Zeeman effect

Note Title 2/2/2005

How does an atom interact with a magnetic field?

$$H = -\vec{\mu}\cdot\vec{B}$$

$\boxed{\text{forget hyperfine structure temporarily}}$

$$\vec{\mu} = \vec{\mu}_\ell + \vec{\mu}_s$$

motion — see hyperfine structure notes

intrinsic spin

$$\vec{\mu}_\ell = -\frac{\mu_0}{\hbar}\vec{L} \qquad \vec{\mu}_s = -2\frac{\mu_0}{\hbar}\vec{S}$$

A note about g-factors →

g-factors basically used to keep track of effect of angular momentum coupling on relationship between $\vec{\mu}$ and $\vec{J}$

Generically, people write:

$$\vec{\mu} = g|\mu| \vec{J}/\hbar$$ where $\vec{J}$ is some angular momentum vector, and $|\mu|$ is a unit magnetic moment

In atomic physics, we like a convention by which the sign of μ is determined by the electric charge, and so we write:

$$\vec{\mu}_e = -g\mu_B \vec{S}/\hbar \quad \text{with } g \simeq 2$$

while, many other areas would write:

$$\vec{\mu}_e = g\mu_B \vec{S}/\hbar \quad \text{with } g \simeq -2$$

Unfortunately, you have to do the work to figure out which convention is being used, as with the nucleus:

$$\mu_{nucleus} = g_I \mu_N \vec{I}/\hbar$$

Consider the "low field limit" – $\mu B \ll \Delta E_{fs}$

rewrite $H = \frac{\mu_0}{\hbar} g_J \vec{J} \cdot \vec{B}$ keep j as a "good" quantum number

where: $g_J \vec{J} = \vec{L} + 2\vec{S}$ defines the Landé g-factor g_J

the easiest way to calculate this is using the projection theorem: $\langle j m_J' | \vec{X} | j m \rangle = \frac{\langle \vec{J} \cdot \vec{X} \rangle_j}{j(j+1)\hbar^2} \langle j m' | \vec{J} | j m \rangle$

which gives:

$$g_J = 1 + \frac{j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}$$

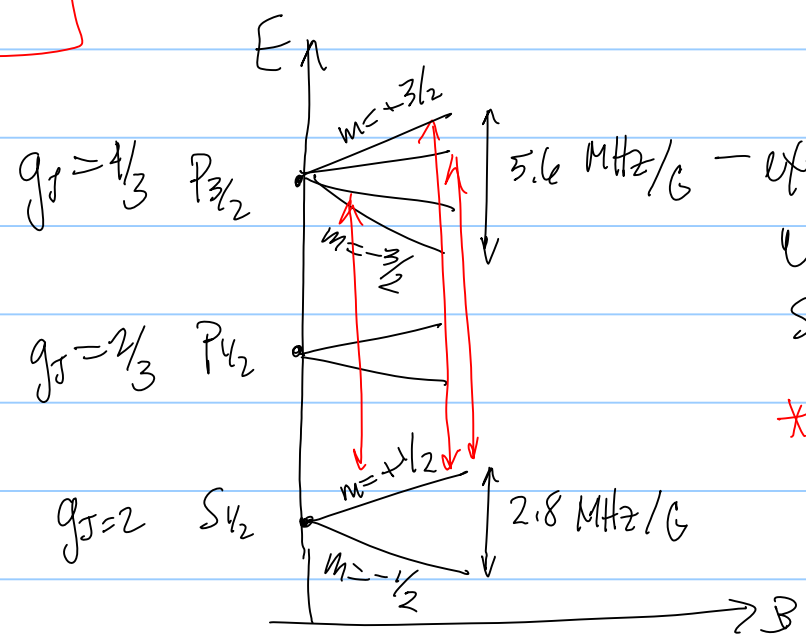
1st order energy shift:

$$\Delta E = \langle n j m | \left(\frac{\mu_0}{\hbar} g_J \vec{J} \cdot \vec{B} \right) | n j m \rangle$$

take $\vec{B}$ along $\hat{z}$ (free to do this if there is nothing else to break symmetry)

$$\Delta E = \frac{\mu_0}{\hbar} g_J B \langle n j m | J_z | n j m \rangle = \mu_0 m g_J B$$

$\boxed{\text{example:}}$



– extra magnetic moment compared with $S_{1/2}$ state comes from $\vec{L}$

* original Lamb shift measurement looked at these transitions and found $\Delta E(B=0) = 11$ GHz

<u>High field limit</u>: $\mu B \gg \Delta E_{fs}$

Paschen-Back effect – treat spin-orbit as a perturbation

We say that $\vec{L}$ and $\vec{S}$ are decoupled, so

$$H = \frac{\mu_0}{\hbar}\left(\vec{L} + 2\vec{S}\right)$$

$$E_{mag} = \langle n l s m_e m_s | H | n l s m_e m_s \rangle = \mu_0 B \left(m_e + 2 m_s\right)$$

this limit is rare for alkali atoms in AMO experiments (would require $B \sim 1000\,T$)

Now, let's put in hyperfine structure:

$$H = -(\vec{\mu}_e + \vec{\mu}_s + \vec{\mu}_I)\cdot\vec{B}$$

$$= \left(\frac{\mu_0}{\hbar}(\vec{L} + 2\vec{S}) - \frac{\mu_N}{\hbar} g_I \vec{I}\right)\cdot\vec{B}$$

low field limit: $\mu B \ll \Delta E_{hfs}$

treat F as a good quantum number and the interaction as a perturbation

$$H = \mu_0 g_F \vec{F}\cdot\vec{B}$$

$$g_F = g_J \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)} +$$

$$g_I \frac{\mu_N}{\mu_0} \frac{F(F+1) + I(I+1) - J(J+1)}{2F(F+1)}$$

very small $\sim \frac{1}{2000}$

$$\Delta E = \mu_0 g_F \langle F m_F | \vec{F}\cdot\vec{B} | F m_F \rangle = \mu_0 B g_F m_F$$

<u>High field</u> : $\mu B \gg \Delta E_{hfs}$ (but $\mu B \ll \Delta E_{fs}$)

officially the Back-Goudsmit regime, but often called Paschen-Back

we say that $\vec{J}$ and $\vec{I}$ are uncoupled, and we treat the hyperfine interaction as a perturbation on top of the Zeeman effect

$$E_{mag} = \langle J F; m_J m_I | \left[\frac{\mu_0}{\hbar}(\vec{L}+2\vec{S}) - \frac{\mu_N}{\hbar} g_I \vec{I}\right]\cdot\vec{B} | J I; m_J m_I \rangle$$

$$= \langle J I; m_J m_I | \left(\mu_0 g_J \vec{J} - \frac{\mu_N}{\hbar} g_I \vec{I}\right)\cdot\vec{B} | J I; m_J m_I \rangle$$

with $\vec{B}$ along $\hat{z}$ again

$$= \mu_0 \left(g_J m_J - \frac{\mu_N}{\mu_0} g_I m_I\right) B$$

small

this limit is reached for 100s of gauss fields

Additional hyperfine energy:

$$H_{hf} = A\,\vec{I}\cdot\vec{J}$$

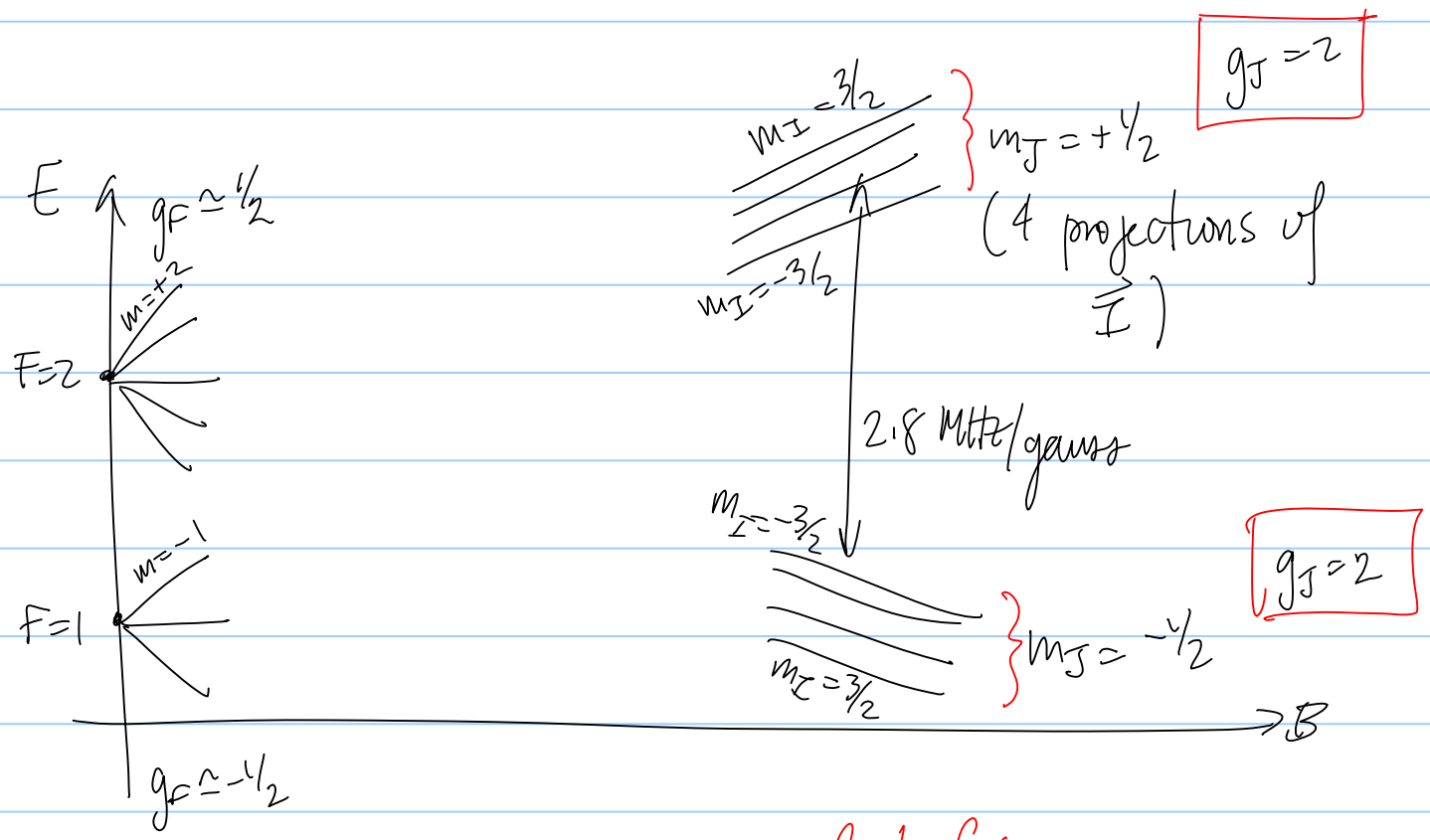
$$\Delta E = \langle m_I m_J | H_{hf} | m_I m_J \rangle$$

$$= A \langle m_I m_J | I_z J_z | m_I m_J \rangle$$

$$= A m_I m_J \cdot \hbar$$

I've been sneaky with atomic units here

Here's our picture so far: (use ^{87}Rb, $I = 3/2$)



low field — high field

what happens in between? (where most AMO experiments take place)

In the intermediate regime, we have to solve the problem exactly!

$$H = A\hbar\,\vec{I}\cdot\vec{J} - \vec{\mu}_J\cdot\vec{B} - \vec{\mu}_I\cdot\vec{B}$$

$$= A\hbar\,\vec{I}\cdot\vec{J} + \underbrace{\frac{\mu_0}{\hbar} g_J \vec{J}\cdot\vec{B} - \frac{\mu_N}{\hbar} g_I \vec{I}\cdot\vec{B}}$$

note that this is not diagonal in the $|F, m_F\rangle$ basis!

w/ $\vec{B}$ along $\hat{z}$

$$= A\hbar\,\vec{I}\cdot\vec{J} + \left(\frac{\mu_0}{\hbar} g_J J_z - \frac{\mu_N}{\hbar} g_I I_z\right) B$$

Now we need the matrix elements of H in the $|m_I m_S\rangle$ basis, then we solve the eigenvalue problem (that means that we write H as a matrix $\rightarrow$ the eigenvectors are our $|F, m_F\rangle$ states and the eigenvalues are the energies)

The answer for the energy of the ground state of alkali atoms is called the <u>Breit-Rabi equation</u>

- we write $\vec{I}\cdot\vec{J} = \vec{I}\cdot\vec{S} = I_z J_z + \frac{1}{2}(I_+ S_- + I_- S_+)$

- and find the matrix elements $\langle m_I' m_S' | H | m_I m_S\rangle$ (since $j = m_j = 0$)

☆ Note! Be cautious – differing conventions on signs of g-factors (especially g_I)

☆ Note! Look up g_I in Arimondo paper, Steck data, or Metcalf summary

the answer for the energy is:

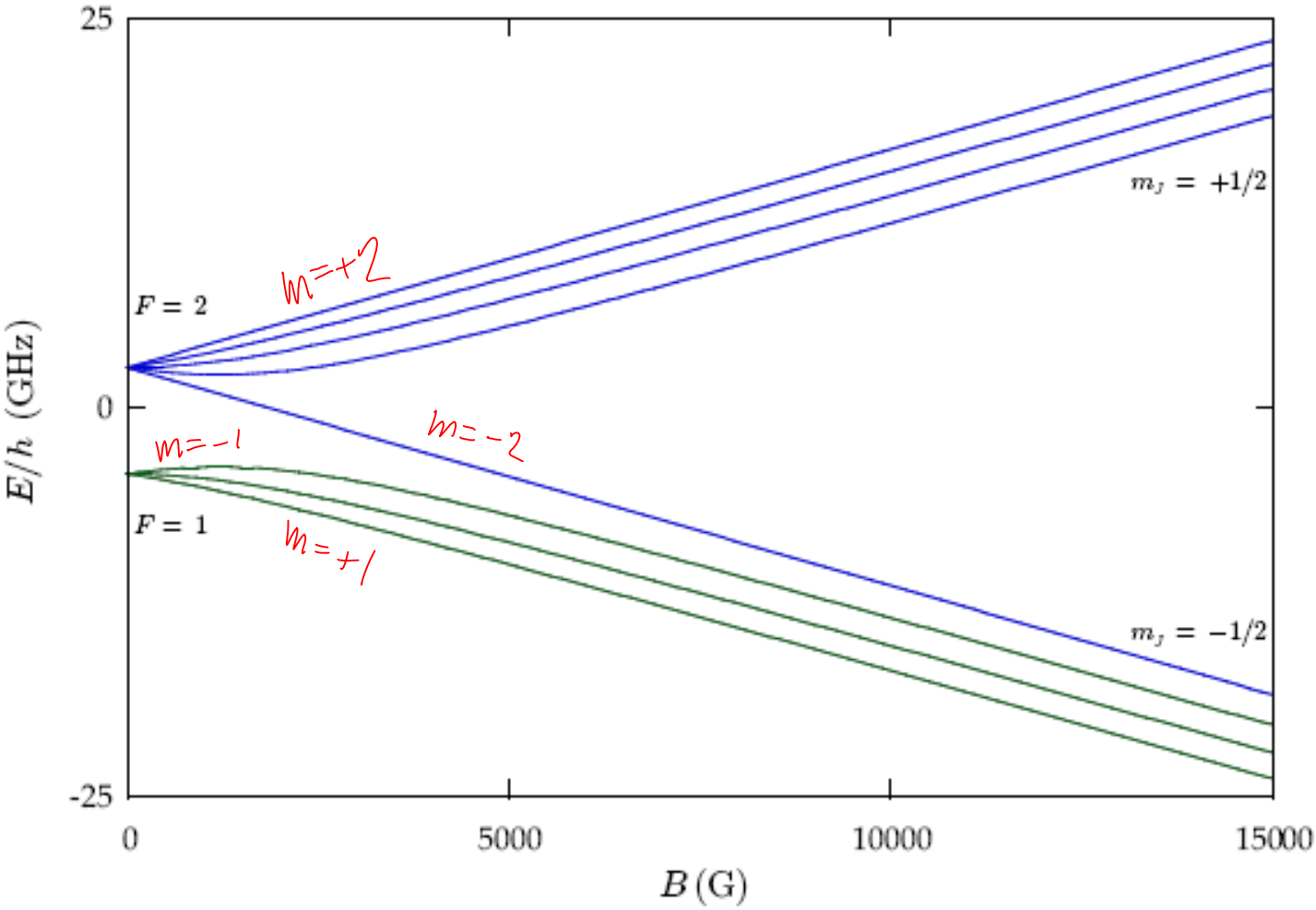
$$E = -\frac{\Delta E_{hfs}}{2(2I+1)} - g_I \mu_N m_F B \pm \frac{\Delta E_{hfs}}{2}\sqrt{1 + \frac{4 m_F x}{2I+1} + x^2}$$

+ : for $F_>$ state
− : for $F_<$ state

where $x = \dfrac{\left(g_J + g_I \frac{\mu_N}{\mu_0}\right)}{\Delta E_{hfs}} \mu_0 B$

remember that $\Delta E_{hfs} = \frac{1}{2} A (2I+1) h$

In the end, ^{87}Rb looks like:



* It is very important to understand that

① The magnetic moment of the atom depends on field

$$\mu = -\frac{\partial E}{\partial B}$$

it's just the slope of that graph!

② The states depend on field

$$|F, m_F\rangle = \alpha(B)\,|m_F - 1/2, 1/2\rangle + \beta(B)\,|m_F + 1/2, -1/2\rangle$$

no longer just Clebsch-Gordan coefficients - field dependent mixing

Finding $E(B, F, m_F)$, $\alpha(B)$, and $\beta(B)$ is your homework problem!

Let's look at Hydrogen:

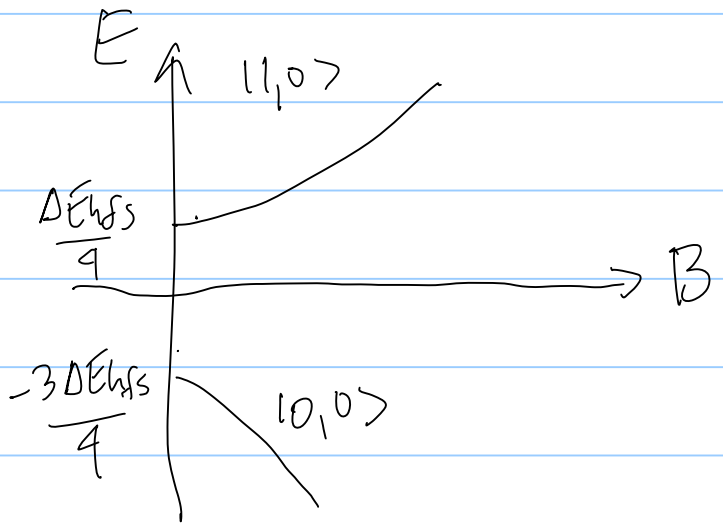
$$E = -\frac{\Delta E_{hfs}}{2(2I+1)} - g_I \mu_N m_F B \pm \frac{\Delta E_{hfs}}{2}\sqrt{1+\frac{4m_F x}{2I+1}+x^2}$$

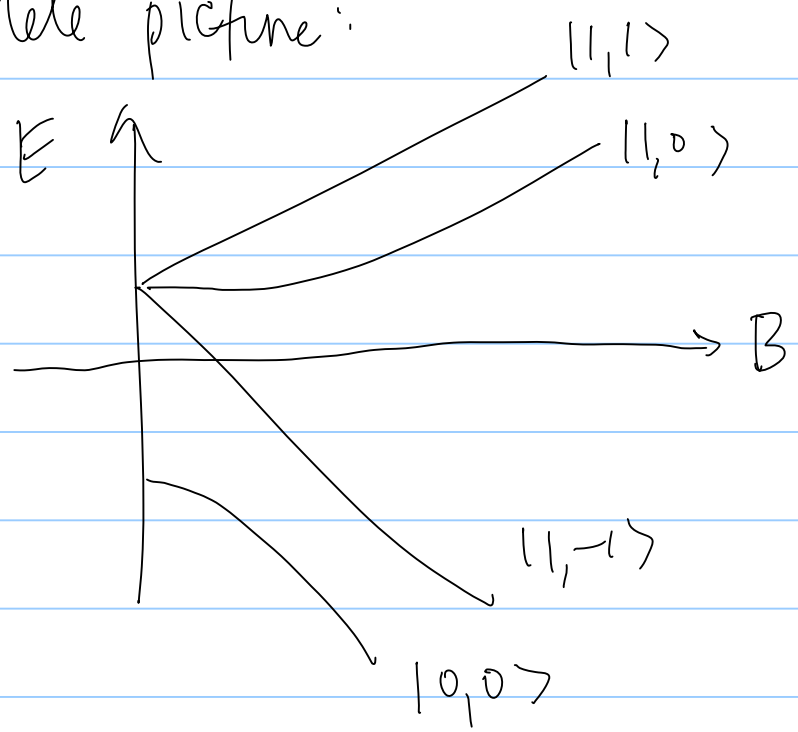
$$x = \frac{\left(g_J + g_I \frac{\mu_N}{\mu_0}\right)}{\Delta E_{hfs}}\mu_0 B$$

$$\approx \frac{g_J}{\Delta E_{hfs}}\mu_0 B = \frac{2\mu_0 B}{\Delta E_{hfs}}$$

For $m_F = 0$ states:

$$E \approx -\frac{\Delta E_{hfs}}{4} \pm \frac{\Delta E_{hfs}}{2}\sqrt{1+\left(\frac{2\mu_0 B}{\Delta E_{hfs}}\right)^2}$$



these have opposite magnetic moments at small fields, which is how the maser rejects |0,0> and keeps |1,0>

Complete picture: