temperature T_F is defined by: $N = \int_0^{k_B T_F} g(\varepsilon) f(\varepsilon, T=0)\, d\varepsilon$

quantum degeneracy is defined by $T < T_F$

- complications for cooling fermions below T_F:
  1. require non-identical particles, otherwise no s-wave collisions (and higher partial waves are threshold suppressed)
     - in the first experiment, 2 spin states were used ($|9/2, 9/2\rangle$ and $|7/2, 7/2\rangle$)
     - can also use a boson "bath" for sympathetic cooling
  2. collisions, which require a change in energy of individual particles, can only occur within kT of T_F at low temperature — all collisions cease at $T=0$!

     * see DeMarco data in the slides

     — this is called "Pauli blocking"

How is degeneracy detected?

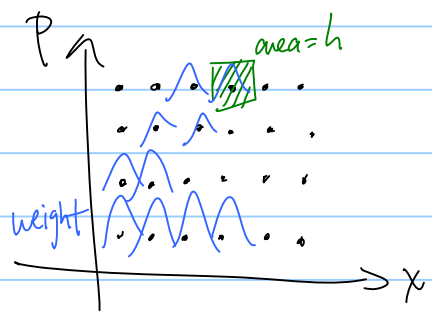
- through the shape of the momentum profile

semiclassically: $n(p) = \frac{1}{(2\pi\hbar)^3} \int d^3\vec{r} \, \frac{1}{e^{[H(\vec{r},\vec{p})-\mu]/kT}+1}$

where $H = p^2/2m + \frac{1}{2}m\omega^2 r^2$

- this comes from the Thomas–Fermi approximation
  - each point in phase space is assigned a definite $(\vec{x},\vec{p})$
  - the distribution of weights is: $w(r,p) = \underbrace{\frac{1}{(2\pi\hbar)^3}}_{\text{minimum volume in phase space a la Heisenberg}} \frac{1}{\frac{1}{z}e^{H(\vec{r},\vec{p})/kT}+1}$

    where $z = e^{\mu/kT}$ is the fugacity



- $n(\vec{p})$ can be written in terms of certain hypergeometric functions...

  $n(p) = f(T, z)$ and is just a function of T and z

  - in fact, z is only a function of T/T_F, and it measures the deviation of $n(p)$ from Gaussian

    $\left(n(p) \propto e^{-p^2/2mkT} \text{ for } z \to 0 \right)$

- So, we fit images to this profile, get η, and therefore T/T_F

  * see DeMarco data in the slides