Survey of Fundamental Device Physics

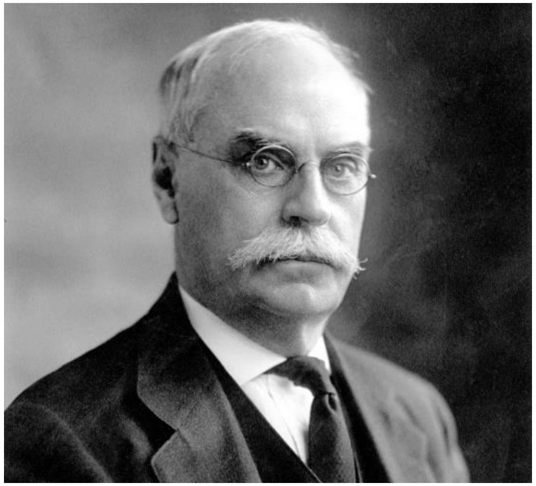
Lecture 4. Eugene V Colla

ILLINOIS

Agenda of the lecture:

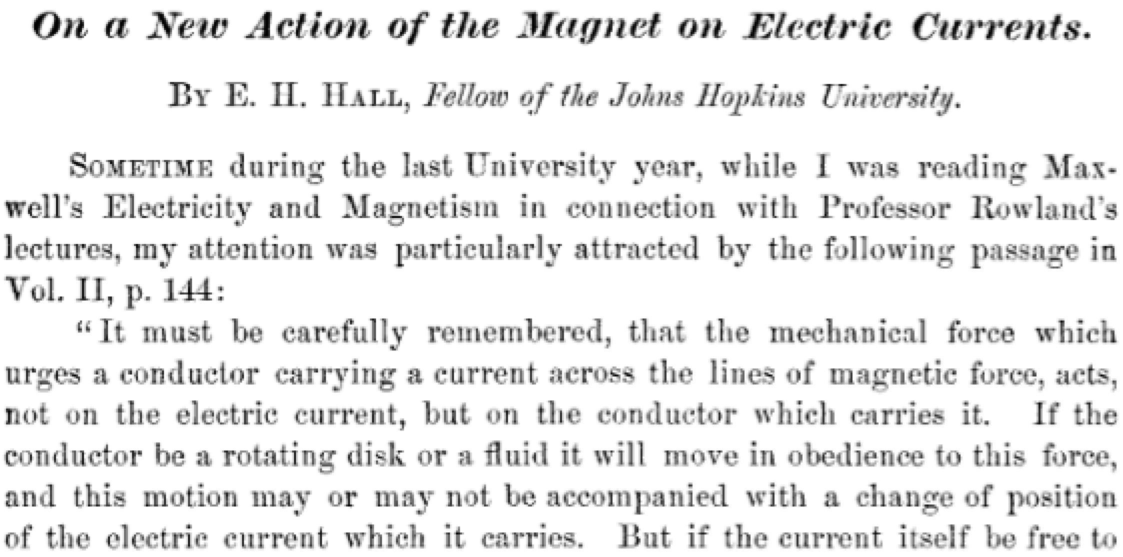
- Hall effect
- Measuring of the magnetic field
- Applications of the Hall effect
- Generating of the magnetic field
- Magnetic shielding
- Applications of the magnetic field

Hall Effect.



Edwin Herbert Hall
(1855-1938)

American Journal of Mathematics, Vol. 2, No. 3 (Sep., 1879), pp. 287-292 (6 pages)

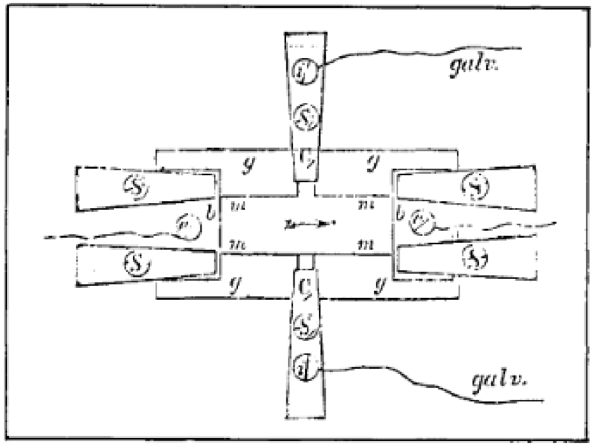


On a New Action of the Magnet on Electric Currents.

By E. H. Hall, *Fellow of the Johns Hopkins University.*

Sometime during the last University year, while I was reading Maxwell's Electricity and Magnetism in connection with Professor Rowland's lectures, my attention was particularly attracted by the following passage in Vol. II, p. 144:

"It must be carefully remembered, that the mechanical force which urges a conductor carrying a current across the lines of magnetic force, acts, not on the electric current, but on the conductor which carries it. If the conductor be a rotating disk or a fluid it will move in obedience to this force, and this motion may or may not be accompanied with a change of position of the electric current which it carries. But if the current itself be free to



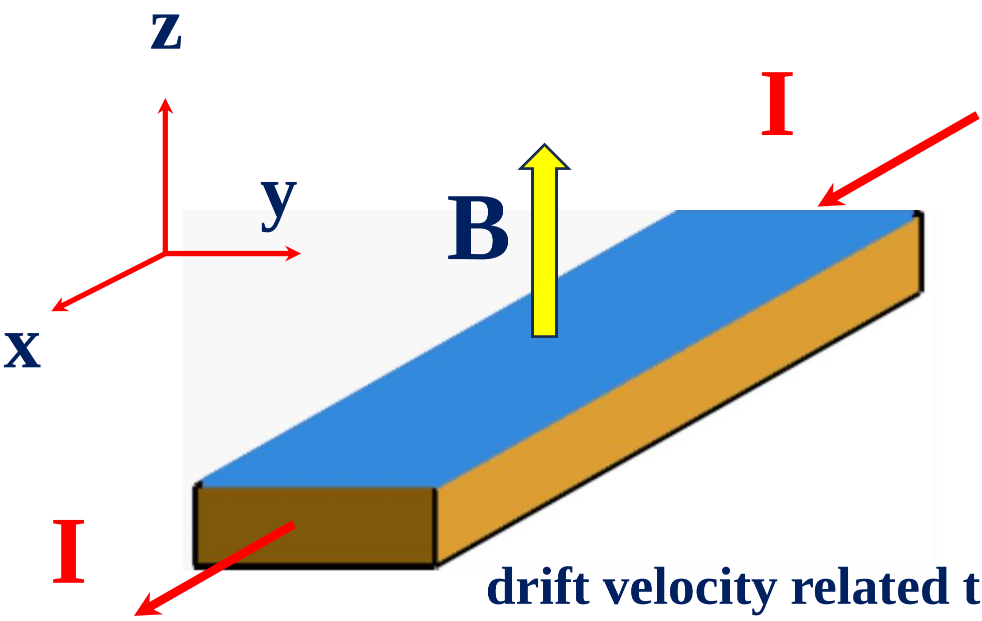
Specimen mounting used by Hall in his early measurements of the transverse potential difference set up in a fixed current carrying conductor subjected to a transverse magnetic field. gggg represents the plate of glass upon which the specimen, in the form of a metal strip mmmm, is mounted…*

******Physics Education, v14, 374 (1979)***

Hall Effect.

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

The current in x direction could be written as: $\vec{I}_x = NqAv_x\hat{x}$



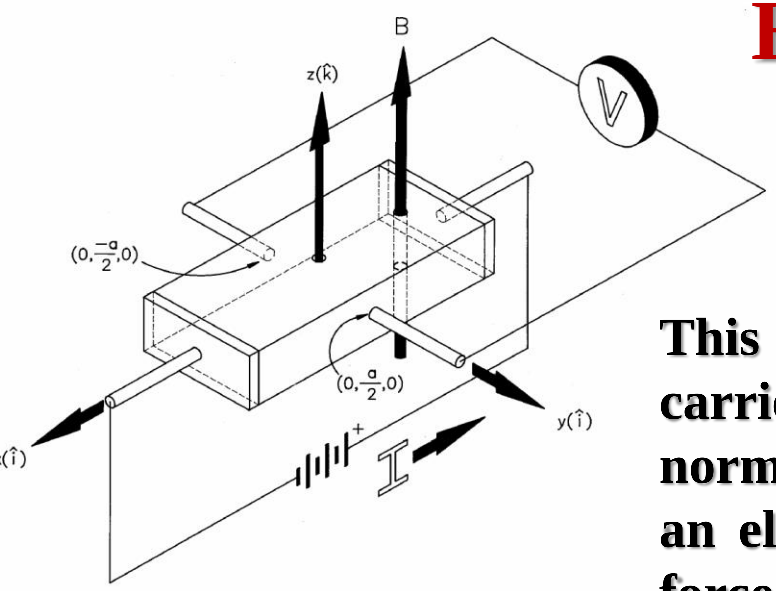
where *N* is the concentration of carriers, *q* is carrier charge and *A* is a cross-section area of the bar and v_x – drift velocity.

drift velocity related to the current as $v_x\hat{x} = \dfrac{\vec{I}_x}{NqA}$

By applying the magnetic field *B* the carriers will be under the Lorentz force

$$\vec{F} = q\vec{v} \times \vec{B} = q\left(\frac{I_x}{NqA}\hat{x}\right) \times B_z\hat{z} = -\frac{I_xB_z}{NA}\hat{y}$$

Hall Effect.

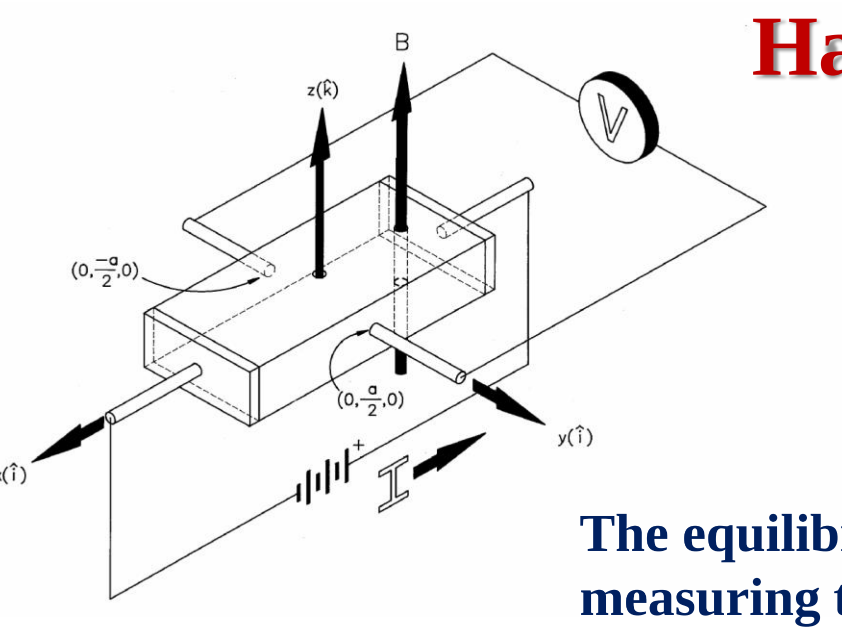


This force will produce the deflection of the carriers resulting in extra charges on the surfaces normal to y axis. Extra charges will give a rise to an electric field E_y. The electric field will exert a force on carriers in the direction opposite the magnetic force. Carriers will flow in y direction until both forces balance:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} = 0 \quad \Longrightarrow \quad qE_y\hat{y} - \frac{I_x B_z}{NA}\hat{y} = 0$$

$$E_y = \frac{I_x B_z}{qNA}$$ equilibrium field

Hall Effect.

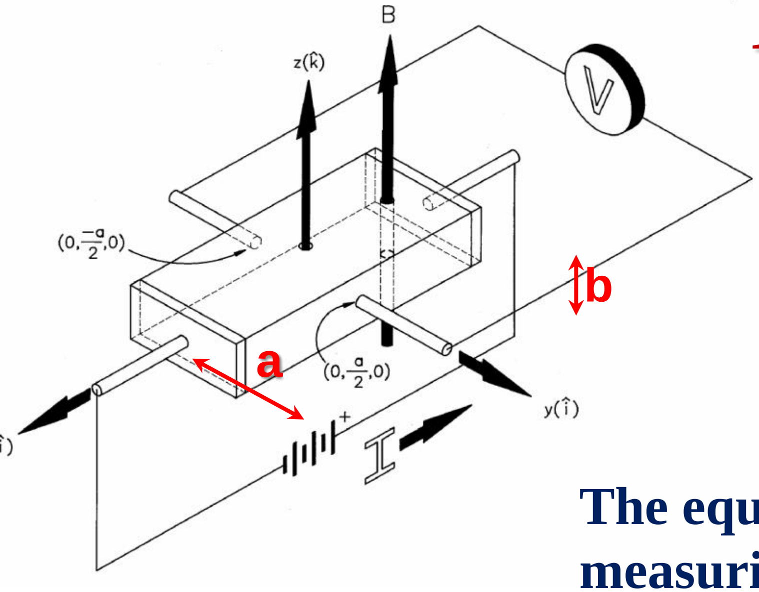


$$qE_y\hat{y} - \frac{I_x B_z}{NA}\hat{y} = 0$$

The equilibrium field E_y could be determined by measuring the potential difference across the sample.

$$V_H = -\int_{-a/2}^{a/2} E_y dy = -E_y a$$

a **– width of the bar**



Hall Effect.

$$qE_y\hat{y} - \frac{I_x B_z}{NA}\hat{y} = 0$$

The equilibrium field *E_y* could be determined by measuring the potential difference across the sample.

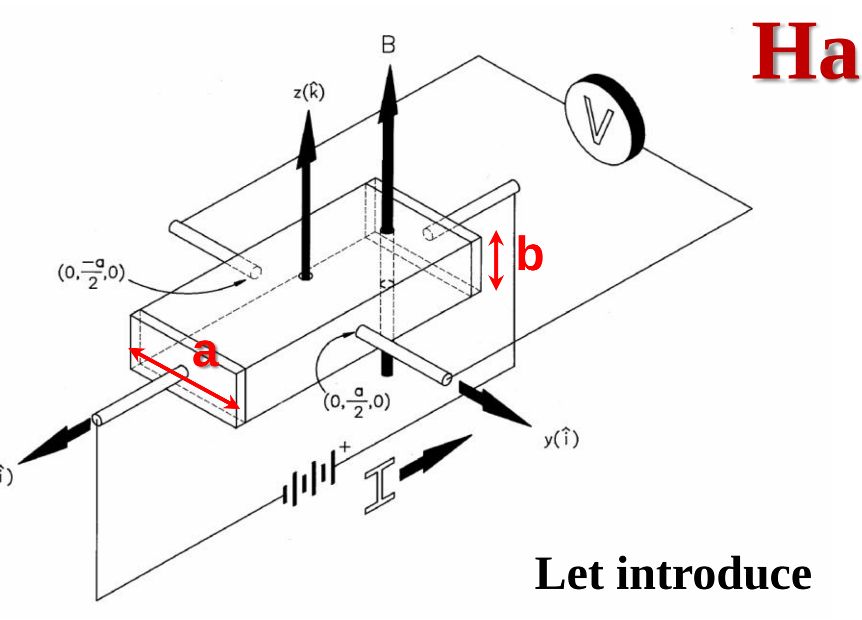
$$V_H = -\int_{-a/2}^{a/2} E_y dy = -E_y a$$

a **– width of the bar**

Finally

$$\boxed{V_H = \frac{I_x B_z}{qNb}}$$

Hall Effect.



$$V_H = \frac{I_x B_z}{qNb}$$

Let introduce $R_H = \frac{1}{Nq}$ as a Hall coefficient.

And Hall voltage $V_H = R_H \frac{I_x B_z}{b}$

Hall voltage is proportional to the magnetic field and Hall effect can be used in equipment for measuring the magnetic filed

Hall Effect. Magnetic Field Sensors.

$$V_H = \frac{I_x B_z}{qNb}$$

$$R_H = \frac{1}{Nq}$$ $[\mathrm{m^3 s^{-1} A^{-1}}]$

$$V_H = R_H \frac{I_x B_z}{b}$$

	Material	n [$\mathrm{m^{-3}}$] (x 10^{28})	R_H [$\mathrm{m^3 A^{-1} s^{-1}}$] (x 10^{-11})
Metals	Ag	5.85	-9.0
	Al	18.06	-3.5
	Be	24.2	+3.4
	Au	5.90	-7.2
	Cu	8.45	-5.5
	Na	2.56	-25
Semiconductors	Ge	2.4 10^{-9}	$-9\mathrm{x}10^{10}$
	Si	$2.5\mathrm{x}10^{-7}$	$-2.5\mathrm{x}10^{9}$
	GaAs	$3\mathrm{x}10^{-7}$	$-2.1\mathrm{x}10^{9}$

Hall Effect. Magnetic Field Sensors. Measuring of the magnetic field.

$$V_H = R_H \frac{I_x B_z}{b} \quad \Longrightarrow \quad B_z = \frac{bV_H}{R_H I_x}$$

Sensitivity of the Hall sensor $$\frac{dV}{dB} \sim R_H$$

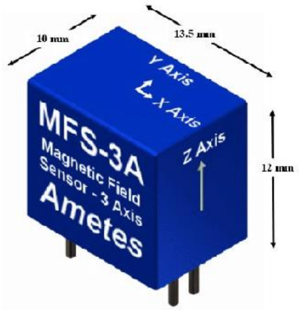
We need to find materials with high R_H

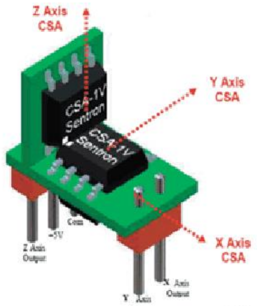
The best for Hall probe applications are the semiconductors:

Material	Eg [eV]	n [cm-3]	$-R_H[\mathrm{cm}^3\mathrm{C}^{-1}]$
Ge	0.67	$2.4\mathrm{x}10^{13}$	$9\mathrm{x}10^{4}$
Si	1.12	$2.5\mathrm{x}10^{15}$	$2.5\mathrm{x}10^{3}$
InSb	0.17	$9\mathrm{x}10^{16}$	70
InAs	0.36	$5\mathrm{x}10^{16}$	125
GaAs	1.42	$3\mathrm{x}10^{15}$	$2.1\mathrm{x}10^{3}$

Hall Effect. Magnetic Field Sensors. Measuring of the magnetic field.

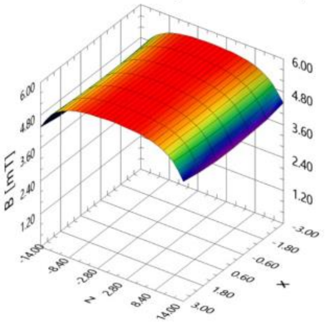
3D sensor from Ametes

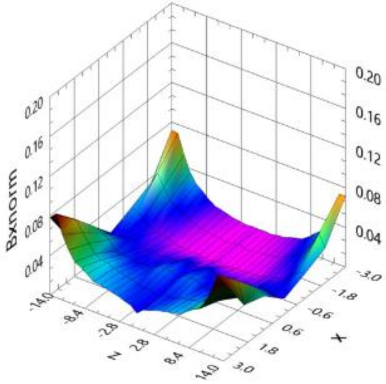


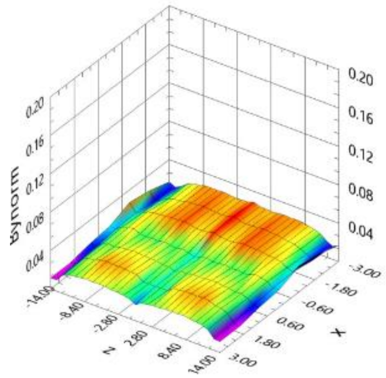


Technical specifications of MFS-3A.

Measurement	Bx, By, Bz
Range	±7.3 mT
Resolution	±20 µT
Sensitivity	280 mV/mT
Accuracy	±3 %
Angular Alignment	±3°
Dimensions	10×13.5×12 mm







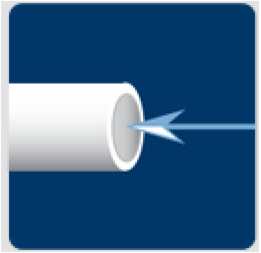
Branko KOPRIVICA et all, "MEASUREMENT OF MAGNETIC FLUX DENSITY OF LARGE-DIAMETER MULTILAYER SOLENOID", 13th International Conference on Applied Electromagnetics - ПЕС 2017

Hall Effect. Magnetic Field Sensors. Measuring of the magnetic field. Gaussmeters.

	F71	F41	475	425
Description	Multi-axis/single-axis benchtop	Single-axis benchtop	Single-axis benchtop	Single-axis benchtop
Probe type	Vector or single-axis measurements	Single-axis measurements	Single-axis measurements	Single-axis measurements
Field ranges (G)	1 mG to 350 kG	1 mG to 350 kG	1 mG to 350 kG	1 mG to 350 kG
Frequency ranges (Hz)	DC to 50 kHz	DC to 50 kHz	DC to 20 kHz	DC to 10 kHz
Accuracy at 1 kG	1.5 G	1.5 G	4.9 G	9.2 G

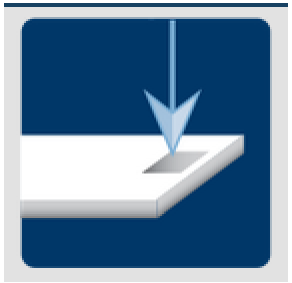
Units (magnetic flux density B) : SI – Tesla T (Wb/m^2); CGS – Gauss G; 1T=10^4G

Hall probes



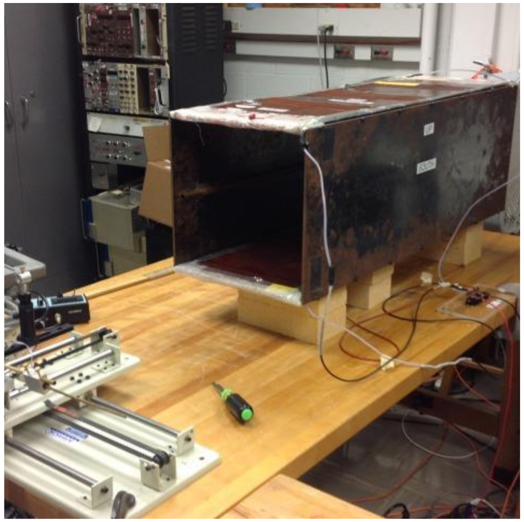
Axial probe

Transverse probe

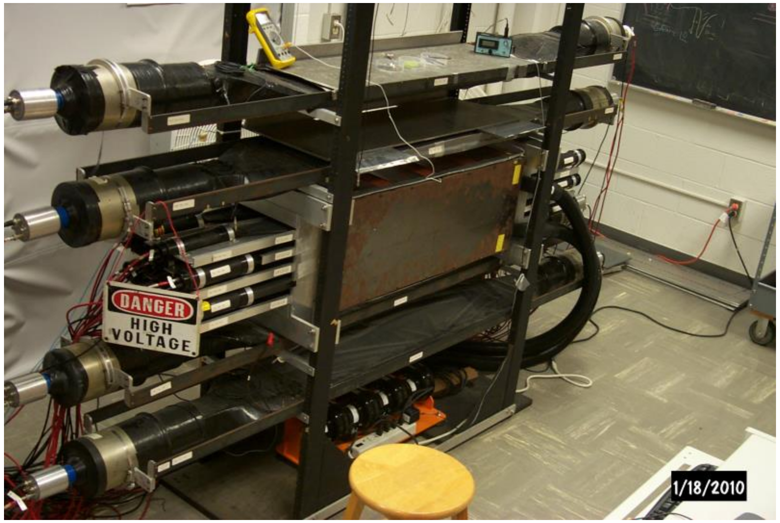


Tangential probe

Hall Effect. Magnetic Field Sensors. Measuring of the magnetic field. P403 Lab.

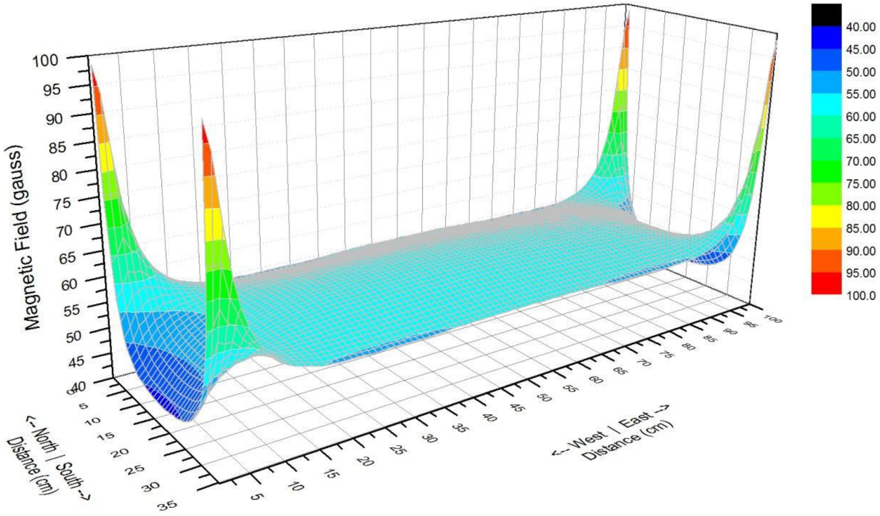


Box magnet for muon experiment setup. Mapping the magnetic field.

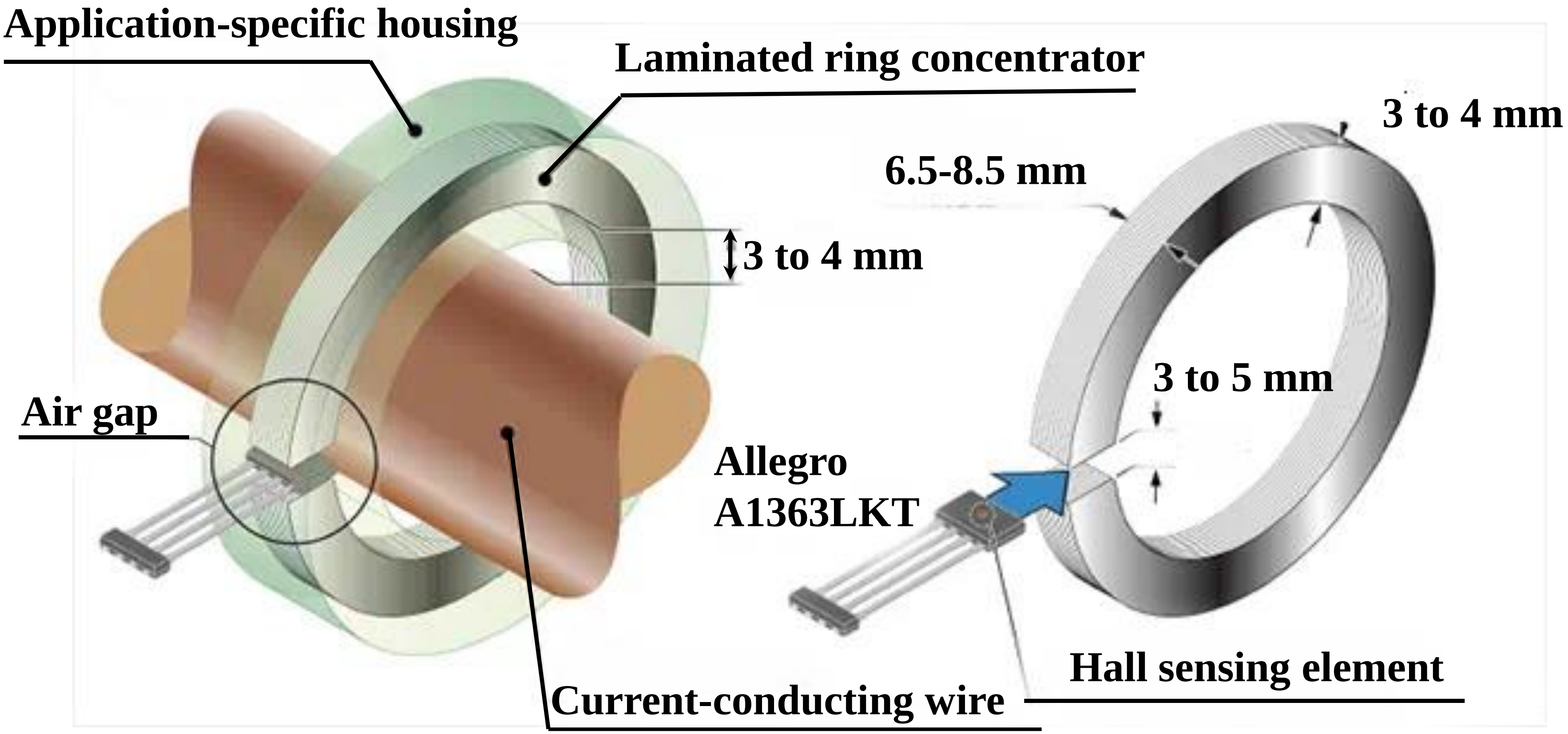


Box magnet hosted several scintillators.

Magnetic field distribution inside of the box magnet measured using Hall probe and gaussmeter



Hall Effect. Applications. Current Sensors.

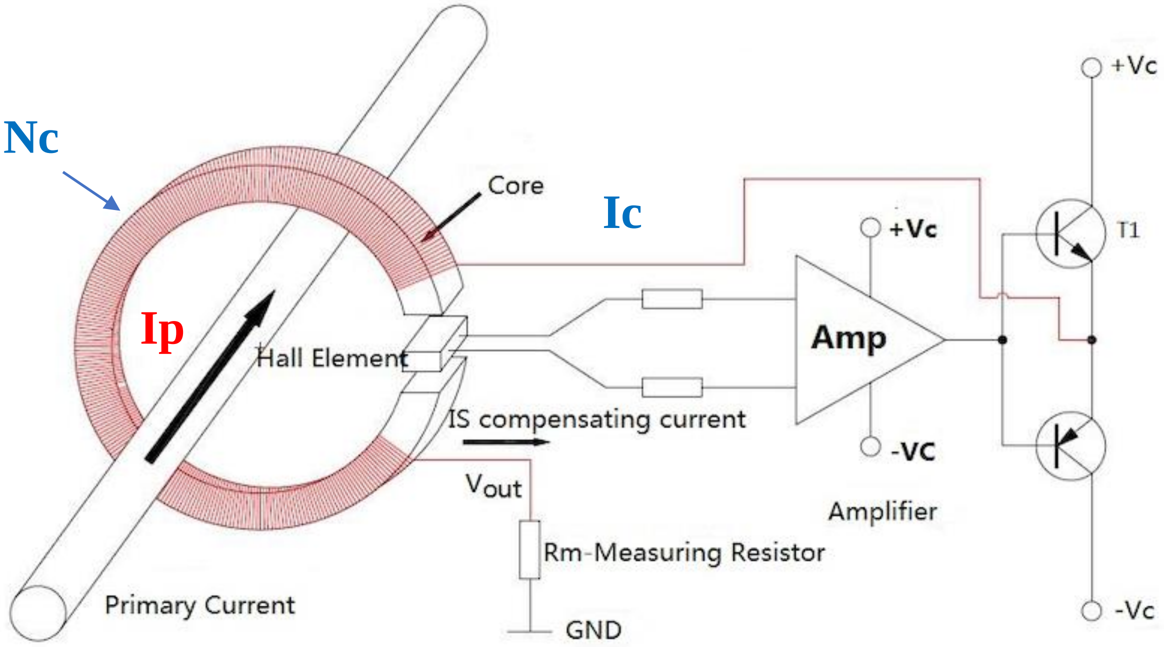


$$B = \frac{\mu\mu_0 I}{2\pi r}$$

$$V_H = \left(R_H \frac{I_x}{b} \right) \left(\frac{\mu\mu_0}{2\pi r} \right) I$$

Courtesy Allegro MicroSystems, LLC

Hall Effect. Applications. Current Sensors.



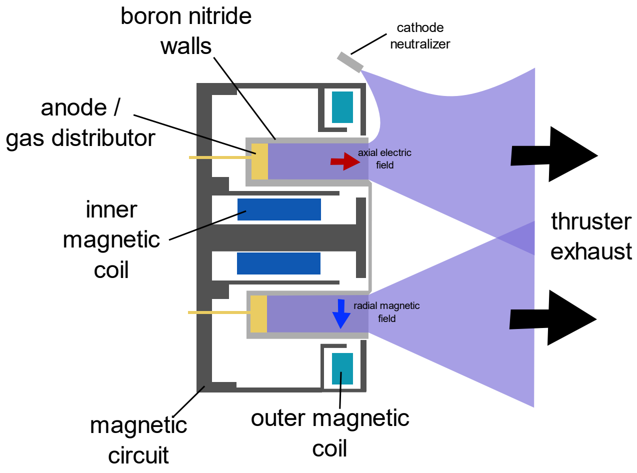
Field created by the primary (measured) current

Compensating field created by the coil winded on the toroid

$$B_p = \frac{\mu\mu_0 I_p}{2\pi r} = \frac{\mu\mu_0 I_c N_c}{2\pi r} \Rightarrow I_p = I_c N_c$$

Courtesy All About Circuits

Hall Effect. Applications. Hall Thrusters.



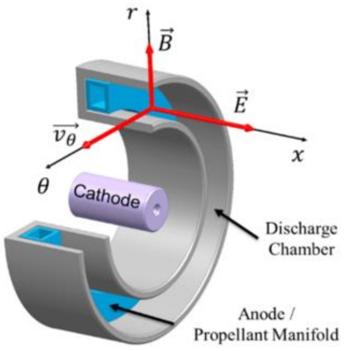
NGHT-1X Engineering Model Hall-Effect Thruster.
Credits: Northrop Grumman

Propellant	Xe
Discharge power, W	до 900
Thrust, mN	до 55
Specific impulse, s	до 2100
Mass, kg	3.1

ФАКЕЛ

SPT-230, Russia

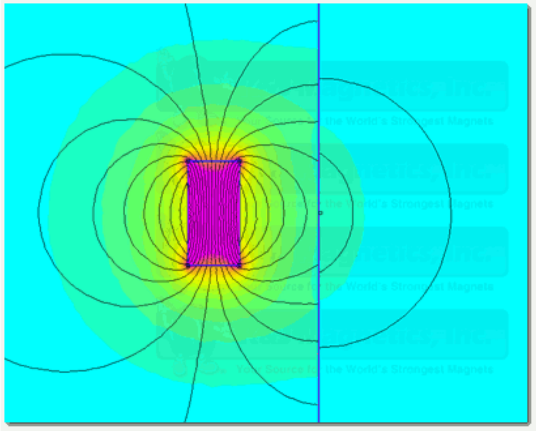
Propellant	Xe
Discharge power, W	до 25000
Thrust, mN	до 1070
Specific impulse, s	до 3200
Mass, kg	25



Hall-effect Thruster schematic.
Credits: NASA

Magnetic Shielding

Static magnetic shielding using high μ materials

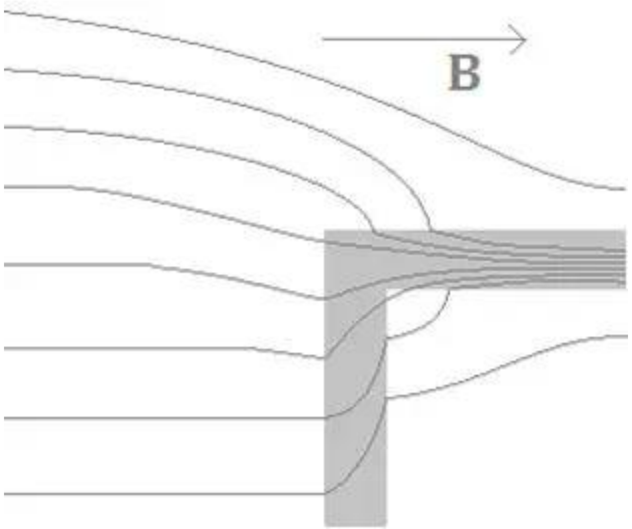


Credit: K&J Magnetics, Inc.

SF=Hint/Hex

Shielding factor.
Usually, quality of shielding (shielding efficiency) can be presented in log units:

SE (db) =20log(Hext/Hint)

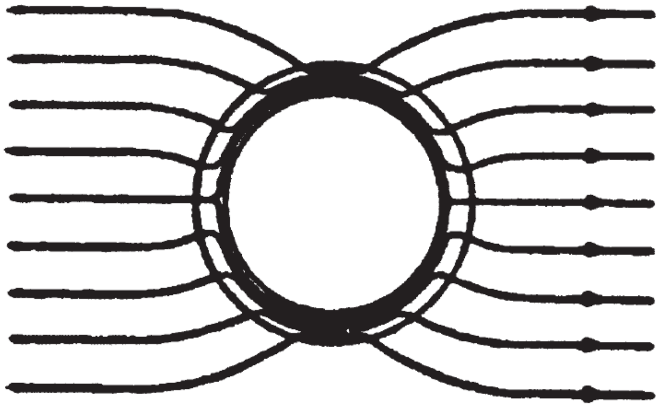


Field distortion observed in shielding cube. If the shield is of high permeability the flux that has leaked through a perpendicular side attenuates again within the shield.

μ metal does not block the magnetic flux but redirects the magnetic lines and as the result decreases the field intensity in shielded area

Magnetic Shielding

Shielding factor



Magnetic lines in and around high μ material shield

For cylindrical shield with l>>r

$$SE^* \approx \frac{\mu t}{2r}$$

μ – permeability of the shield material
r – outer radius of the shield
t – thickness of the shield

For sphere $SE^* \approx \frac{4\mu t}{3D} + 1$

D – outer diameter of the sphere

For cube $SE^* \approx \frac{4\mu t}{5a} + 1$

SE (db) =20log(H_{ext}/H_{int})

* Shielding efficiency in linear scale (no in db)

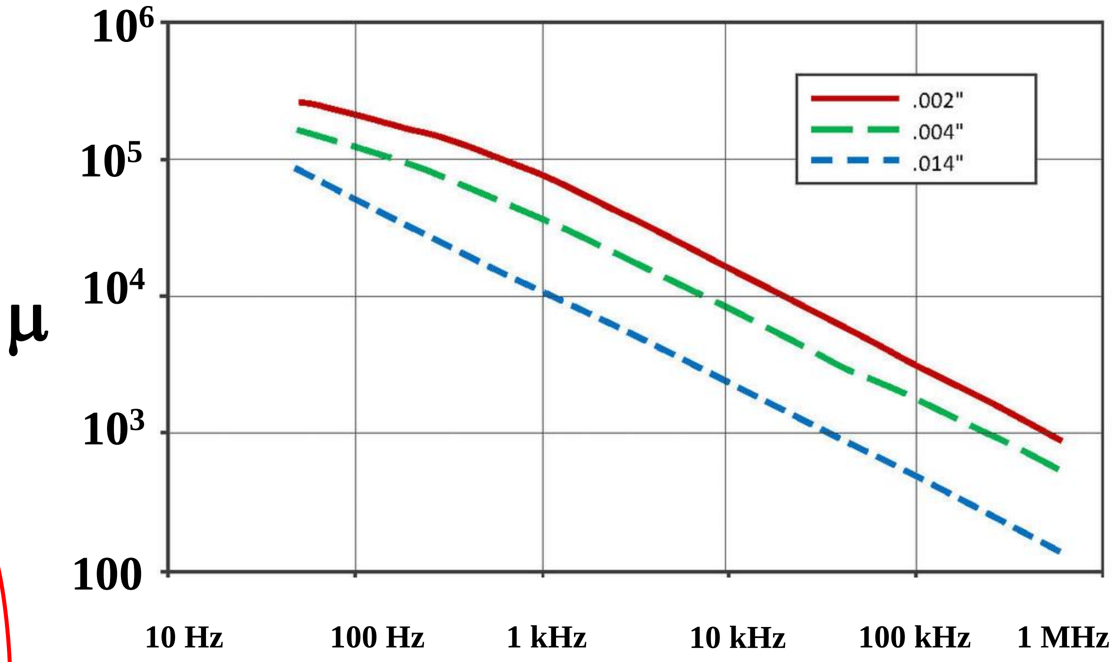
A. Goldman, Handbook of Modern Ferromagnetic Materials, Kluwer Academic Publishers 1999

Magnetic Shielding

Magnetic shielding. High μ materials. Frequency response.

Material	μ
Free space	1.000 000 00
Air	1.00000037
Al	1.00002
Cu	0.99999
96%, 4% Si (non-oriented)	7,000
97 Fe, 3% Si (grain oriented)	100,000
50% Co, 50% Fe (Permendur)	5,000
79% Ni, 16% Fe, 5% Mo (Super Malloy)	1,000,000
97% Fe, 3% Si (monocrystal)	3,800,000

Ferromagnetic materials



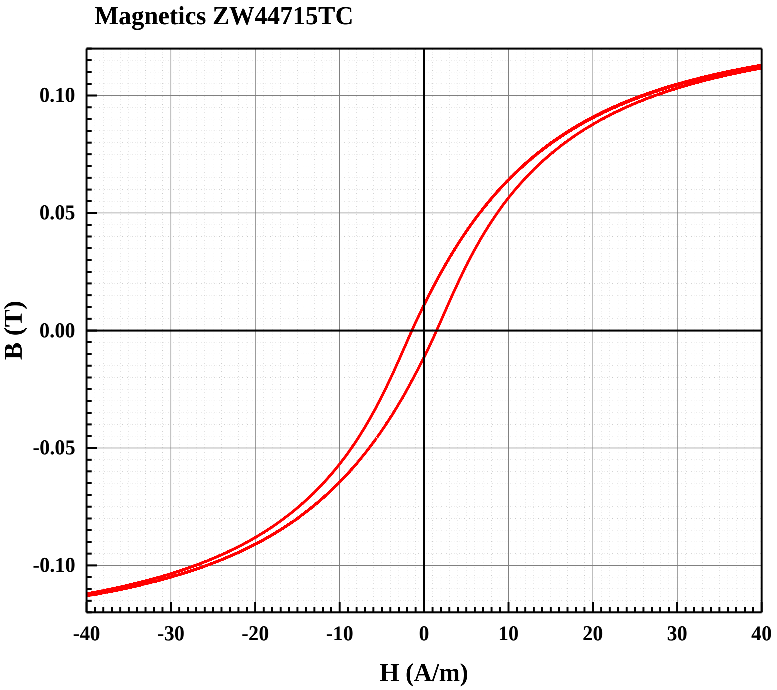
Frequency dependence of the permeability of μ metal

Courtesy of MuMetal

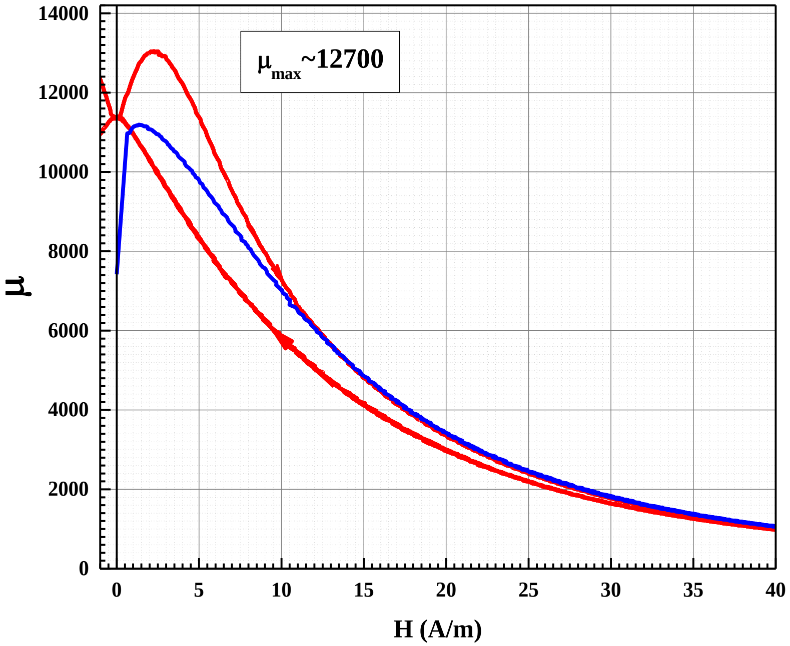
Magnetic Shielding

Shielding using high μ materials. Limitations.

Independent on shield design SE is proportional permeability of the shielding material



Magnetics ZW44715TC



$$\mu = \frac{dB}{dH}$$

μ depends on the applied magnetic field, and it results in efficiency of the material for shielding application.

Courtesy of Physics 401 course

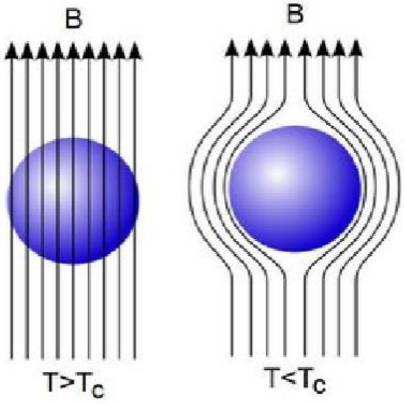
Magnetic Shielding

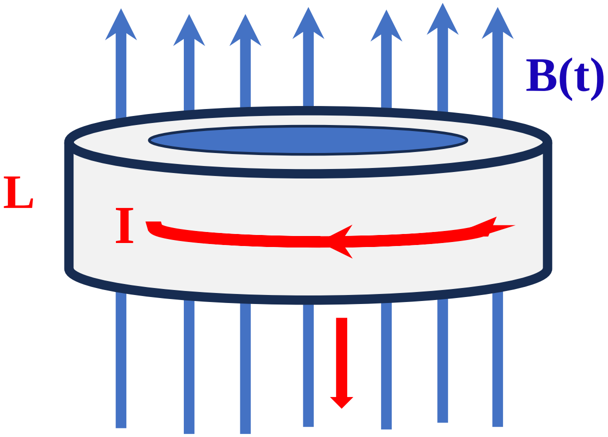
Magnetic shielding using diamagnetic material materials. Superconductors.

Fritz Walther Meissner
1882-1974

Robert Ochsenfeld
1901 –1993

Meissner, W. and Ochsenfeld, R.„Ein neuer Effekt bei Einfritt der Supraleitfähigkeit“; Naturwissenschaften, 21, 787-788 (1933)





$\Phi_e(t)$ - magnetic flux due to external field B(t)

$\Phi_i(t) = LI(t)$ - flux generated by the superconducting current I, L – inductance of the ring

$\Phi_t(t) = \Phi_e + LI(t)$ - total flux

$$E(t) = -\frac{d\Phi_t}{dt} = -\frac{d\Phi_e}{dt} - L\frac{dI}{dt} = 0 \qquad \text{Faraday law}$$

E(t)=0 - emf equals zero because of the zero resistivity of the ring

$$\frac{d\Phi_e}{dt} = -L\frac{dI}{dt} \Longrightarrow \Phi_e(t) = -LI(t)$$

Magnetic Shielding

Magnetic shielding using diamagnetic material materials. Superconductors.

Limitations: critical temperature

Material	T_c (K)	Year
Hg	4.1	1911
Pb	7.2	1913
Nb	9.2	1930
$NbN_{0.96}$	15.2	1950
Nb_3Sn	18 1	1954
$Nb_3(Al_{3/4}\ Ge_{1/4})$	20–21	1966
$Nb_3\ Ga$	20 3	1971
$Nb_3\ Ge$	23 2	1973
$Ba_x\ La_{5-x}\ Cu_5\ O_y$	30–35	1986
$YBa_2\ Cu_3\ O_{7-\delta}$	95	1987
$Bi_2\ Sr_2\ Ca_2\ Cu_3\ O_{10}$	110	1988
$Tl_2\ Ba_2\ Ca_2\ Cu_3\ O_{10}$	125	1988
$HgBa_2\ Ca_2\ Cu_3\ O_{8+\delta}$	133	1993

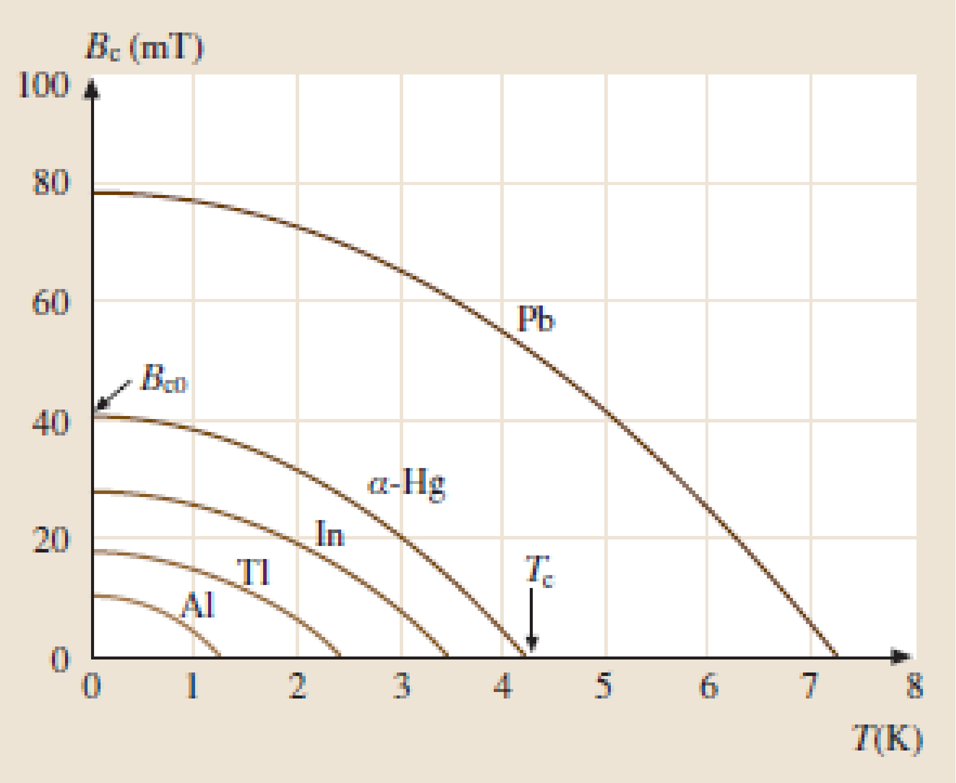
High T_c

Charles P. Poole, Jr. et all, Superconductivity, Academic Press, 2007

Magnetic Shielding

Magnetic shielding using diamagnetic material materials. Superconductors.

Limitations: critical magnetic field.



$$B_c(T) = B_{c0}\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

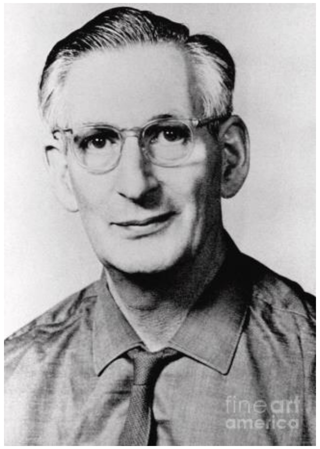
Pb: B_c at 4.2 K ~50 mT

Magnetic Shielding

Magnetic shielding using diamagnetic material materials. Superconductors.

Limitations: penetration depth.

Fritz Wolfgang London
1900-1954



Heinz London
1907-1970

London Equations and penetration depth

$$\frac{\partial \vec{j}_s}{\partial t} = \frac{n_s e^2}{m}\vec{E} \qquad \nabla \times \vec{j}_s = -\frac{n_s e^2}{m}\vec{B}$$

$$\downarrow$$

$$\nabla^2 \vec{B} = \frac{1}{\lambda^2}\vec{B}$$

where

$$\boxed{\lambda_s \equiv \sqrt{\frac{m}{\mu_0 n_s e^2}}}$$

London penetration depth

m, e	**-**	**mass and charge of electron**
N_s	**-**	**density of superconducting carriers**
J_s	**-**	**superconducting current density**
E, B	**-**	**electrical and magnetic fields**

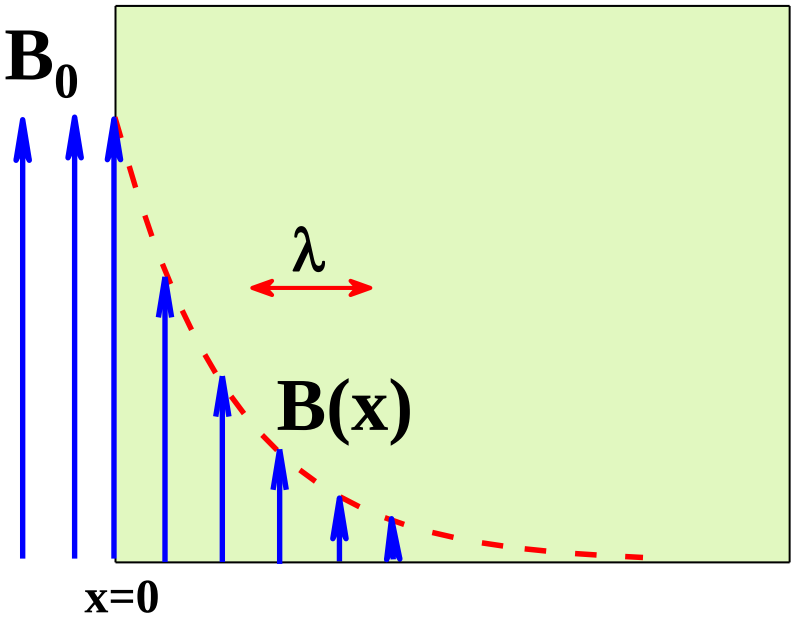
Magnetic Shielding

Magnetic shielding using diamagnetic material materials. Superconductors.

Limitations: penetration depth.

If the magnetic field is uniform on boundary of superconductor equals B_0 the magnetic field inside of the superconductor is:

$$B(x) = B_0 \exp\left(-\frac{x}{\lambda}\right)$$



λ is temperature dependent parameter

$$\lambda(T) = \frac{\lambda(0)}{\sqrt{\left[1-\left(\frac{T}{T_c}\right)^4\right]}}$$

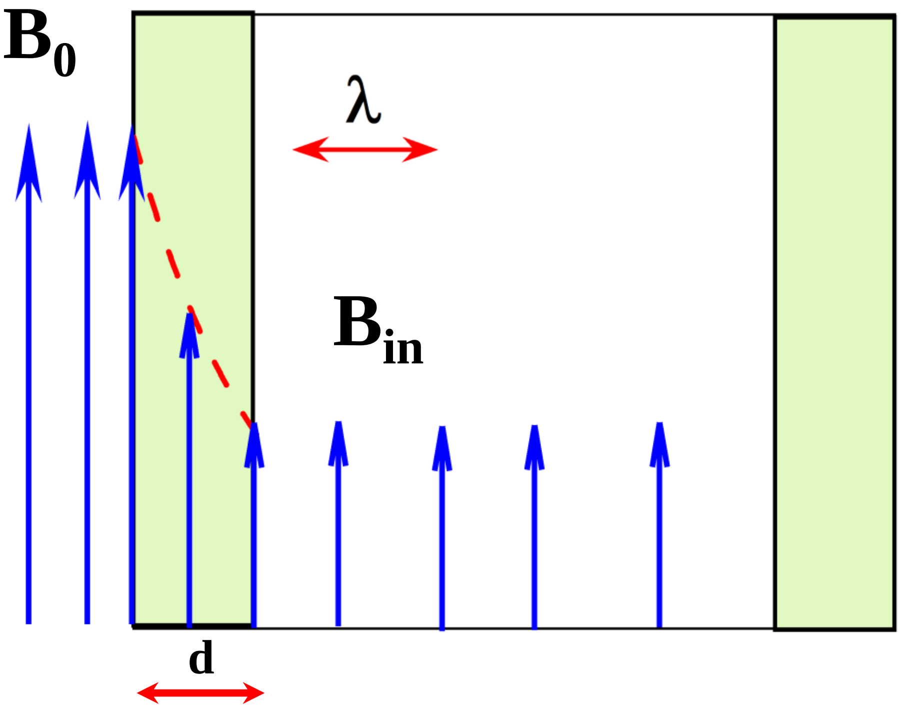
Magnetic Shielding

Magnetic shielding using diamagnetic material materials. Superconductors.

Limitations: penetration depth.

$$\lambda(T) = \frac{\lambda(0)}{\sqrt{\left[1 - \left(\frac{T}{T_c}\right)^4\right]}}$$

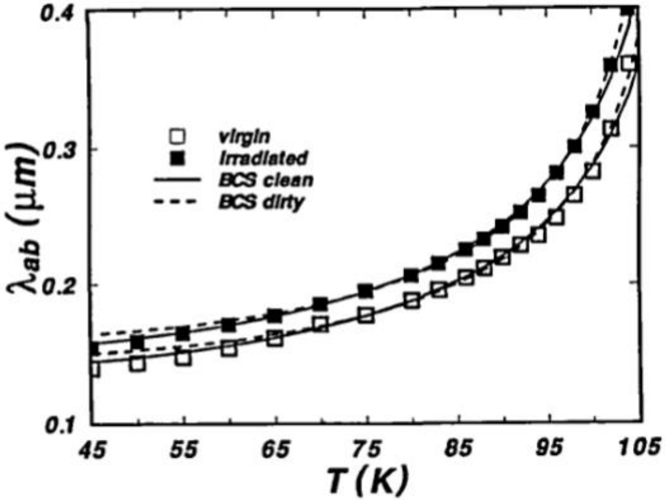
Material	London penetration depth λ_L(nm)
Sn	34
Al	16
Pb	37
Cd	110
Nb	39



The model of superconducting shielding box with wall thickness d

Magnetic Shielding

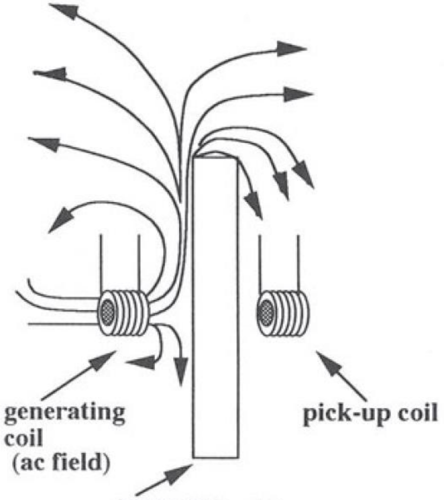
Magnetic shielding using high Tc Superconductors.

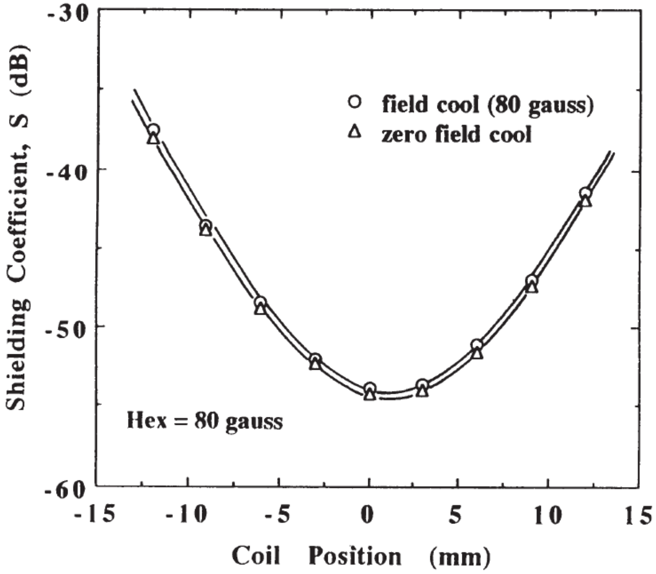


λ(T) as a function of T for virgin and neutron irradiated BiPb-2223 materials

J.G. Ossadson at all, "Effects of neutron irradiation on the London penetration depth for polycrystalline Bi1.8Pb0.3Sr2Ca2Cu$_3$O$_{10}$ superconductor"; Proceedings of the 4th International Conference and Exhibition: World Congress on Superconductivity, 1, 347, (2013)

Setup and experimental results of investigation of the penetration depth of HTS BiPb-2223 material





Shigefoshi Ohsidma et all, "Magnetic Shielding Effect of a Double Sheet of Fe-Ni and BSCCO"; Advances in Superconductivity VI, 1325, (Springer 1994)

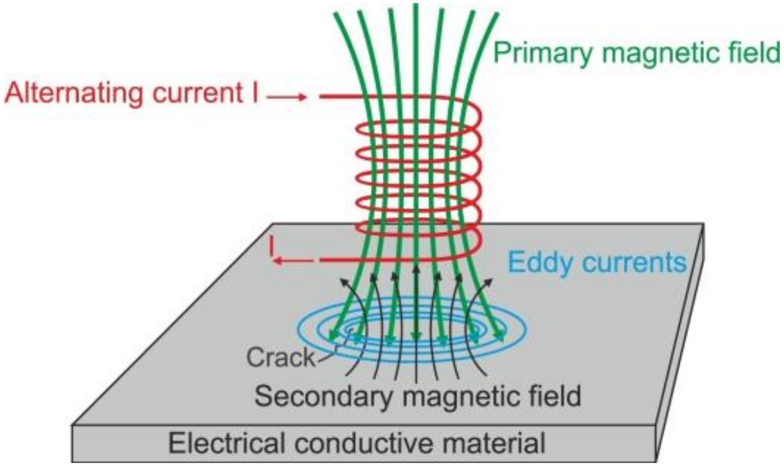
Magnetic Shielding

AC - Magnetic shielding. Eddy currents.

Dominique François
Jean Arago
1786-1853

AC magnetic field will create AC magnetic flux and according the Faraday law will generate the emf in conductive material

$$emf = -\frac{d\Phi_b}{dt}$$



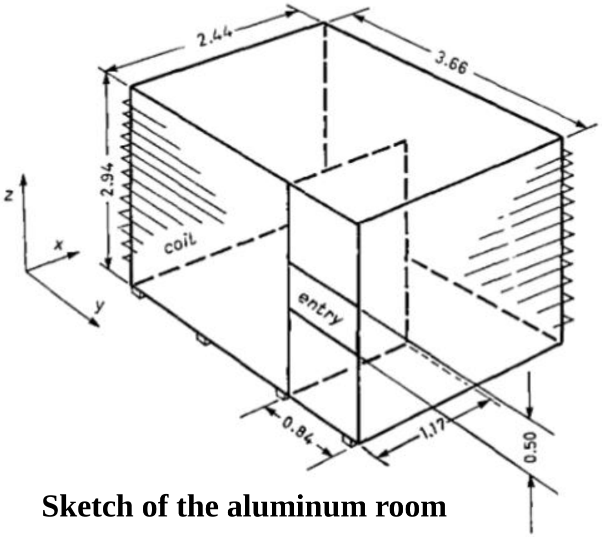
Induced by the variation of the flux emf will generate in conductive material the Eddy currents. Eddy currents will generate the secondary magnetic field directed against the primary magnetic field. The value of the Eddy current depends on the complex impedance of the conductive material and the frequency of the primary field

Magnetic Shielding

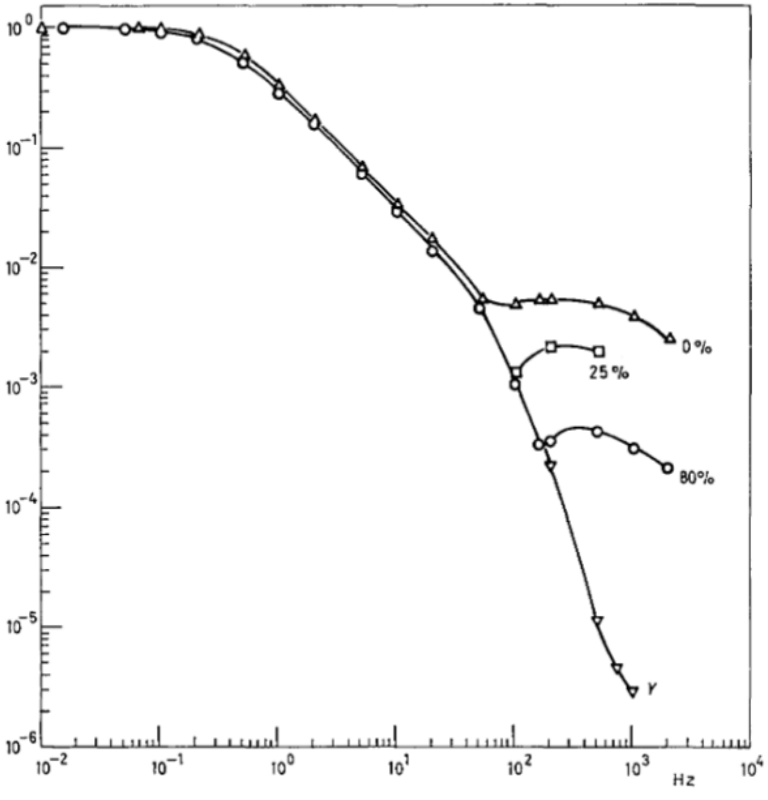
AC - Magnetic shielding. Eddy currents. An Example.

G. Strolnk et all, “An Eddy-Current-Shielded Room with a Partially Closed Entrance”, IL Nuovo Cimento, v2, 195, (1983)

Eddy current shielded room was designed for biomagnetic measurements



Sketch of the aluminum room



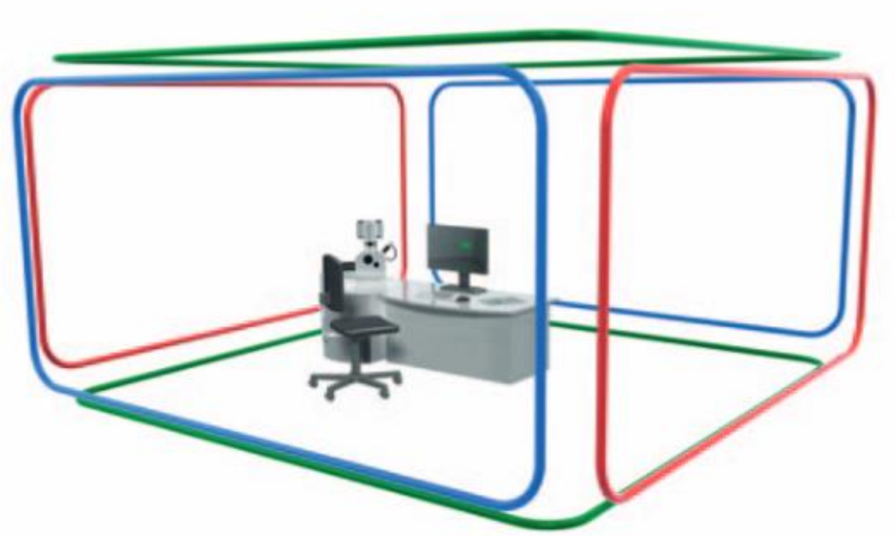
The attenuation Sz vs. frequency with 0%, 25% and 80% of the doorway covered with thick aluminum plates.

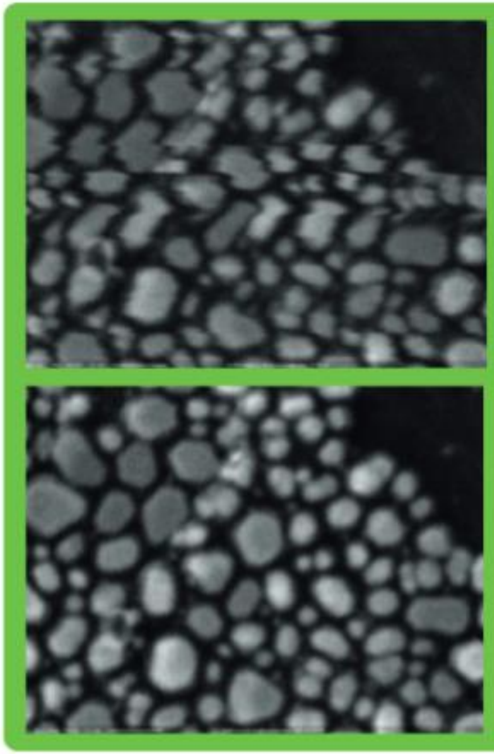
Magnetic Shielding

Magnetic Field Compensation.

To eliminate the effect of the environmental magnetic field we can use some an active compensating system.

Bilz® VIBRATION TECHNOLOGY, INC.



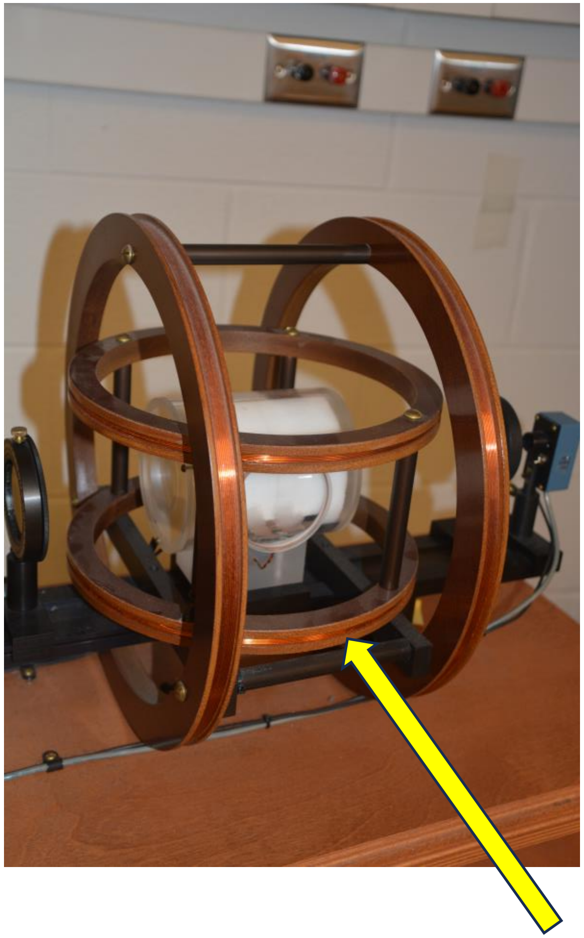


REM image without (above) and with (below) magnetic field compensation

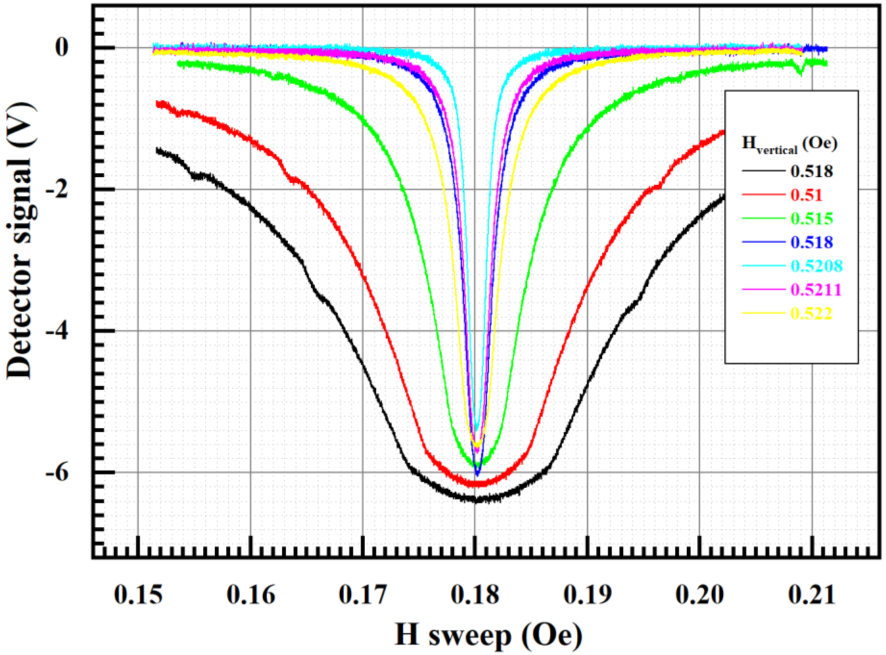
3 axis automatic real-time compensation of low frequency magnetic field disturbance
n Frequency range DC to 1,000 Hz (1kHz)
n Fluxgate magnetic field sensor with sub Nano Tesla resolution
n Controller mode: AC, DC, AD+DC
n 40 db typical suppression of 50 Hz disturbance
n Compensation coil connection capability
n Measured value and alarm display

Magnetic Shielding

Magnetic Field Compensation. P403 – Optical Pumping.

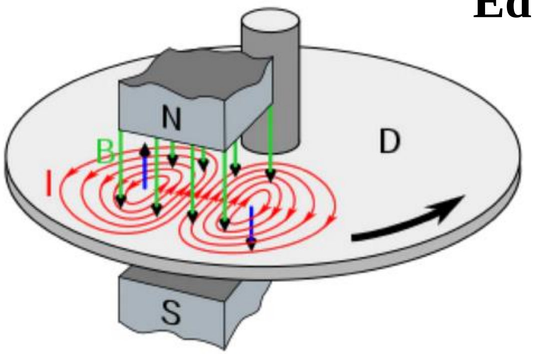

Vertical Coil

The experiment is very sensitive to the magnetic field and even the component of Earth magnetic filed should be compensated. Vertical component can be compensated by using vertical Helmholtz coils.



Magnetic Damping and Braking

Eddy Currents Brakes.

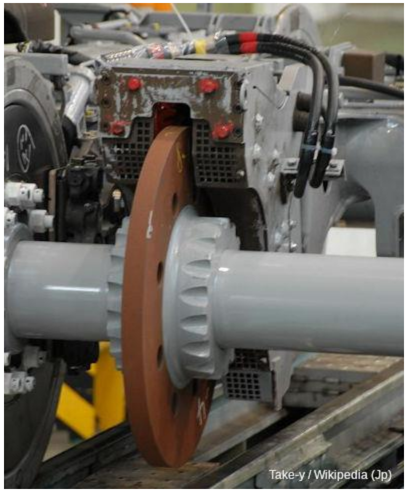


According the Faraday law the magnetic field generated by the eddy currents will work opposite the initial flux change. The magnetic field associated with the eddy currents will act to oppose the initial change in flux, by Lenz's law

Credit to Science Ready HSC Resources

Magnetic brakes:

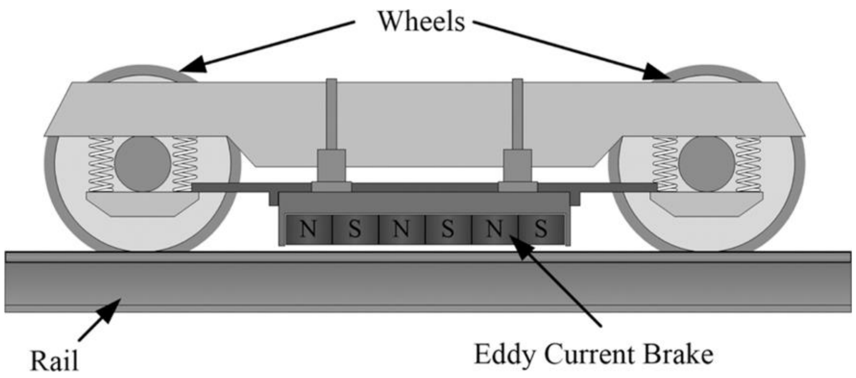
1. No mechanical friction
2. No power required
3. Kinetic energy will release in Joule heating provided by Eddy currents


Take-y / Wikipedia (Jp)

circular eddy-current brake from the Shinkansen 700 train

Magnetic Damping and Braking

Linear Eddy Currents Brakes.



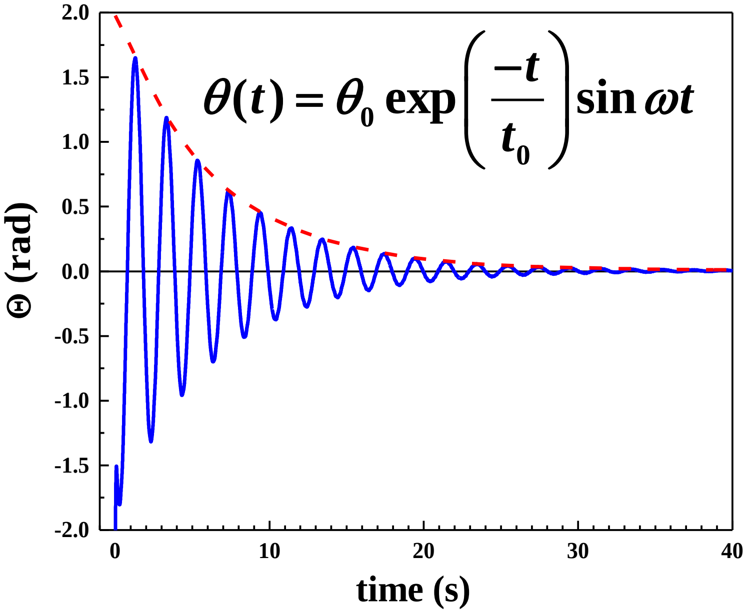
Railway eddy current brake

Credit to “The Institution of Engineering and Technology”

S. Kitanov and A. Podol’ski, The Open transportation Journal, 2, 19 (2008)

Magnetic Damping and Braking

Eddy Currents. Damping. Physics 401 Torsional Oscillator experiment.



$$\theta(t) = \theta_0 \exp\left(\frac{-t}{t_0}\right) \sin \omega t$$

$$I\frac{d^2\theta}{dt^2} + K\theta + R\frac{d\theta}{dt} = 0$$

Damping coefficient

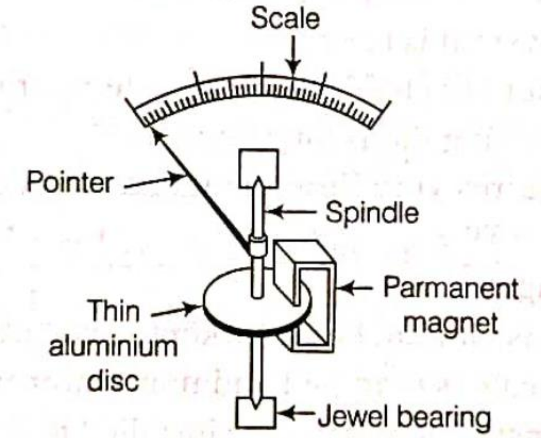
Log decrement $\delta = \ln\left(\frac{\theta_n}{\theta_{n+1}}\right) = \frac{2\pi}{\omega t_0}$

Magnetic Damping and Braking

Eddy Currents. Damping. More examples.

The Eddy Current Damper is a rate-limiting device that has been successfully applied in solar array and antenna deployment.

Credit to satsearch



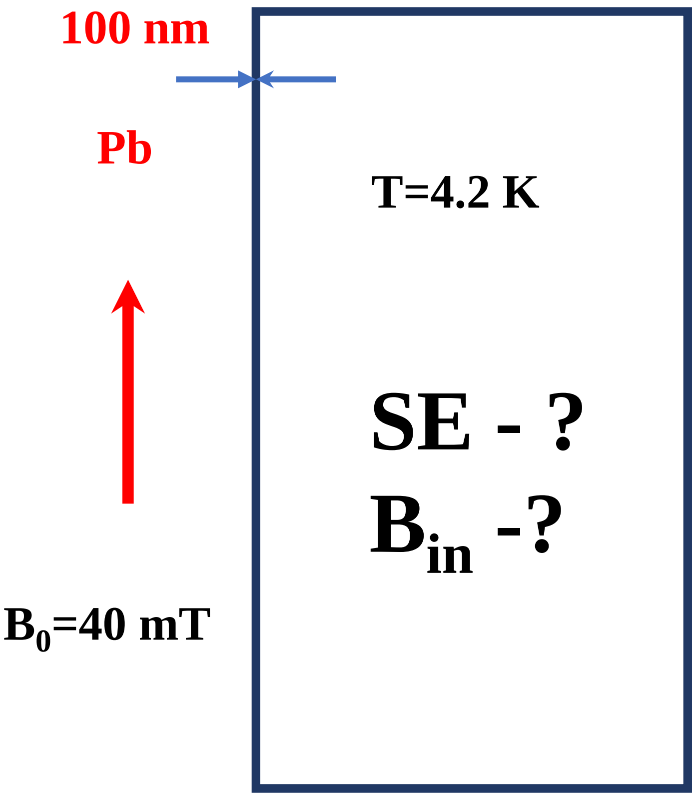
Eddy Current Damping System

Damping system in analog measuring instruments

Homework:

Here is the box shielded by the Pb in superconducting state. Box is faced to magnetic field of 40mT and is at T=4.2K. The thickness of shielding material is 100 nm.

Calculate the residual field in the cavity and SE – shielding efficiently



100 nm

Pb

T=4.2 K

SE - ?

B_{in} -?

B_0=40 mT