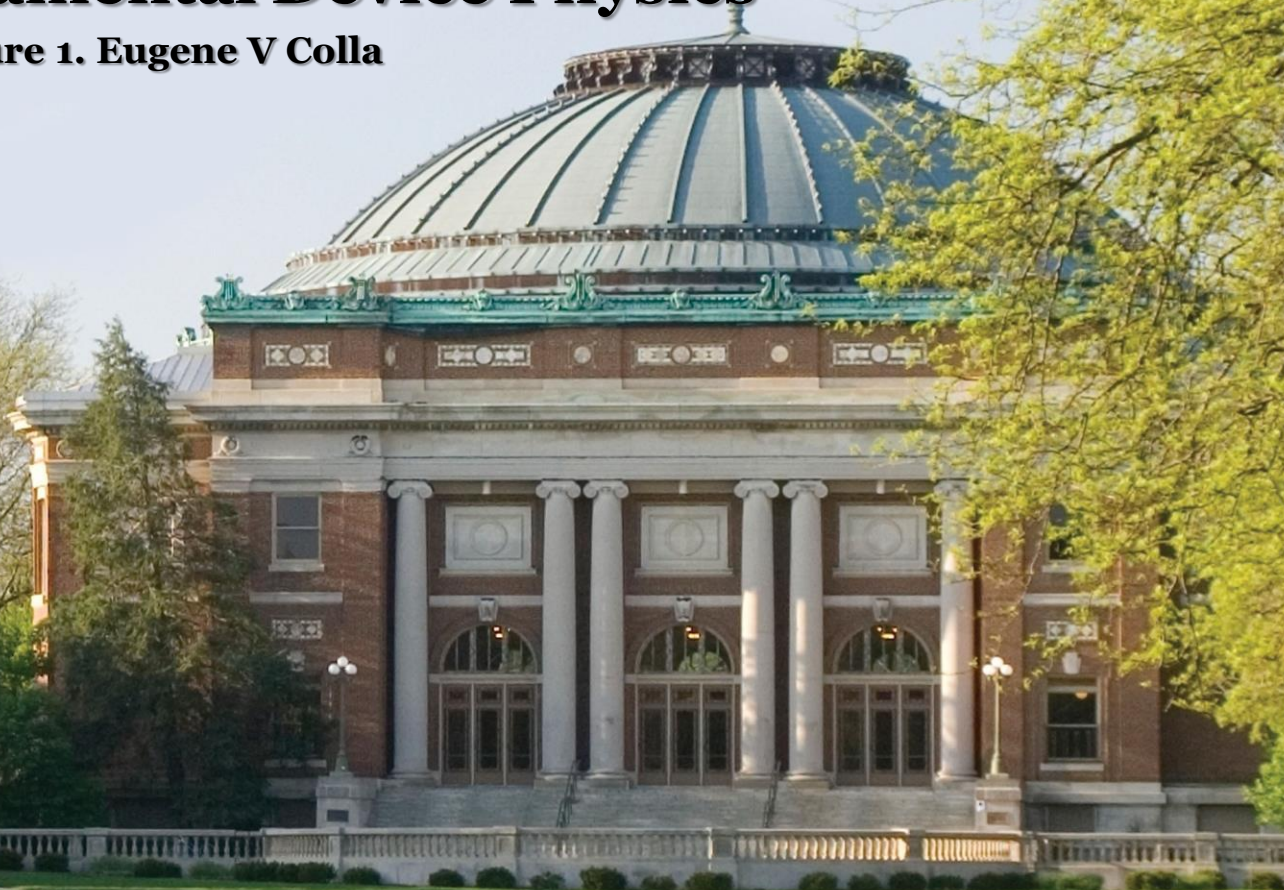


UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN

Physics 525

Survey of Fundamental Device Physics

Lecture 1. Eugene V Colla

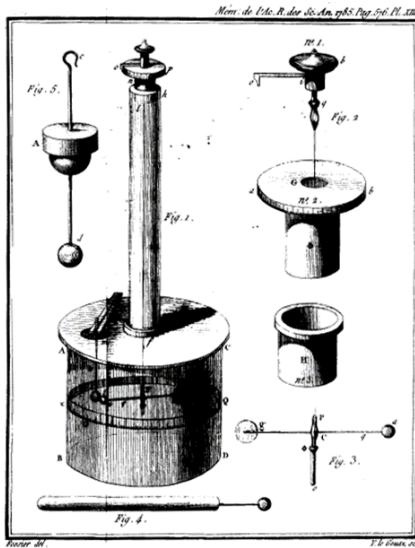


Unit 6. Magnetic fields etc. Maxwell Equations

Agenda

- 1. From Coulomb, Gauss, Biot and Savart, Ampere, Faraday Laws to Maxwell Equations**
- 2. Lorentz Forces and Charged Particles Accelerating**
- 3. Electromagnetic Spectrum and Waves propagations**

Basics of Electricity and Magnetism



Coulomb's Law

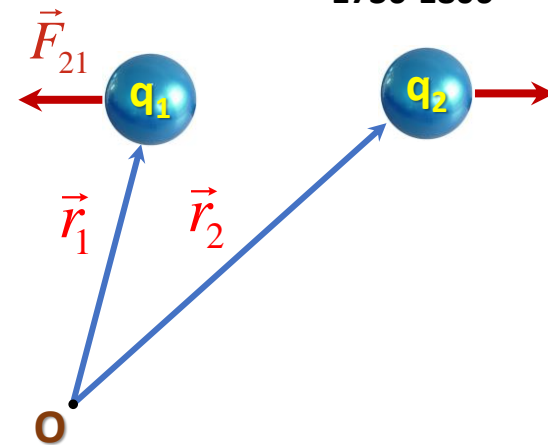
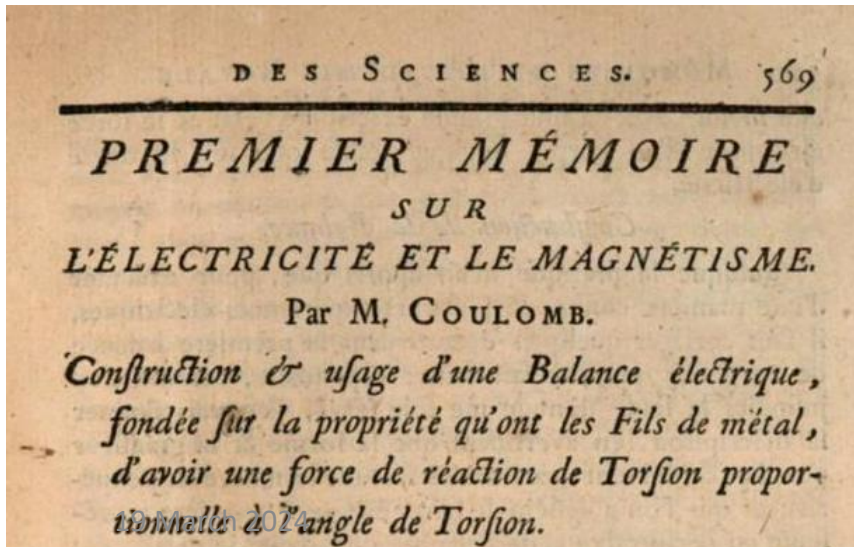
$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} q_1 q_2 \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3}$$

$$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$$



Coulomb's torsion balance (1785)

Charles Augustin de Coulomb
1736-1806



Basics of Electricity and Magnetism

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} q_1 q_2 \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3} \quad \text{and} \quad \vec{F}_{21} = -\vec{F}_{12} \quad \text{3rd Newton Law}$$

Electrical Field definition:

$$\vec{E}_0 = \lim_{q \rightarrow 0} \frac{\vec{F}}{q} \quad \longrightarrow \quad \vec{E}_0 = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^3} \hat{r}$$

Or for continuous distribution with charge density

$$\rho(\vec{r}) = dQ / d\tau \quad \text{over volume } \tau$$

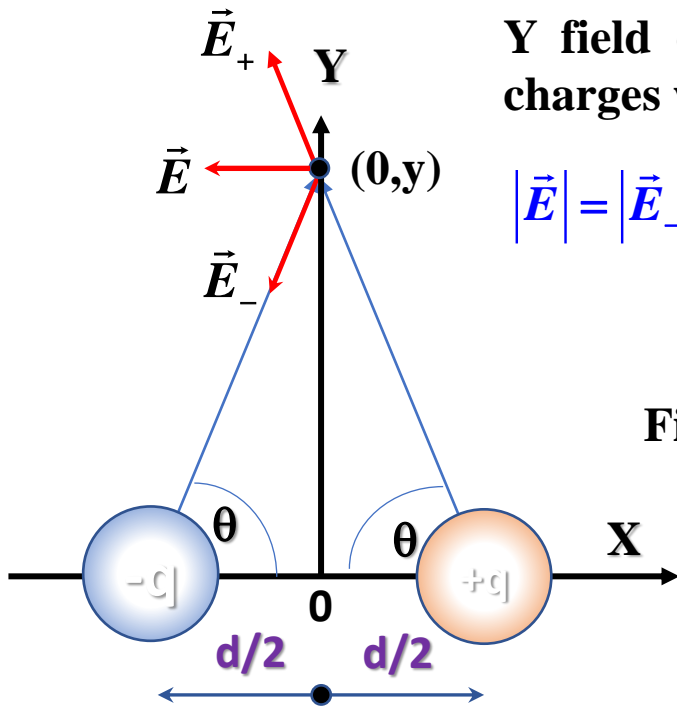
$$\vec{E}_0(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{i=N} q_i \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|^3}$$

$$\vec{E}_0(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{(\vec{r} - \vec{r}')_i}{|\vec{r} - \vec{r}'|^3} \rho(r') d\tau$$

Basics of Electricity and Magnetism.

Example: Electrical field generated by the dipole

Calculating the field at $(0,y)$ point. Based on symmetry the Y field components generated by negative and positive charges will compensate each other.



$$|\vec{E}| = |\vec{E}_-| + |\vec{E}_+| = 2 \cdot \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{y^2 + \left(\frac{d}{2}\right)^2} \cdot \cos \Theta \quad \cos \Theta = \frac{d}{2\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}$$

Finally:

$$|\vec{E}| = \frac{q}{4\pi\epsilon_0} \cdot \frac{d}{\left(y^2 + \left(\frac{d}{2}\right)^2\right)^{\frac{3}{2}}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{\left(y^2 + \left(\frac{d}{2}\right)^2\right)^{\frac{3}{2}}}$$

Where p is dipole moment $p=qd$

In the case if $y \gg d$

$$|\vec{E}| \approx \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{y^3}$$

Basics of Electricity and Magnetism

Electrical field can be presented as the gradient on scalar electrical potential V_0

$$\vec{E}_0 = -\text{grad}(V_0) = -\nabla V_0$$

Gauss Law

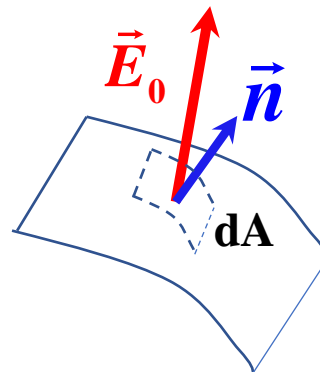
Introducing the electric flux as:

$$d\Phi = \vec{E}_0 \cdot \vec{n} \times dA$$

\vec{n} - normal to the surface unit vector,
 dA - element of surface.



Johann Carl Friedrich Gauss
1777-1855



After integrating over the surface

Basics of Electricity and Magnetism

$$\Phi(E) = \int_A \vec{E}_0 \cdot \vec{n} \times dA = \frac{Q}{\epsilon_0}$$

Introducing the electrical inductance $\vec{D} = \epsilon_0 \vec{E}$ the equation could be rewritten as:

$$\Phi(E) = \int_A \vec{D}_0 \cdot \vec{n} \times dA = Q = \int_V \rho(v) dv$$

Where $\rho(v)$ is the volume distribution of the charge. Now we apply

the divergence theorem $\int_A \vec{D} \cdot d\vec{A} = \int_V \nabla \cdot D dv$ ($d\vec{A} = \vec{n} \times dA$)

Finally, we will have the differential form of Gauss law (1st Maxwell equation)

$$\nabla D = \rho$$

1

Basics of Electricity and Magnetism

For magnetic field induction B assuming that no magnetic charges exist* we can write the similar equation on the magnetic flux:

$$\Phi(\vec{B}) = \int_A \vec{B} \cdot d\vec{A} = 0$$

Here is the integral through close surface A and applying the divergence theorem we can get the second Maxwell equation in derivative form:

$$\nabla \cdot \vec{B} = 0 \quad (2)$$

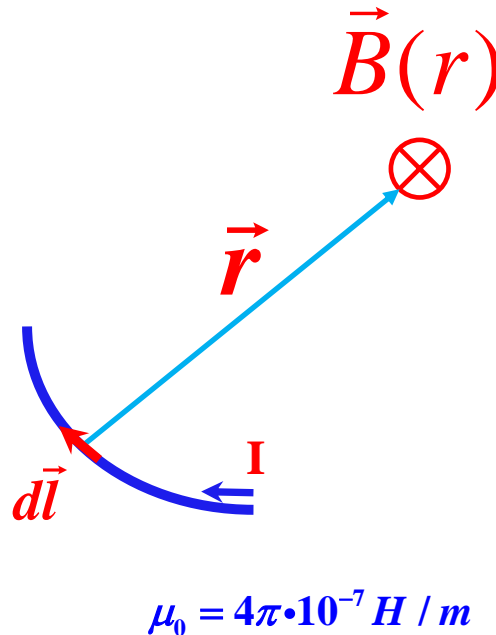
**In 1931 Paul Dirac (P.A.M. Dirac, Proc. Roy. Soc. A 133, 60) did show that magnetic charges (monopole) could exist in the nature but up to now there is no experimental confirmation of this theory.*

Basics of Electricity and Magnetism



Jean-Baptiste Biot
1774-1862

Biot and Savart law



Félix Savart
1791-1841

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{r}}{r^3}$$

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ H / m}$$

$d\vec{B}(\vec{r})$ - magnetic field contribution to $\vec{B}(\vec{r})$ created by element of the circuit $d\vec{l}$ carrying the current I

To calculate the net magnetic field generated by the whole wire with current I we need to take integral

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \vec{r}}{|\vec{r}|^3}$$

Basics of Electricity and Magnetism

Ampere law



André-Marie Ampère
1775-1936

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \sum_{i=1}^{i=N} I_i \quad I_i - \text{current components}$$

Applying the Stok's theorem* and taking bin account that

$I = \int_s \vec{J} \cdot \vec{n} dS$ (\vec{J} current, \vec{n} - vector perpendicular to the element of the surface dS) we get the fourth Maxwell equation in vacuum

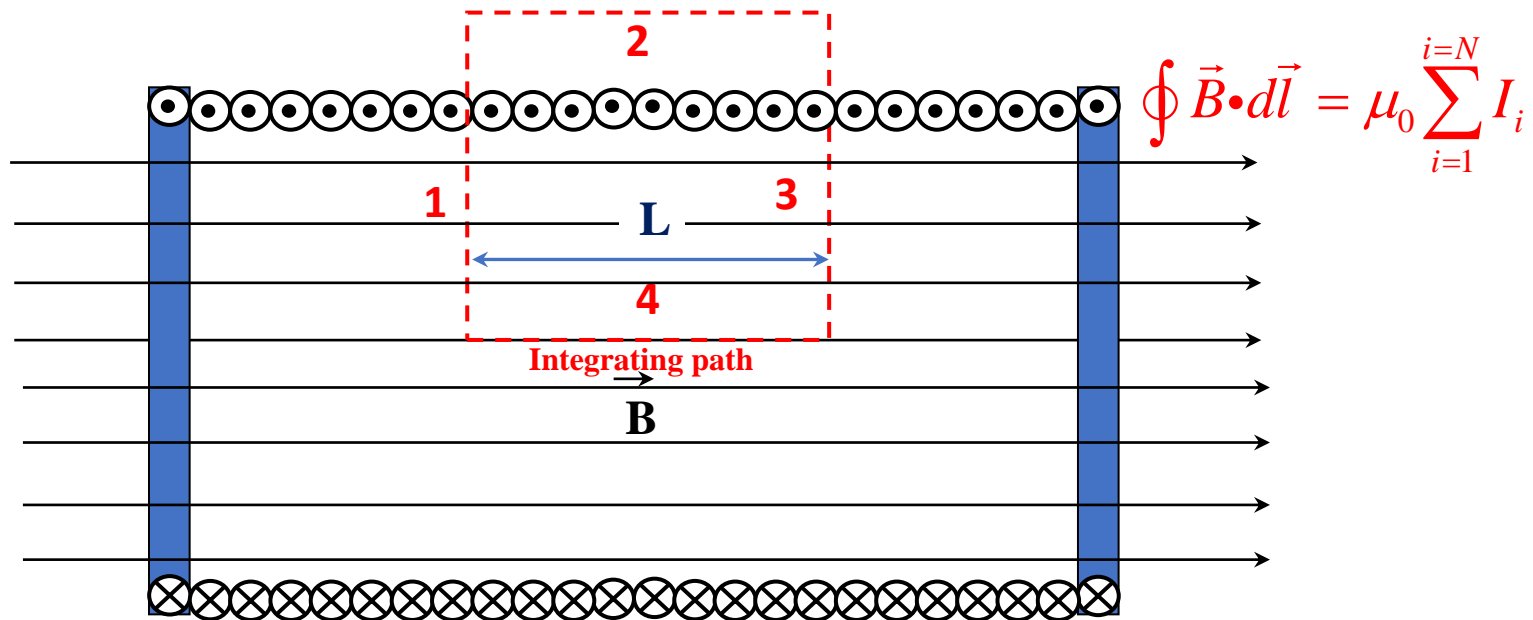
$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

4

* Stok's theorem: $\oint \vec{A} \cdot d\vec{l} = \int \nabla \vec{A} \cdot \vec{n} dS$

Basics of Electricity and Magnetism

Ampere law. Example: calculating the magnetic field created by solenoid.



Assuming that solenoid is long enough and the field outside is zero

$$\oint \vec{B} \cdot d\vec{l} = 0 + 0 + 0 + LB = \mu_0 NI$$

1: $\vec{B} \perp \vec{l}$ 2: $\vec{B} = 0$ 3: $\vec{B} \perp \vec{l}$

$$B = \frac{\mu_0 NI}{L} = \mu_0 nI$$

Basics of Electricity and Magnetism

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

Maxwell found that this equation is not complete as it does not account for the current of charging the capacitor. This current can be calculated as:

$$I = \frac{dQ}{dt} = \epsilon_0 A \frac{dE}{dt} \quad \text{where } A \text{ is the area of the capacitor plate and } \epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$$

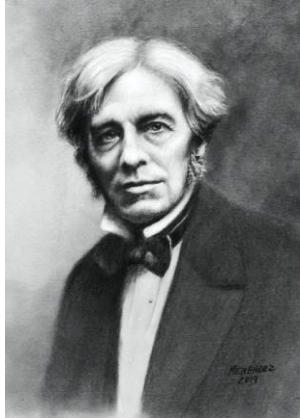
$$\text{or current density } \vec{J} = \epsilon_0 \frac{d\vec{E}}{dt}$$

Now the Ampere Law and fourth Maxwell equation can be modified as:

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{d\vec{E}}{dt} \right)$$

4

Basics of Electricity and Magnetism



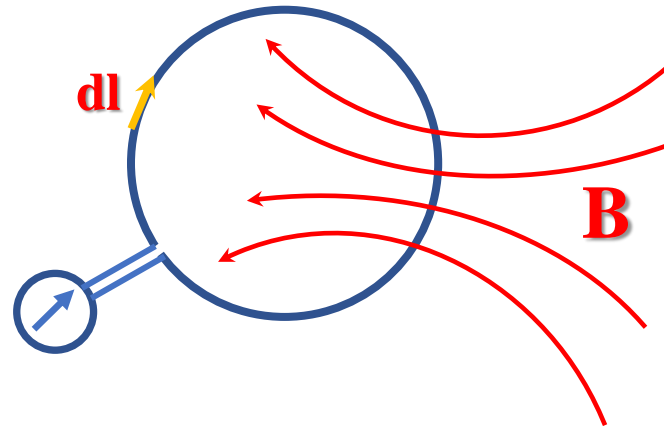
Michael Faraday
1791-1867

Faraday Law

$$Emf = -\frac{d\Phi}{dt}$$

Emf – electromotive force and Φ is the magnetic flux

$$d\Phi = d\vec{B} \cdot d\vec{s}$$



$$Emf = \oint \vec{E} \cdot d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

now we can apply the Stok's theorem and will get the differential form of the Faraday law – third Maxwell equation

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

3

Basics of Electricity and Magnetism

Faraday law. Example: transformer

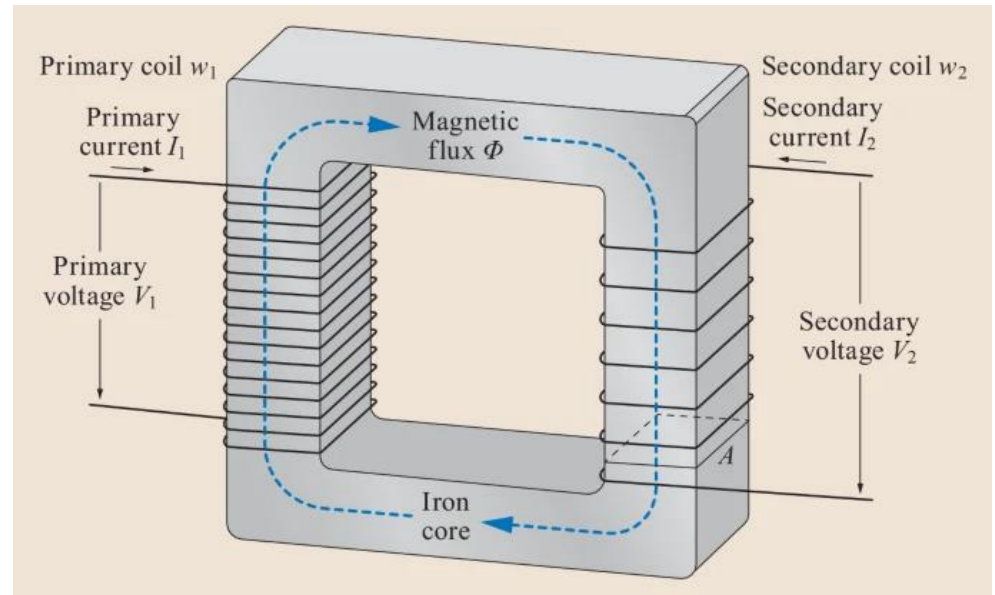
Primary coil driven by the primary voltage V_1 and according the Faraday Law

$$V_1 = -N_1 A \frac{d\Phi_1}{dt}$$

The same for the secondary coil

$$V_2 = -N_2 A \frac{d\Phi_2}{dt}$$

Because of the high μ value of the iron core the magnetic flux is almost totally contained in iron core and $\Phi_1 \approx \Phi_2$



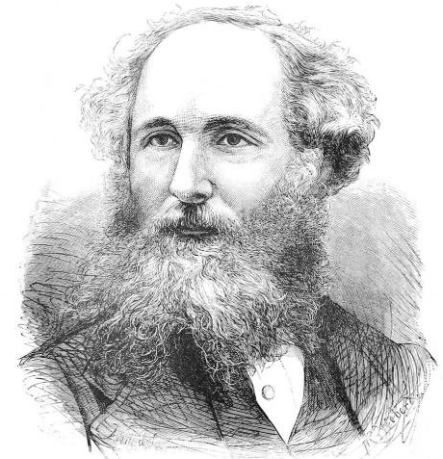
This results in ration between V_1 and V_2 as

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

N_1, N_2 numbers of turns of primary and secondary coils; A – cross sectional area of the iron core

Basics of Electricity and Magnetism

Maxwell equations describe how the magnetic and electric field can be generated by charges and currents. J C Maxwell published them in 1861-1862.



James Clerk Maxwell
1831-1879

$$\nabla \vec{D} = \rho \quad (1)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3)$$

$$\nabla \vec{B} = \mathbf{0} \quad (2)$$

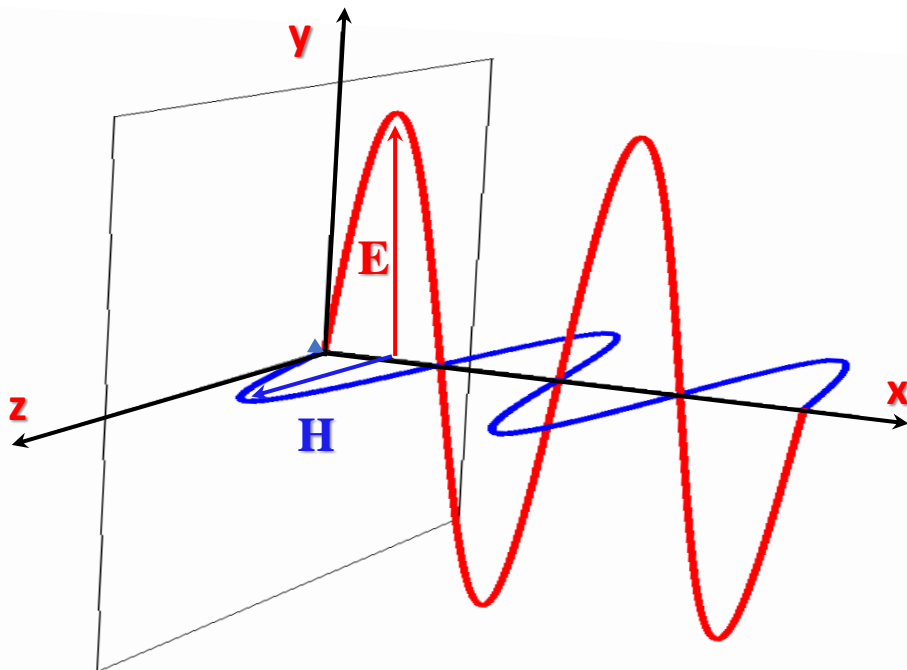
$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (4)$$

Basics of Electricity and Magnetism

Maxwell equations . Electromagnetic waves.

$$\nabla \vec{D} = \rho \quad (1) \quad \nabla \vec{B} = \mathbf{0} \quad (2) \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3) \quad \nabla \times \vec{H} = \mathbf{J} + \frac{\partial \vec{D}}{\partial t} \quad (4)$$

In vacuum $\rho = 0$ (no electric charges) and $\mathbf{J} = \mathbf{0}$ (no current) . Consider the plane wave propagating in **Z** direction. In this case $\mathbf{E}_y = \mathbf{E}_z = \mathbf{0}$ and $\mathbf{H}_x = \mathbf{H}_z = \mathbf{0}$



$$\frac{\partial E_y}{\partial x} = -\mu \frac{\partial H_z}{\partial t} \quad \text{from (3)}$$

$$\frac{\partial H_z}{\partial x} = -\epsilon \frac{\partial E_y}{\partial t} \quad \text{from (4)}$$

where $\mu = \mu_0 \mu_r$ $\epsilon = \epsilon_0 \epsilon_r$

In vacuum $\mu_r = 1$ and $\epsilon_r = 1$

Basics of Electricity and Magnetism

Maxwell equations . Electromagnetic waves.

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t}$$

$$\frac{\partial H_y}{\partial z} = -\epsilon \frac{\partial E_x}{\partial t}$$

Combining these leads to:



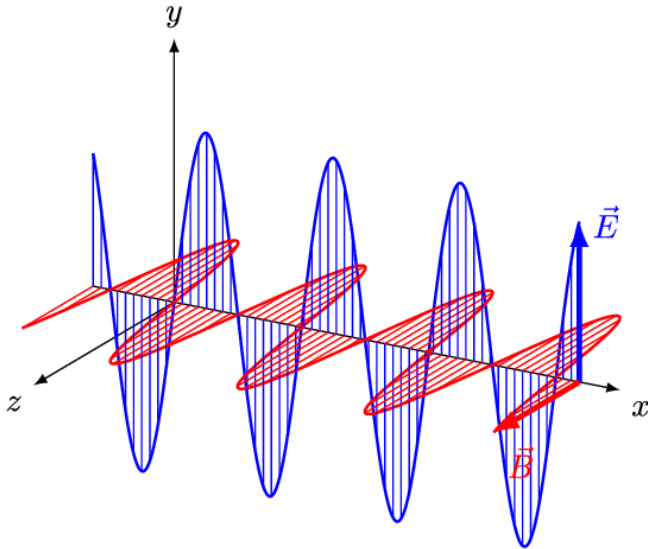
$$\frac{\partial^2 E_x}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 H_z}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 H_z}{\partial t^2}$$

where $v = \frac{1}{\sqrt{\mu\epsilon}}$

Looking of the solutions was these equations in form:

$$E_x = E_{x0} \cos(\omega t - kx) \quad H_y = H_{y0} \cos(\omega t - kx)$$



From this solution we've got the parameters of the traveling wave:

Phase velocity $v_p = v = \frac{1}{\sqrt{\mu\epsilon}}$

Wave vector $k = \frac{\omega}{v} = \frac{2\pi}{\lambda}$

Basics of Electricity and Magnetism

Maxwell equations . Electromagnetic waves.

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 E_x}{\partial t^2} \quad \frac{\partial^2 H_y}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 H_y}{\partial t^2} \quad E_x = E_{x0} \cos(\omega t - kx)$$
$$H_y = H_{y0} \cos(\omega t - kx)$$

$$v_p = v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\epsilon_0}} \frac{1}{\sqrt{\mu_r\epsilon_r}} = \frac{c}{\sqrt{\mu_r\epsilon_r}} \quad \mu = \mu_0\mu_r \quad \epsilon = \epsilon_0\epsilon_r$$

μ_0 is the free space permeability, ϵ_0 is the free space permittivity

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{(8.85 \cdot 10^{-12})(4\pi \cdot 10^{-7})}} \cong 3.00 \cdot 10^8 \text{ m / s}$$

c - speed of the light in free space

Creating a Static Magnetic Field using different Current Carrying Coils

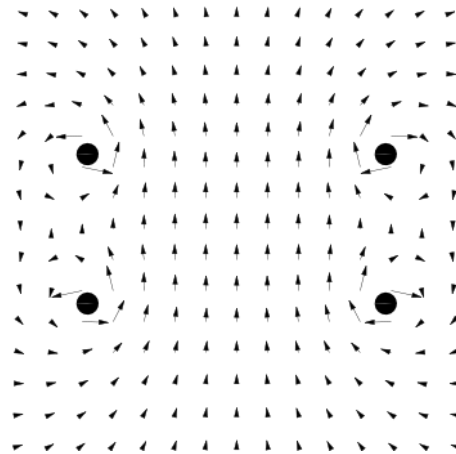
- a. Helmholtz coils**
- b. Solenoids**

Helmholtz coils



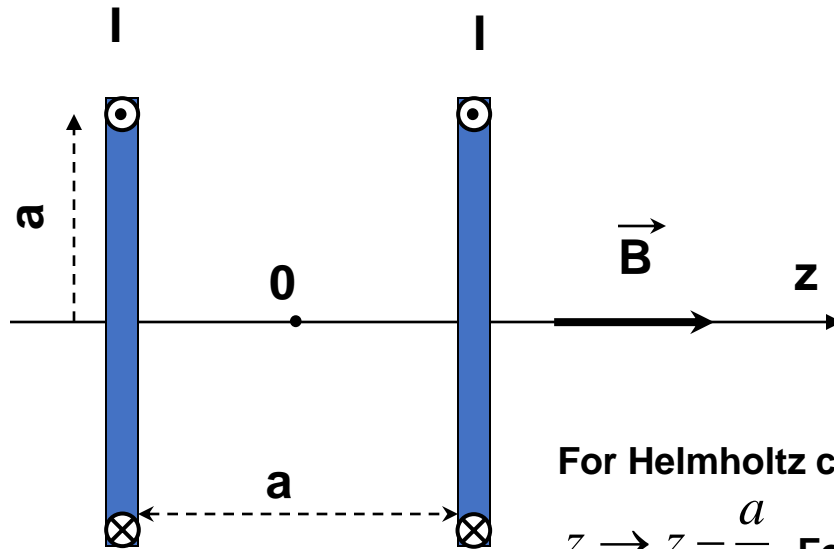
**Hermann Ludwig
Ferdinand von
Helmholtz
(1821-1894)**

Credit to E. Colla P401



**Magnetic field vector in a plane
bisecting the current loops.
(courtesy Wikipedia)**

Helmholtz coils. Magnetic field along the axis.



N turns

For single loop:

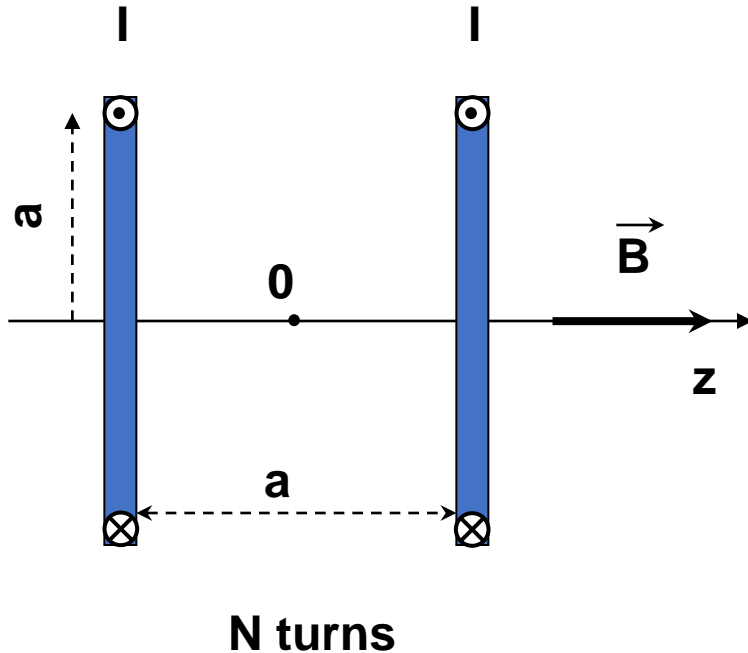
$$\vec{B} = \left\{ \frac{\mu_0 I}{2} \frac{a^2}{(z_1^2 + a^2)^{\frac{3}{2}}} \right\} \hat{z}$$

For Helmholtz coils total current equals NI ,

$$z \rightarrow z - \frac{a}{2} \quad \text{For right hand coil and}$$

$$z \rightarrow z + \frac{a}{2} \quad \text{for left hand coil}$$

Helmholtz coils. Magnetic field along the axis.



Finally:

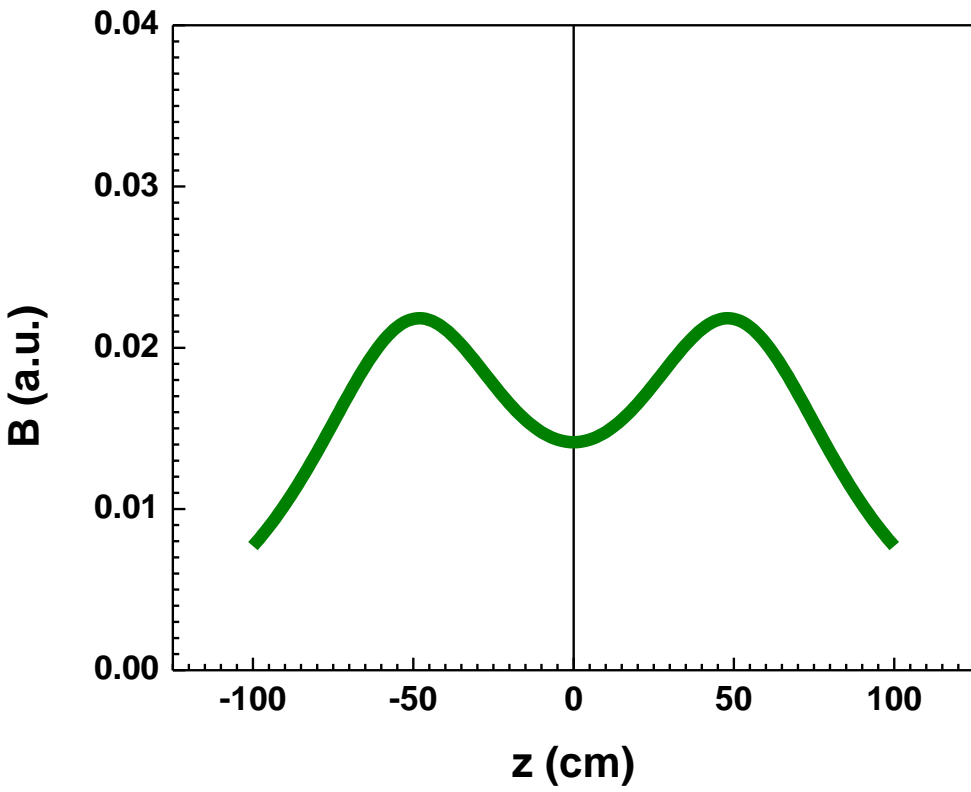
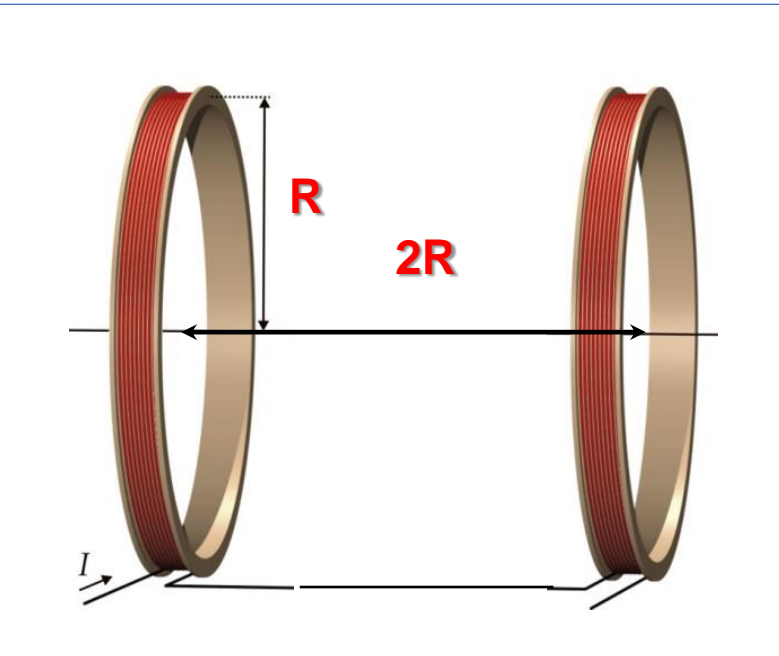
$$\vec{B} = \frac{\mu_0 N I a^2}{2} \left\{ \frac{a^2}{\left[\left(z + \frac{a}{2} \right)^2 + a^2 \right]^{\frac{3}{2}}} + \frac{a^2}{\left[\left(z - \frac{a}{2} \right)^2 + a^2 \right]^{\frac{3}{2}}} \right\} \hat{z}$$

or

$$\vec{B} = \frac{\mu_0 N I}{2a} \left\{ \frac{1}{\left[\left(\frac{z}{a} + \frac{1}{2} \right)^2 + 1 \right]^{\frac{3}{2}}} + \frac{1}{\left[\left(\frac{z}{a} - \frac{1}{2} \right)^2 + 1 \right]^{\frac{3}{2}}} \right\} \hat{z}$$

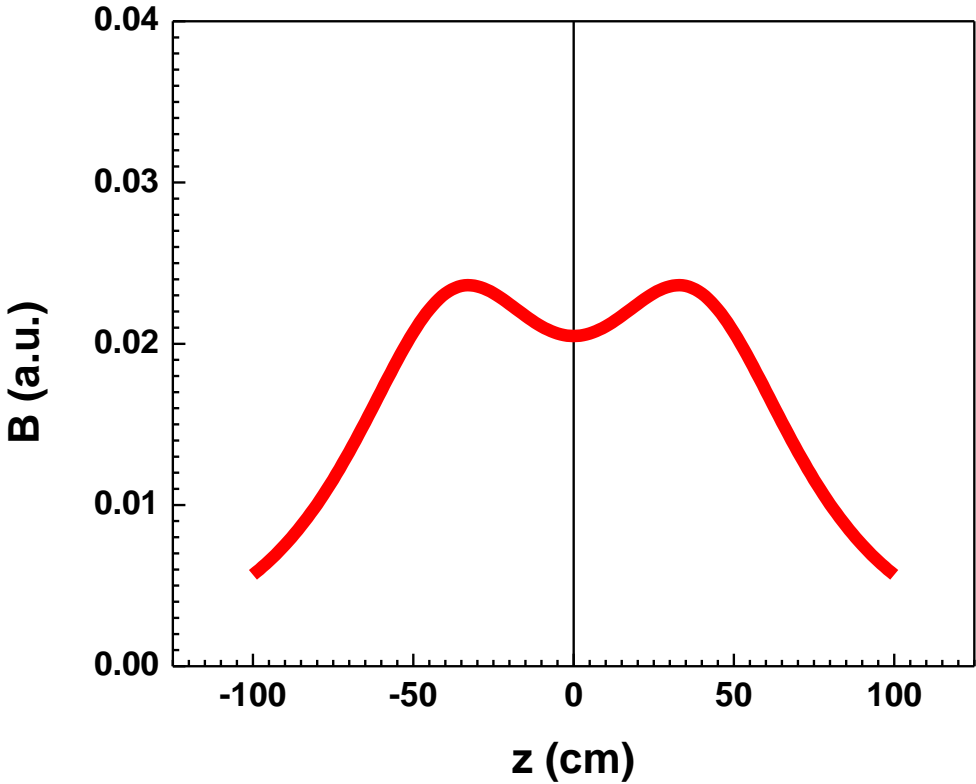
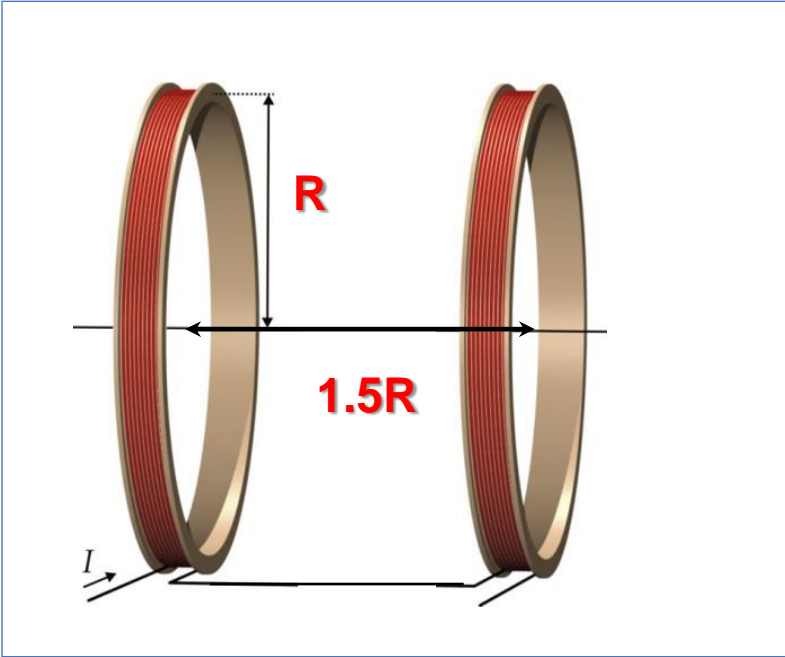
Helmholtz coils. Distance between the coils

1. $a=2R$



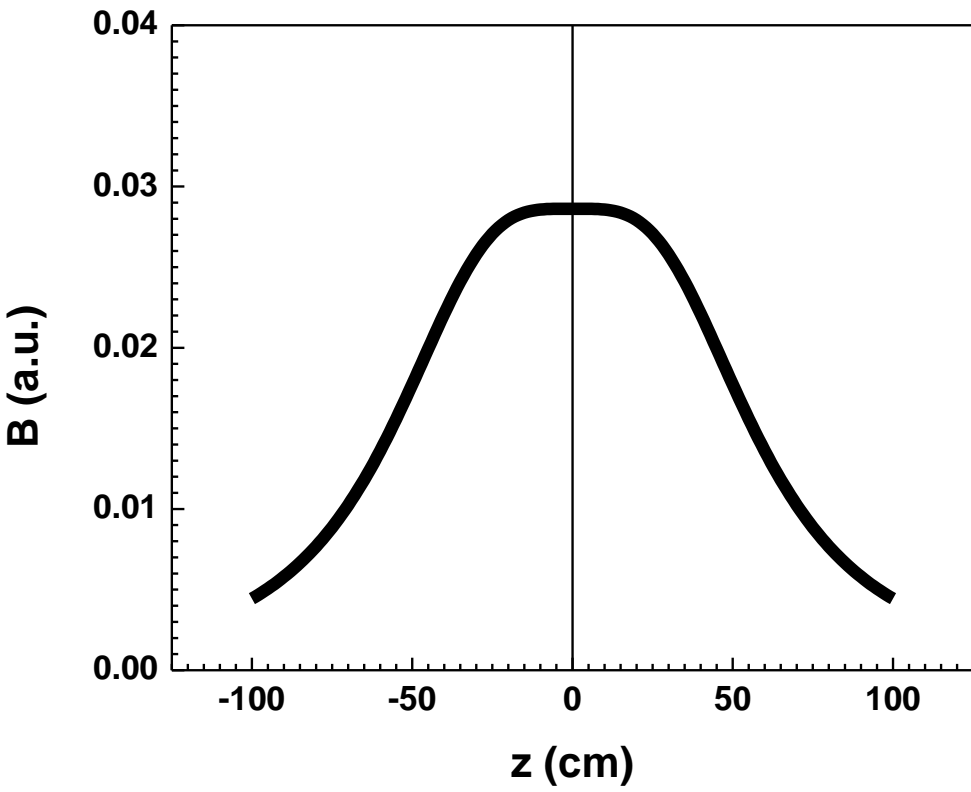
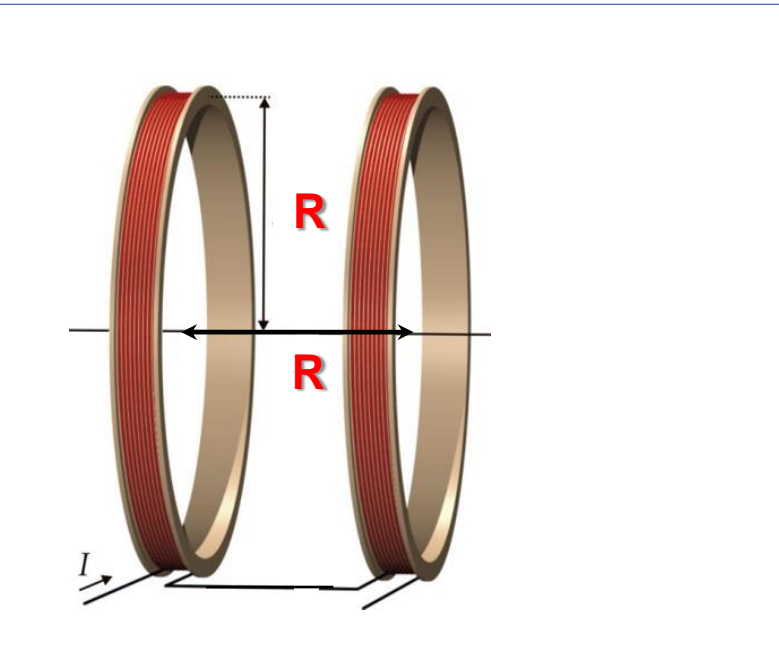
Helmholtz coils. Distance between the coils

1. $a=1.5R$



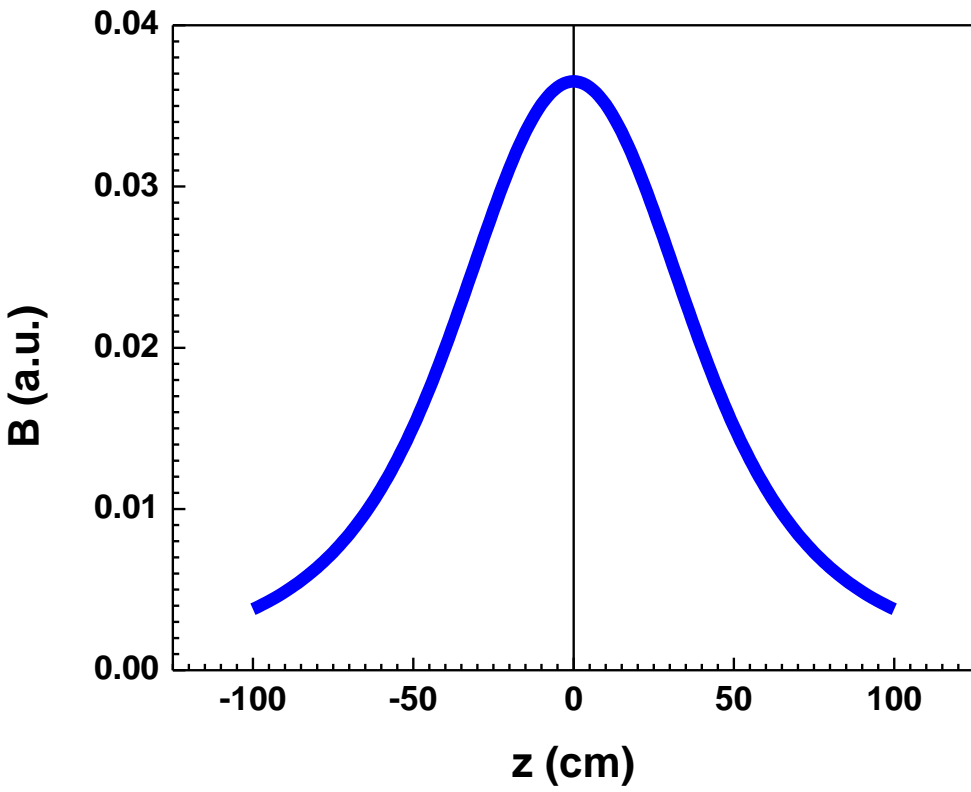
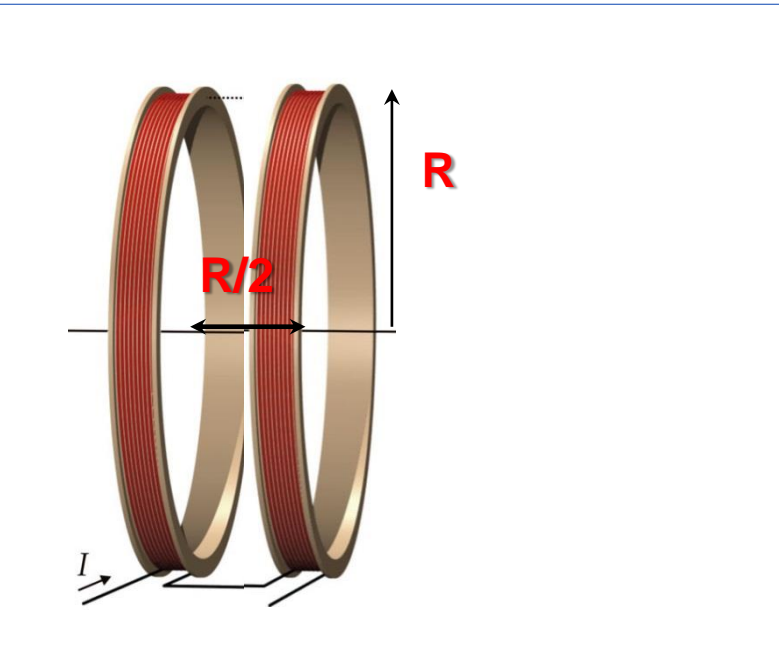
Helmholtz coils. Distance between the coils

3. $a=R$

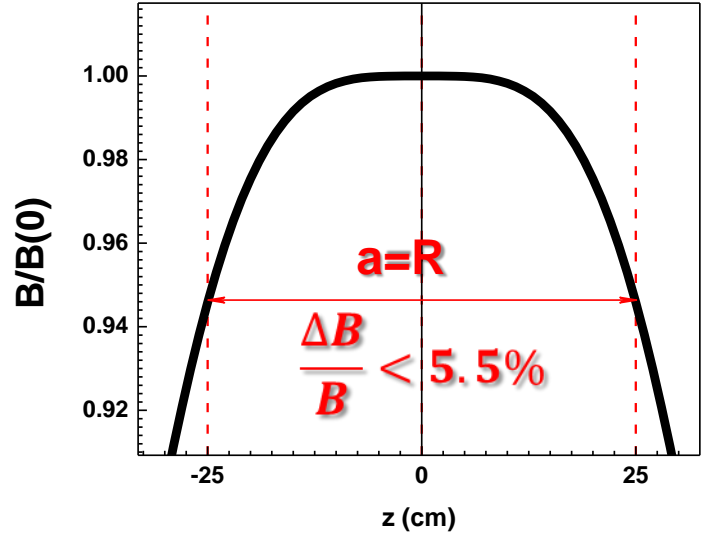
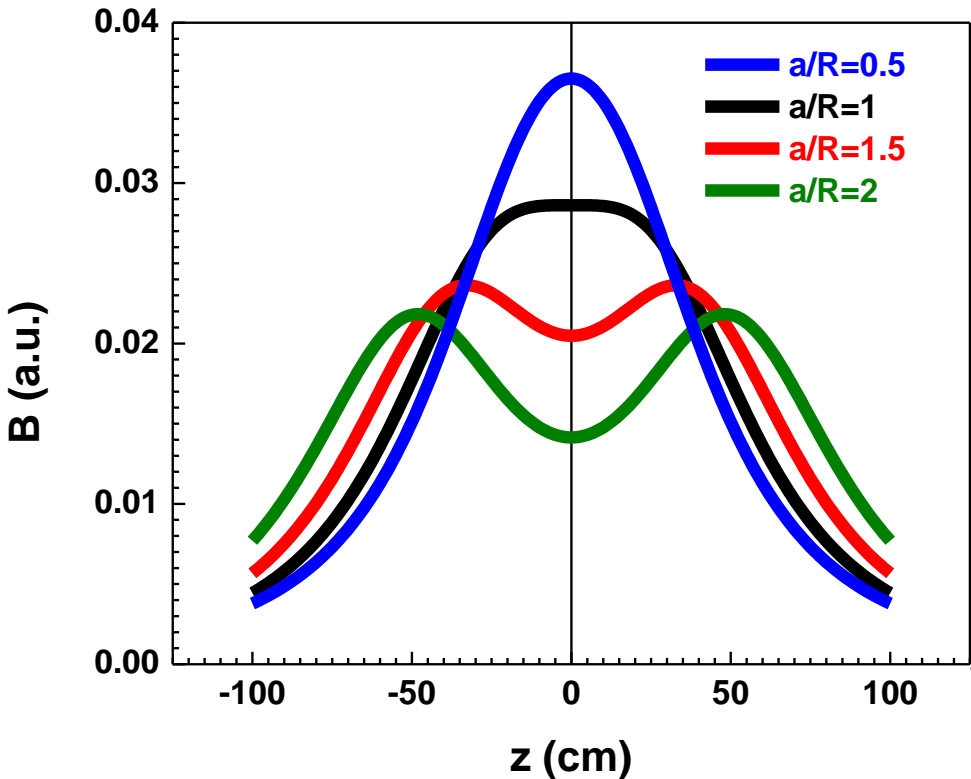


Helmholtz coils. Distance between the coils

4. $a=0.5R$



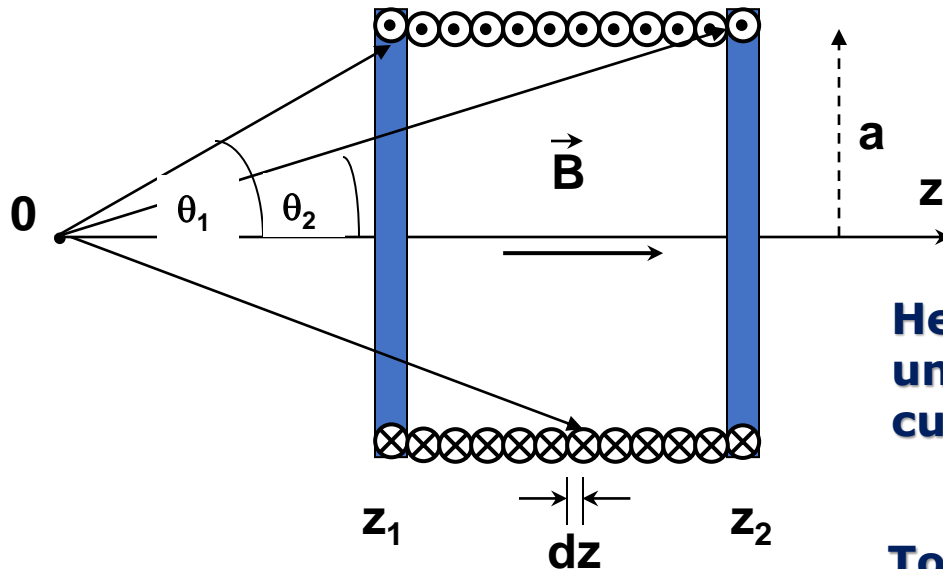
Helmholtz coils. Distance between the coils



In the z range $-a/4 \div a/4$ the field uniformity is better than 0.5%

Solenoids. Magnetic field along the axis.

Magnetic field generated by length dz :

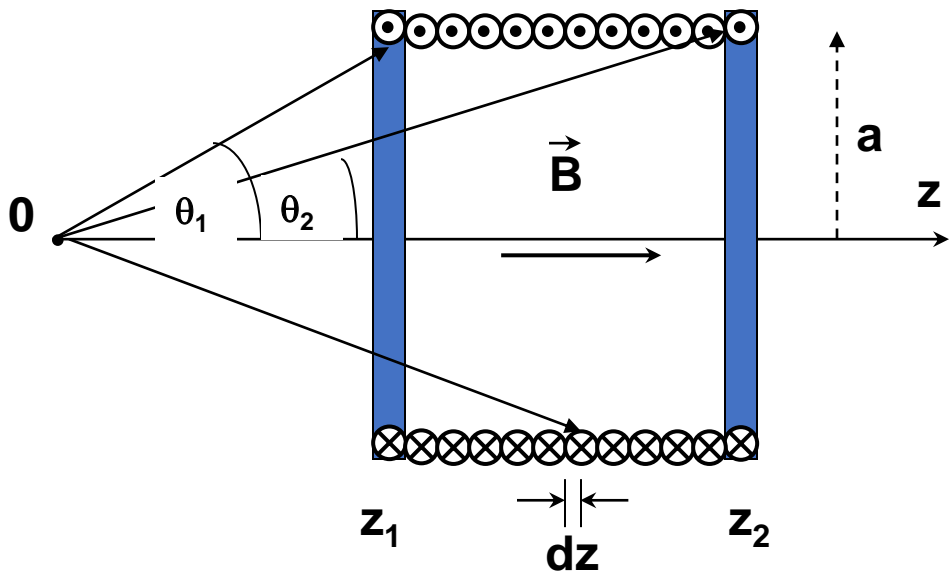


$$\vec{B} = \left\{ \frac{\mu_0 n I dz}{2} \frac{a^2}{(z_1^2 + a^2)^{\frac{3}{2}}} \right\} \hat{z}$$

Here n is number of turns per unit length and I – solenoid current

To calculate the magnetic field generated by the whole length of the solenoid we need to perform the integrating from z_1 to z_2

Solenoids. Magnetic field along the axis.



Field from current loop

$$\vec{B} = \left\{ \frac{\mu_0 n I dz}{2} \frac{a^2}{(z_1^2 + a^2)^{\frac{3}{2}}} \right\} \hat{z}$$

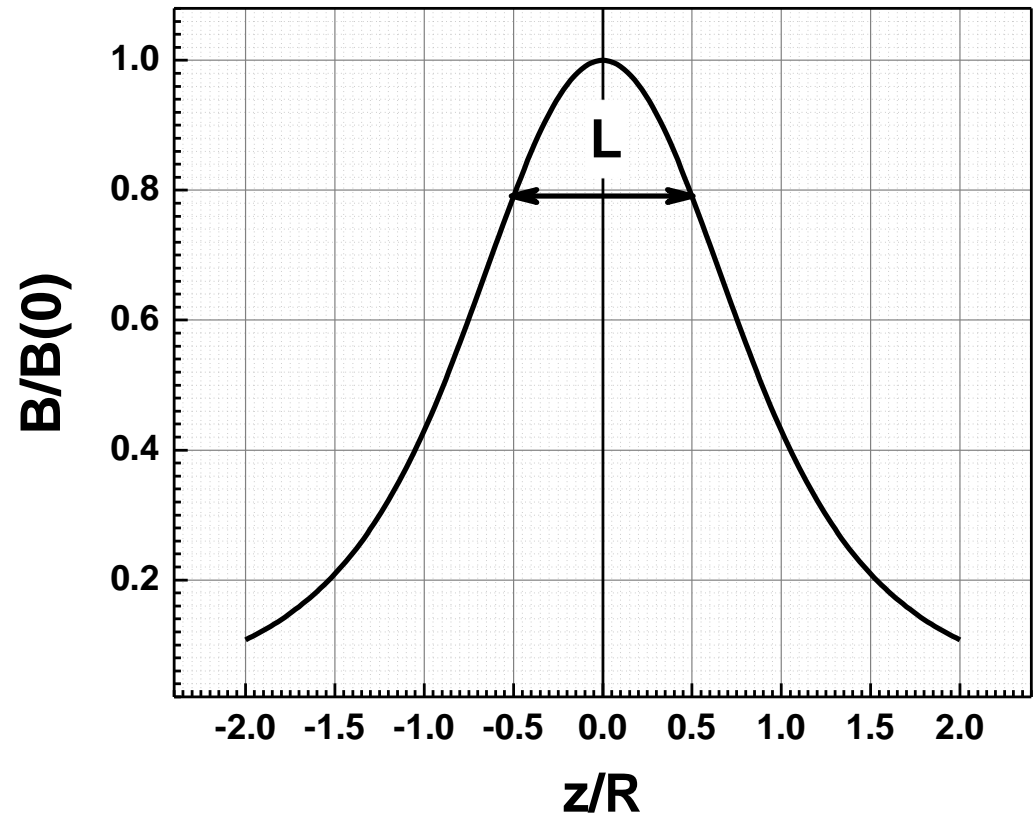
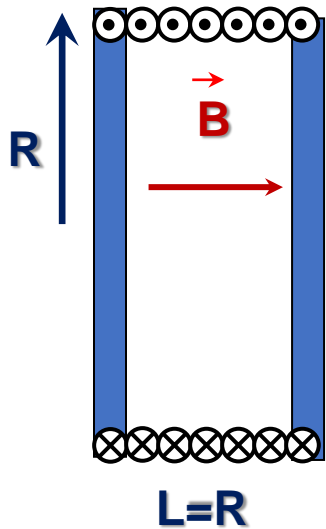
n – turns per unit length
I – solenoid current

Making the changing variables $z = \frac{a}{\tan \theta}$

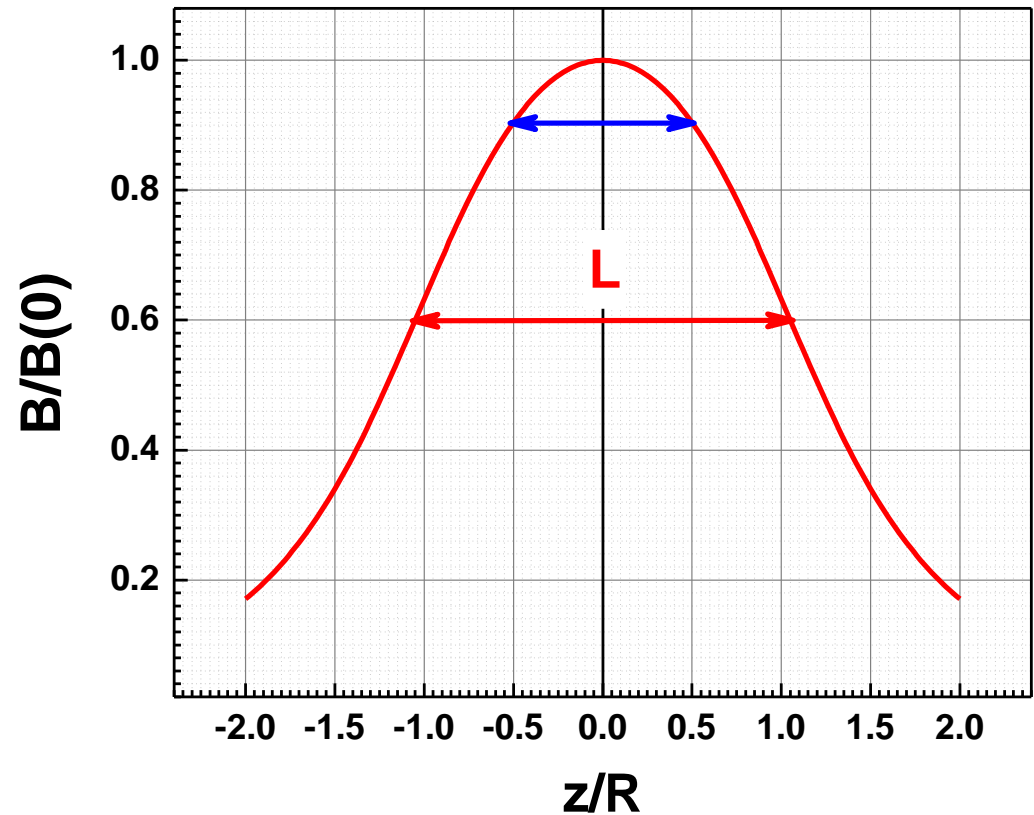
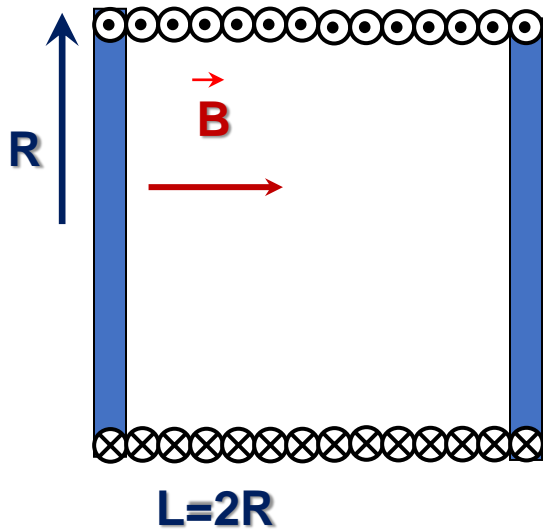
$$\vec{B} = -\frac{\mu_0 n I}{2} \int_{\theta_1}^{\theta_2} \sin \theta d\theta \hat{z} = \frac{\mu_0 n I}{2} [\cos \theta_1 - \cos \theta_2] \hat{z}$$

where $\cos(\theta_1) = \frac{z_1}{\sqrt{a^2 + z_1^2}}$; $\cos(\theta_2) = \frac{z_2}{\sqrt{a^2 + z_2^2}}$

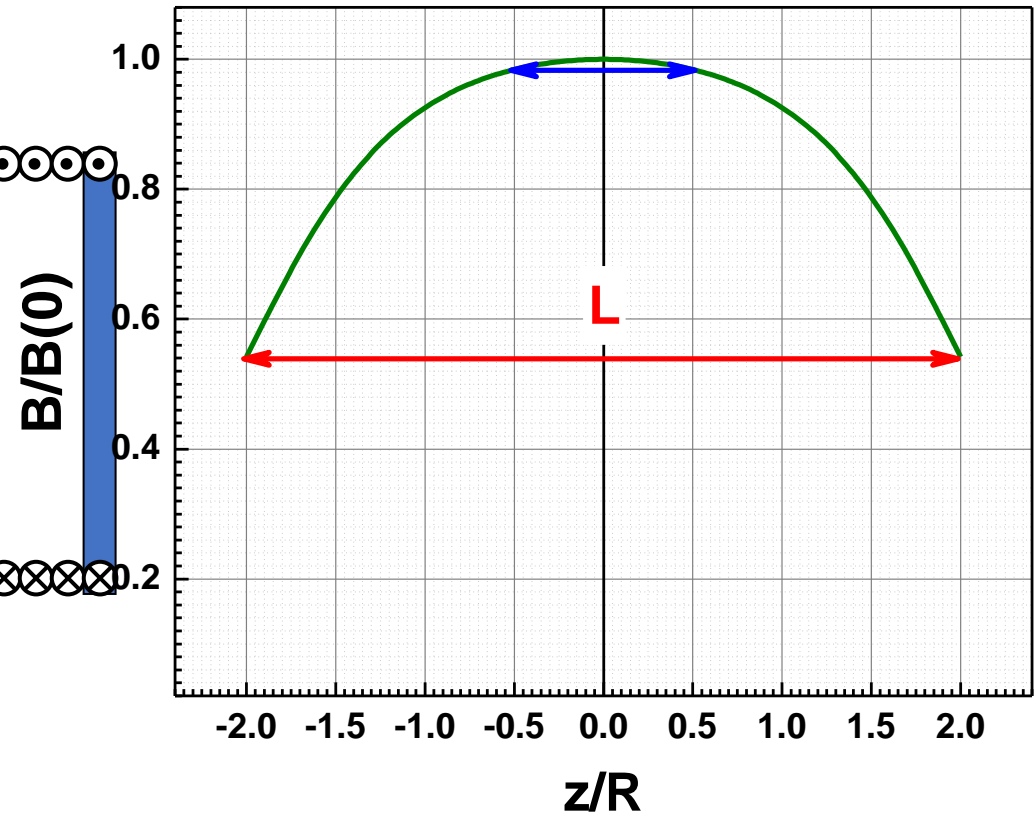
Solenoids. How uniform the field is.



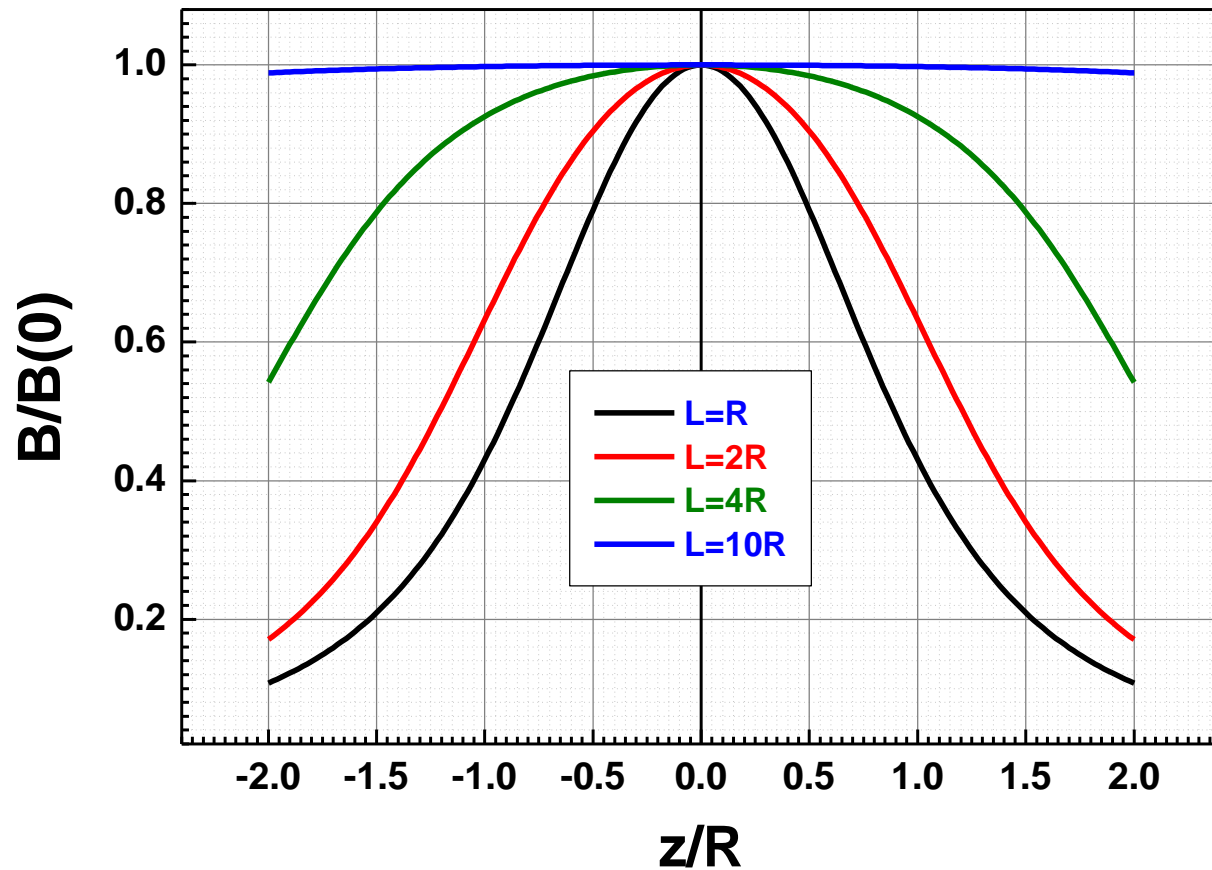
Solenoids. How uniform the field is.



Solenoids. How uniform the field is.

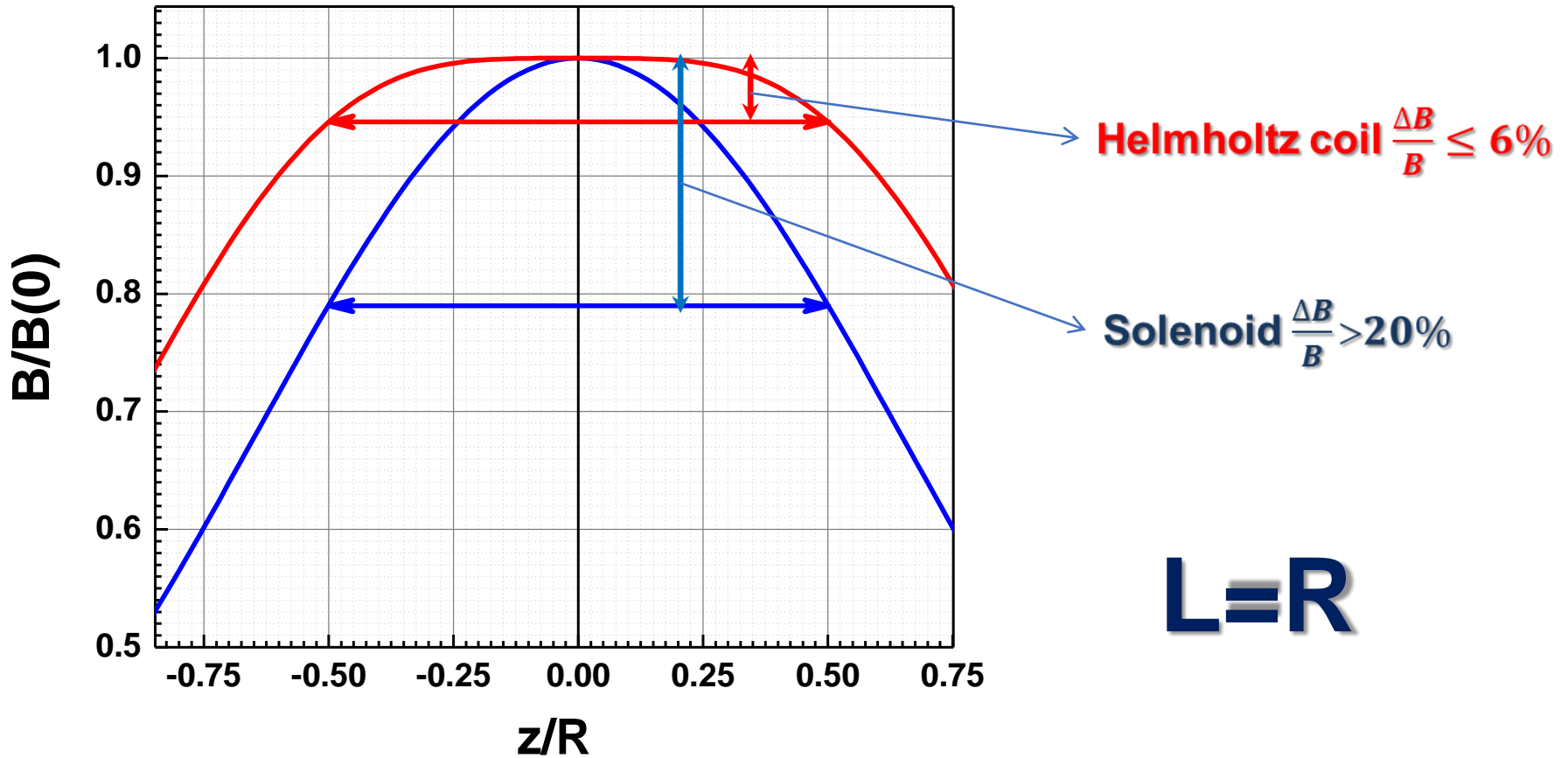


Solenoids. How uniform the field is.



To create the uniform field in solenoid you need you need to wind a long coil with $L \gg R$

Solenoids vs. Helmholtz coil.



Basics of Electricity and Magnetism



Hendrik Antoon Lorentz
1853-1928

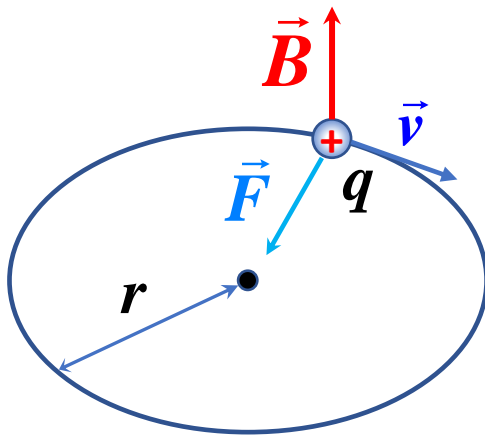
Lorentz Force

Lorentz force is force provided by the electrical field \vec{E} and magnetic field \vec{B} on the moving with velocity \vec{v} charged particle carrying the charge q

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

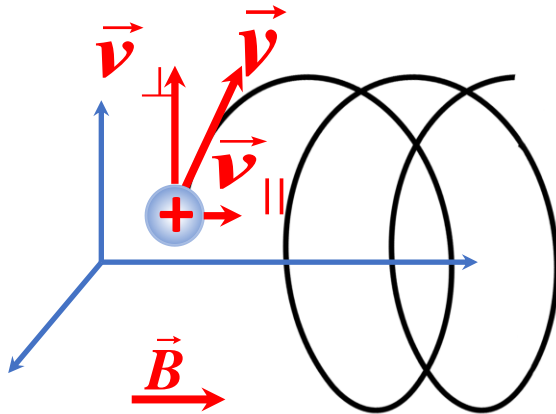
Motion of the charge in the magnetic field

Velocity \vec{v} is perpendicular to the vector of magnetic field \vec{B} . Resulting force \vec{F} will work as centripetal force and the trajectory of the particle will be a circle and the radius r of it can be calculated as:



$$qvB = \frac{mv^2}{r}; r = \frac{mv}{qB}$$

Basics of Electricity and Magnetism



In case if the particle velocity is not exactly perpendicular to the direction of the magnetic field the trajectory of the particle will be a spiral with radius

$$r = \frac{mv_{\perp}}{qB} \quad \text{and it will move with the velocity } \vec{v}_{\parallel}$$

Basics of Electricity and Magnetism. Applications.

Lorentz Force and charged particle accelerators. X-rays tube.

Nobel prize in Physics 1901

"in recognition of the extraordinary services he has rendered by the discovery of the remarkable rays subsequently named after him"

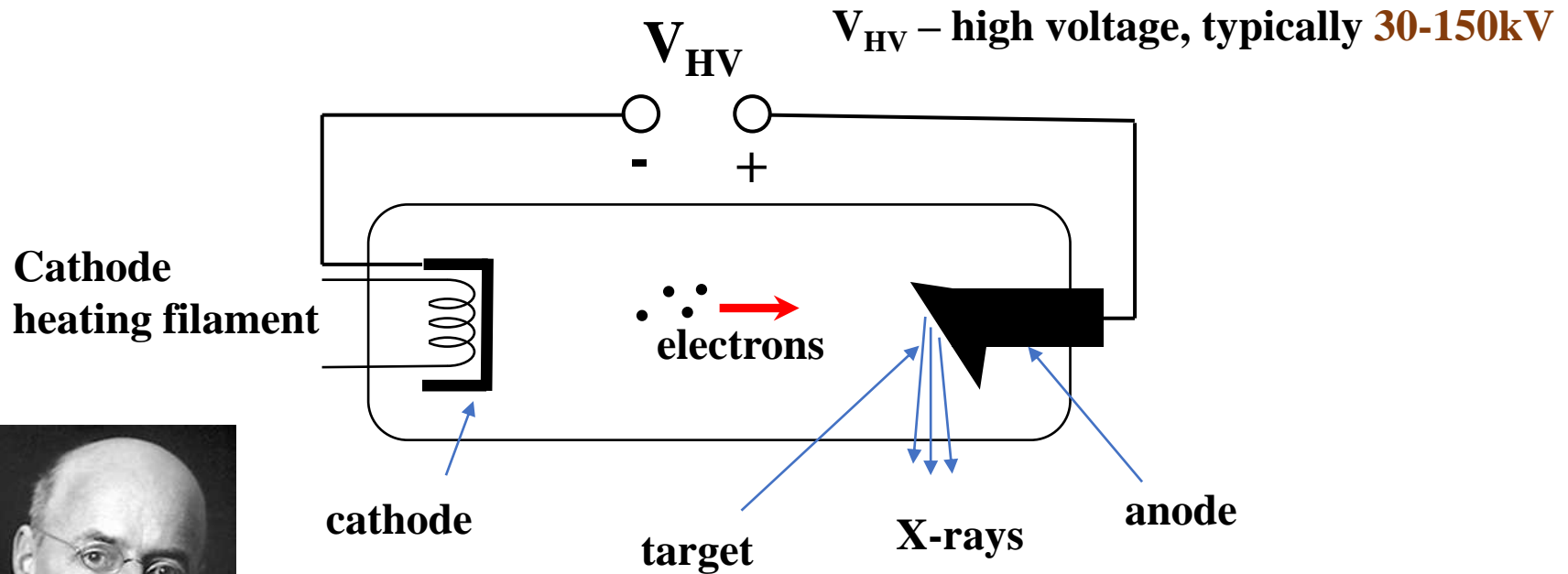


Wilhelm Conrad Röntgen
1846-1923



Basics of Electricity and Magnetism. Applications.

Lorentz Force and charged particle accelerators. X-rays tube.



Owen Willans Richardson
1879-1959

$$J_{cv} = AT^2 \exp\left(\frac{-W}{kT}\right)$$

Richardson equation: J_{cv} cathode emission current, W – work function of the cathode material, T – temperature, A – constant

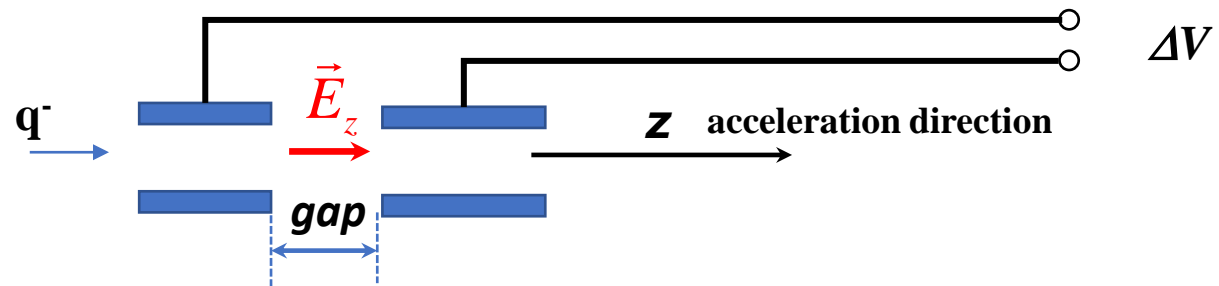
Basics of Electricity and Magnetism. Applications.

Lorentz Force and charged particle accelerators. Linear accelerator.



Rolf Widerøe
1902-1996

“Drift Tube Linac” 1927



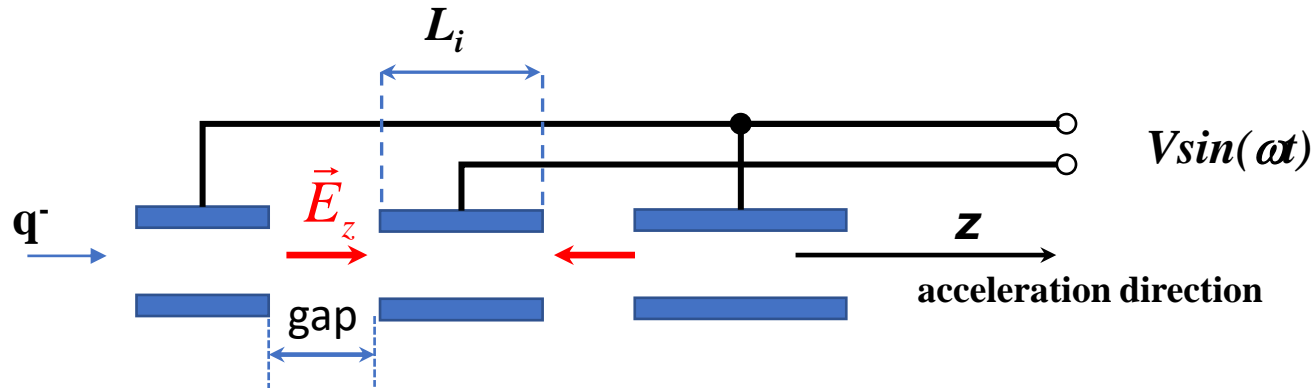
The acceleration works in between the electrodes in *gap*. Increment of the

kinetic energy dW can be calculated as: $dW = q \frac{\partial E_z}{\partial z}$ and total energy earned

by particle traveling across the gap: $\Delta W = q \int \frac{\partial E_z}{\partial z} dz = q \Delta V$

Basics of Electricity and Magnetism. Applications.

Lorentz Force and charged particle accelerators. Linear accelerator.

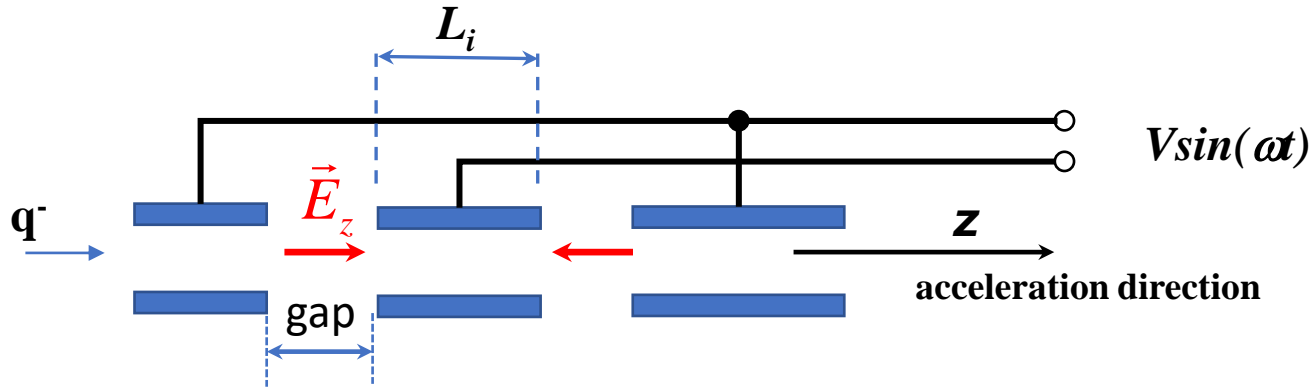


Now we have several steps of acceleration (two gaps in this figure) and we applying now ac electrical field $V \sin(\omega t)$ and $\Delta W = qV \sin(\omega t)$ and it means that not all particle will be accelerated but only those which entered the gap in proper phase. The next step of acceleration will be done while traveling across the next gap and to be successfully accelerated the particles should come the second gap in correct phase and this imply the proper distance L in between two gaps. The time of traveling to the next gap t_i should equal to half period of the applied rf voltage:

$$t_i = \frac{T}{2} = \frac{\pi}{\omega} = \frac{L_i}{v_i} \quad , \text{ where } v_i \text{ is the speed of the approaching the next gap}$$

Basics of Electricity and Magnetism. Applications.

Lorentz Force and charged particle accelerators. Linear accelerator.



The time of traveling to the next gap t_i should equal to half period of the applied rf voltage:

$$t_i = \frac{T}{2} = \frac{\pi}{\omega} = \frac{L_i}{v_i}, \text{ where } v_i \text{ is the speed of the approaching the next gap}$$

→ $L_i = \frac{\pi}{\omega} v_i$; and in relativistic case $v_i = \beta_i c$ where c is speed of the light

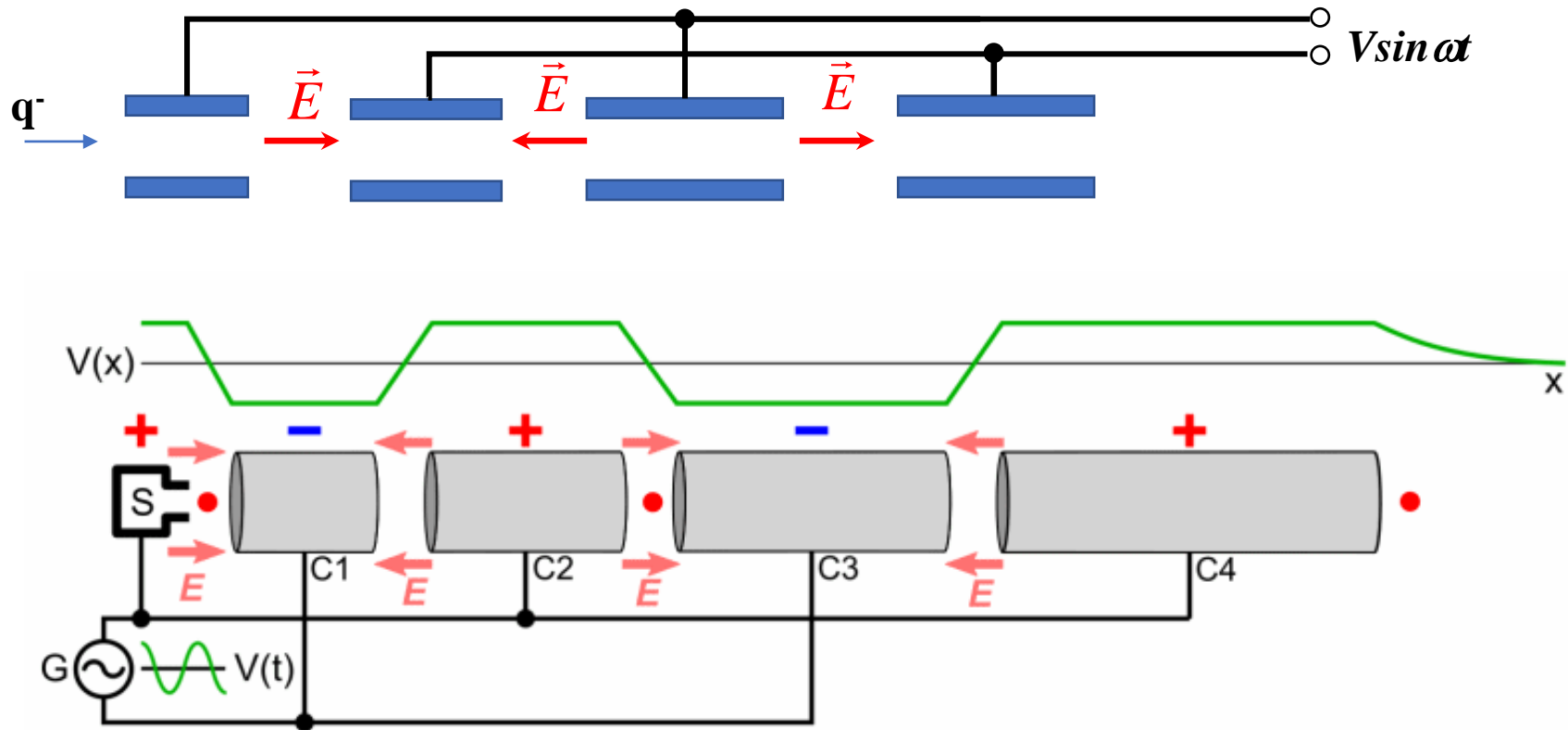
in a vacuum and $\beta_i = \sqrt{1 - \frac{1}{\gamma_i^2}} = \sqrt{1 - \left(\frac{E_0}{E}\right)^2}$; γ_i - Lorentz factor; E_0 - rest energy

E – total energy $E = E_0 + W$; W kinetic energy of accelerated particle

For electron $E_0 = 511 \text{ keV}$

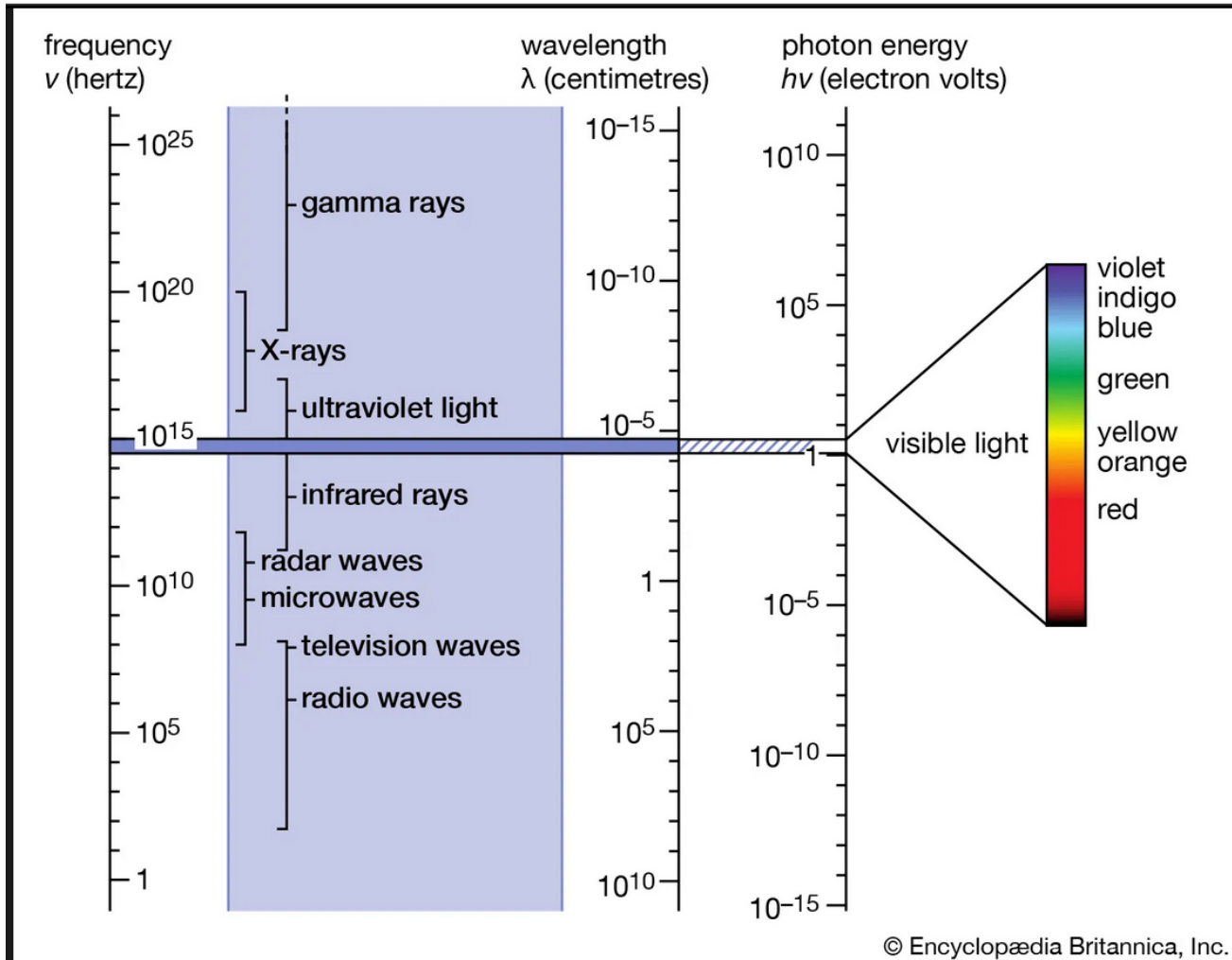
Basics of Electricity and Magnetism. Applications.

Lorentz Force and charged particle accelerators. Linear accelerator.



Basics of Electricity and Magnetism. Applications.

Generating od electromagnetic waves of different frequencies.

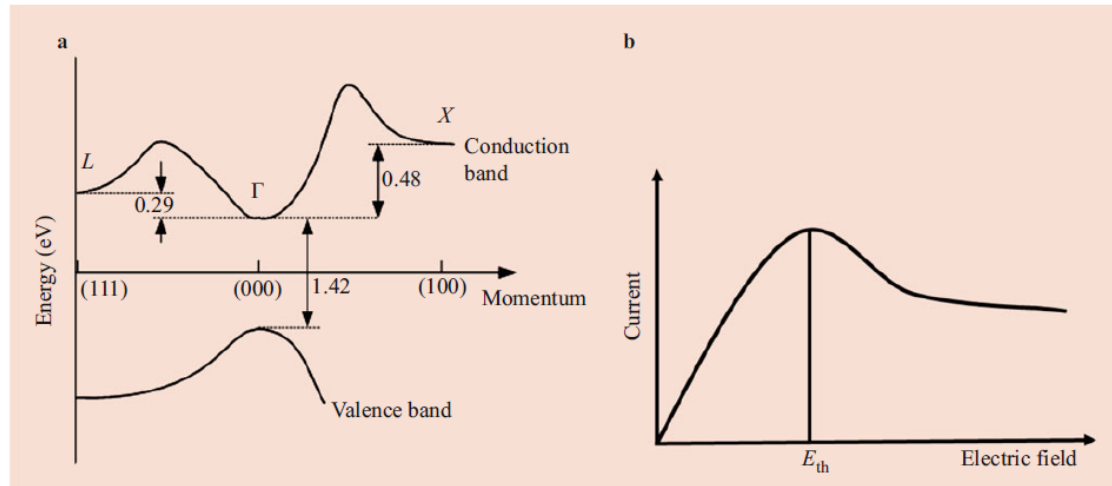


Basics of Electricity and Magnetism. Applications.

Generating of electromagnetic waves. Microwaves. Gunn Diode.



J. B. Gunn
1928-2008



Energy band structure of GaAs showing the band gap and the energy separations between the different valleys and (b) current versus electric field characteristics of the bulk material showing the threshold field E_{th} above which negative differential conductance appears

Frequency range: 10GHz ÷ 1THz
Output power ~200mV

Applications:
airborne collision avoidance system
Car radar detector

Basics of Electricity and Magnetism. Applications.

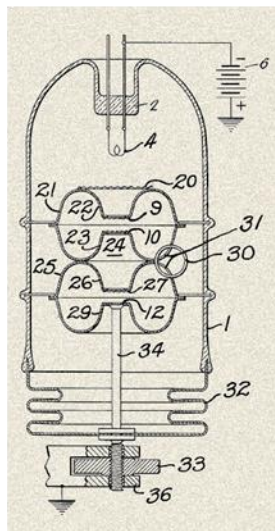
Generating of electromagnetic waves. Microwaves. Klystron.



Russell Varian
1898-1959



Sigurd Varian
1901-1961



Patented May 20, 1941

2,242,275

UNITED STATES PATENT OFFICE

2,242,275

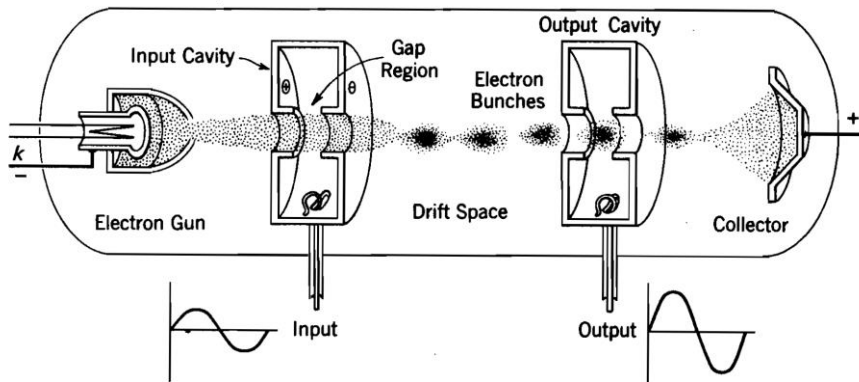
ELECTRICAL TRANSLATING SYSTEM AND METHOD

Russell H. Varian, Stanford University, Calif., assignor to The Board of Trustees of The Leland Stanford Junior University, Stanford University, Calif., a corporation of California

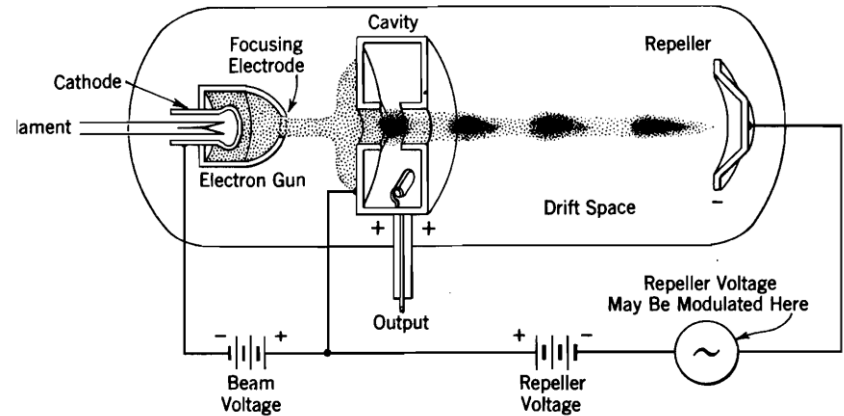
Application October 11 1937 Serial No. 162 955

Basics of Electricity and Magnetism. Applications.

Generating of electromagnetic waves. Microwaves. Klystron.



Single transit klystron



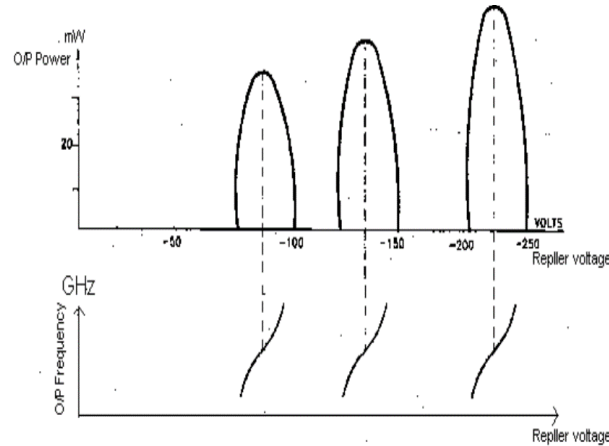
Reflection klystron

Advantages: well defined frequencies,
high power output

Basics of Electricity and Magnetism. Applications.

Generating of electromagnetic waves. Microwaves. Klystron.

2K25



GENERAL CHARACTERISTICS

Frequency Range 8,500 to 9,660 MHz

Cathode Oxide-coated,
indirectly heated

Heater Voltage 6.3Volts

Heater Current 0.44 Amperes

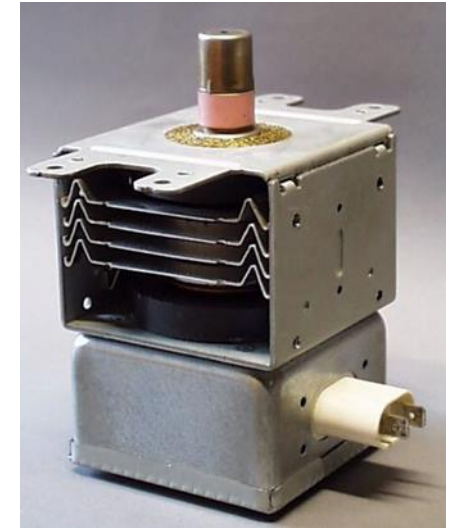
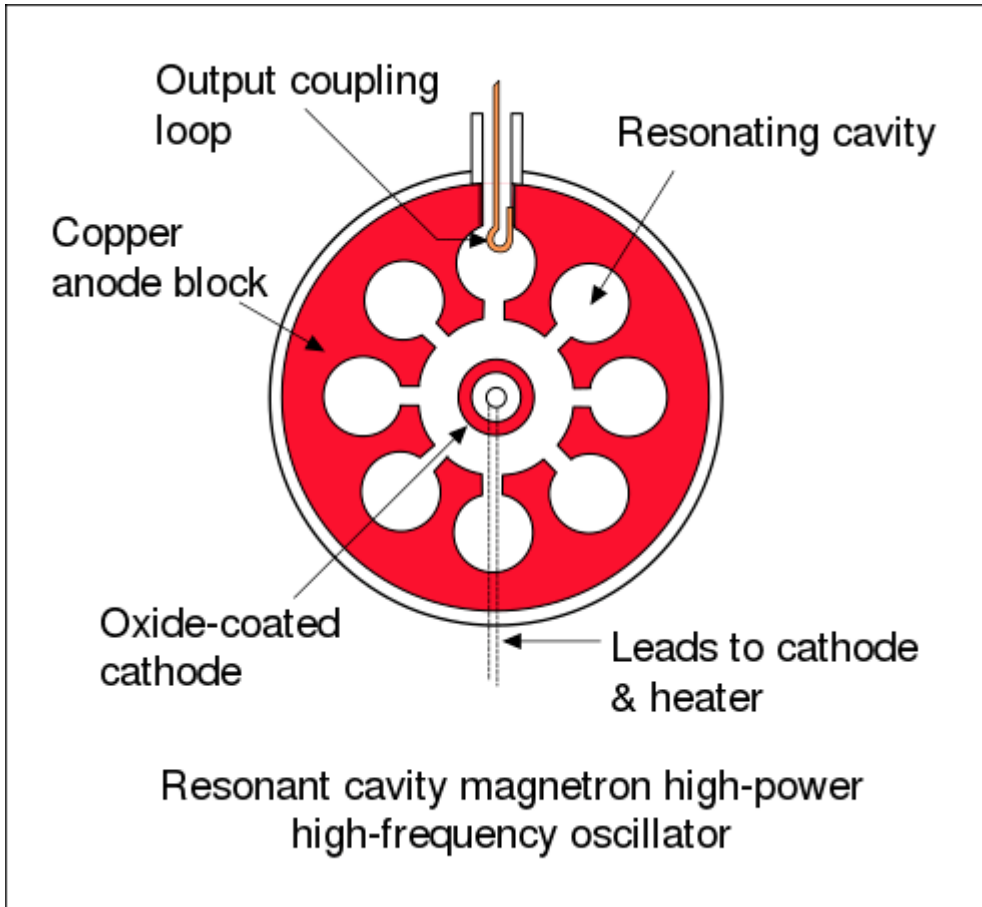
Power output 25 mW

400 kW klystron used for spacecraft communication at the Canberra Deep Space Communications Complex.

courtesy of Wikipedia

Basics of Electricity and Magnetism

Generating of electromagnetic waves. Microwaves. Magnetron.



Microwave oven magnetron; typical power 0.7-1.5kW

courtesy of Wikipedia

Homework

Quadrupole static electrical charges configuration with charges locations:

$a/2, a/2, 0$ -1

$-a/2, a/2, 0$ +1

$-a/2, -a/2, 0$ -1

$a/2, -a/2, 0$ -1

Calculate the electrical field distribution along the lines:

$a/2, a/2, z$; $-a/2, a/2, z$ and $0, 0, z$

