Isotope shifts

PHYS514

Note Title 12/28/2004

Various effects; we'll consider 2:
① reduced mass
② finite nuclear volume

... many other things we're leaving out
"hyperfine anamolies" — ex. nucleus has internal structure, so our multipole expansion is not right for very small r ($r <$ few fm)

① always negative

$$E_n = \frac{-e^2}{4\pi\varepsilon_0 a_\mu}\frac{Z^2}{2n^2} \propto \frac{M}{m+M} = \frac{1}{\frac{m}{M}+1} \sim \left(1 - \frac{m}{M}\right)$$

correction

heavier isotopes more tightly bound

② Nuclear charge has a finite volume — potential is not $1/r$ for small r

Loosely classified as a hyperfine effect, since it is a nuclear effect

We can make a guesstimate of this effect (real calculation would require a serious nuclear model)

Assume charge is uniformly distributed in a sphere of radius R

$$R = r_0 A^{1/3} \quad r_0 \simeq 1.2\,\text{fm}$$

potential energy of the electron:
$$V(r) = \begin{cases} \dfrac{Ze^2}{4\pi\varepsilon_0 2R}\left(\dfrac{r^2}{R^2} - 3\right) & r \leq R \\ -\dfrac{Ze^2}{4\pi\varepsilon_0 r} & r > R \end{cases}$$

to take off difference with H_0

$$H = H_0 + H' \qquad H' = \begin{cases} \dfrac{Ze^2}{(4\pi\varepsilon_0)2R}\left(\dfrac{r^2}{R^2} + \dfrac{2R}{r} - 3\right) & r \leq R \\ 0 & r \geq R \end{cases}$$

Hydrogenic Hamiltonian

$$\Delta E = \langle \psi_{n\ell m} | H' | \psi_{n\ell m} \rangle$$
$$= \frac{Ze^2}{4\pi\varepsilon_0 2R} \int_0^R |R_{n\ell}(r)|^2 \left(\frac{r^2}{R^2} + \frac{2R}{r} - 3\right) r^2 dr$$

for $r < R$, $R_{n\ell}(r) \sim R_{n\ell}(0)$ — only non-zero for s-states

$$\Delta E \sim \frac{Ze^2}{4\pi\varepsilon_0}\frac{R^2}{10}|R_{n0}(0)|^2 \sim \frac{Ze^2}{4\pi\varepsilon_0}\frac{2\pi}{5}R^2|\psi_{n00}(0)|^2$$

$$\Delta E \sim \frac{e^2}{4\pi\varepsilon_0}\frac{2}{5}R^2\frac{Z^4}{a_\mu^3 n^3}$$

larger mass (larger R) is less bound, $\Delta E \propto R^2$

→ Note that this formula only works for Hydrogen... see Foot for an equation for many electron atoms