Superconductivity - Overview

Alexey Bezryadin

Department of Physics
University of Illinois at Urbana-Champaign

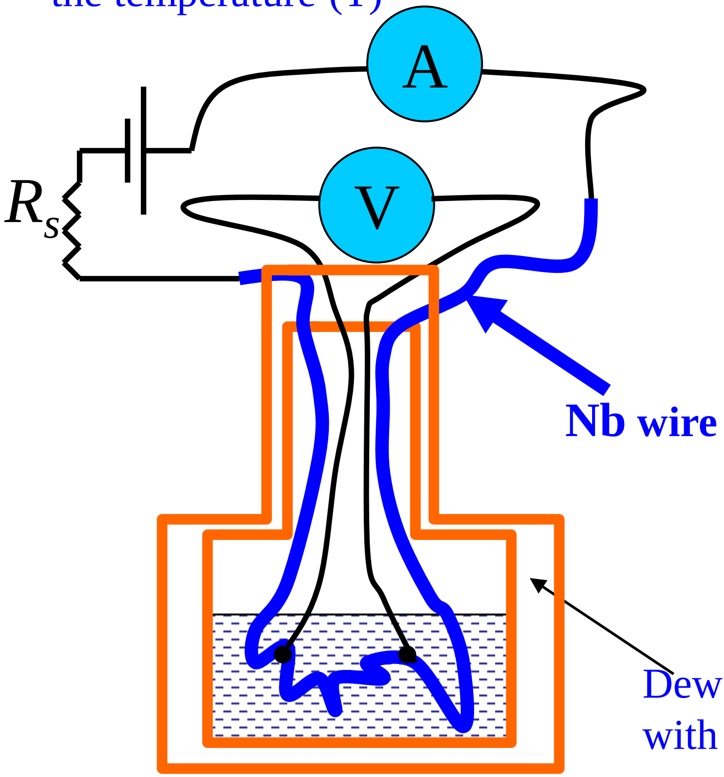
How to measure superconduciting transitions

Electrical resistance of some metals drops to zero below a certain temperature which is called "critical temperature“ (H. K. O. 1911)

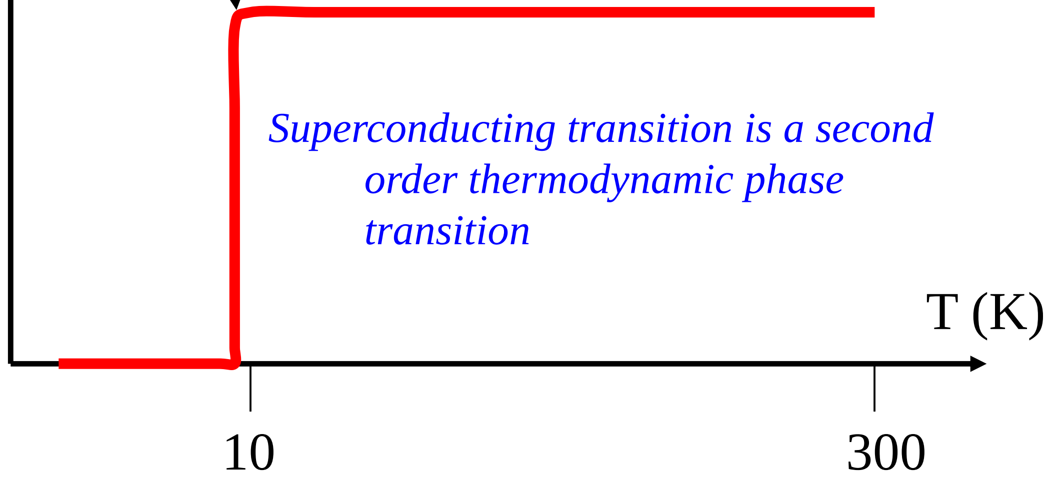
How to observe superconductivity

1. Take Nb (niobium) wire
2. Connect to a voltmeter and a current source
3. Immerse into helium Dewar (T=4.2 K boiling point)
4. Measure electrical resistance (R) versus the temperature (T)

Heike Kamerling Onnes

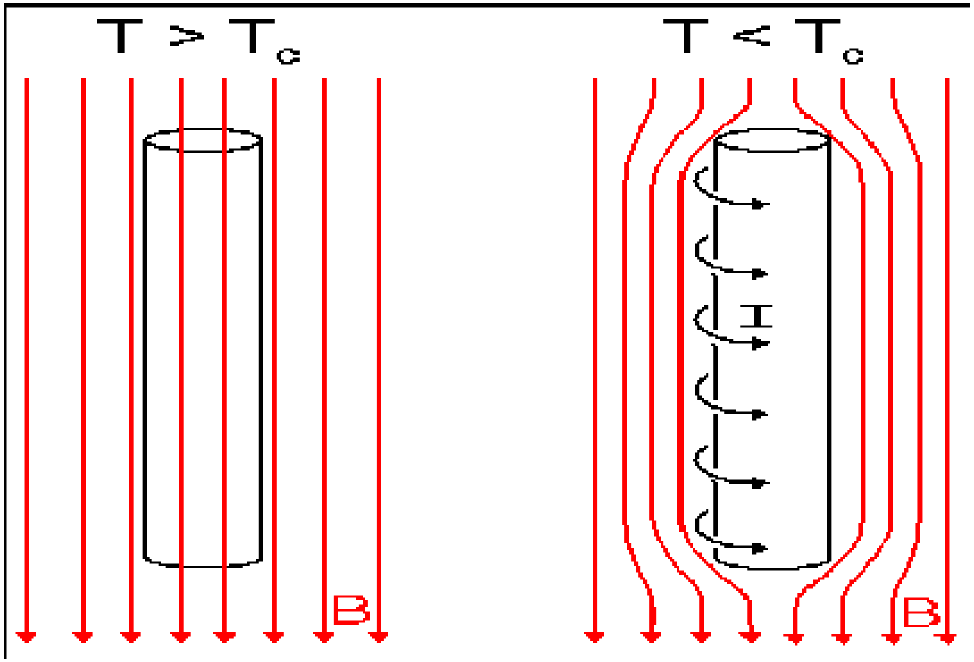


Dewar
with liquid Helium (4.2K)

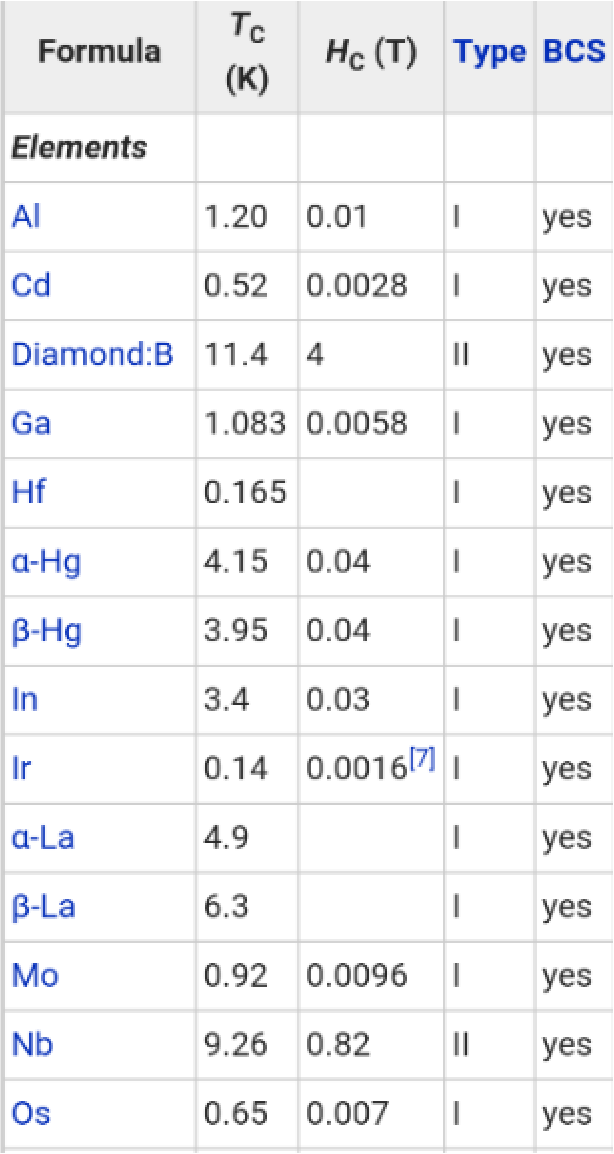


Superconducting transition is a second order thermodynamic phase transition

Meissner effect – the key signature of superconductivity



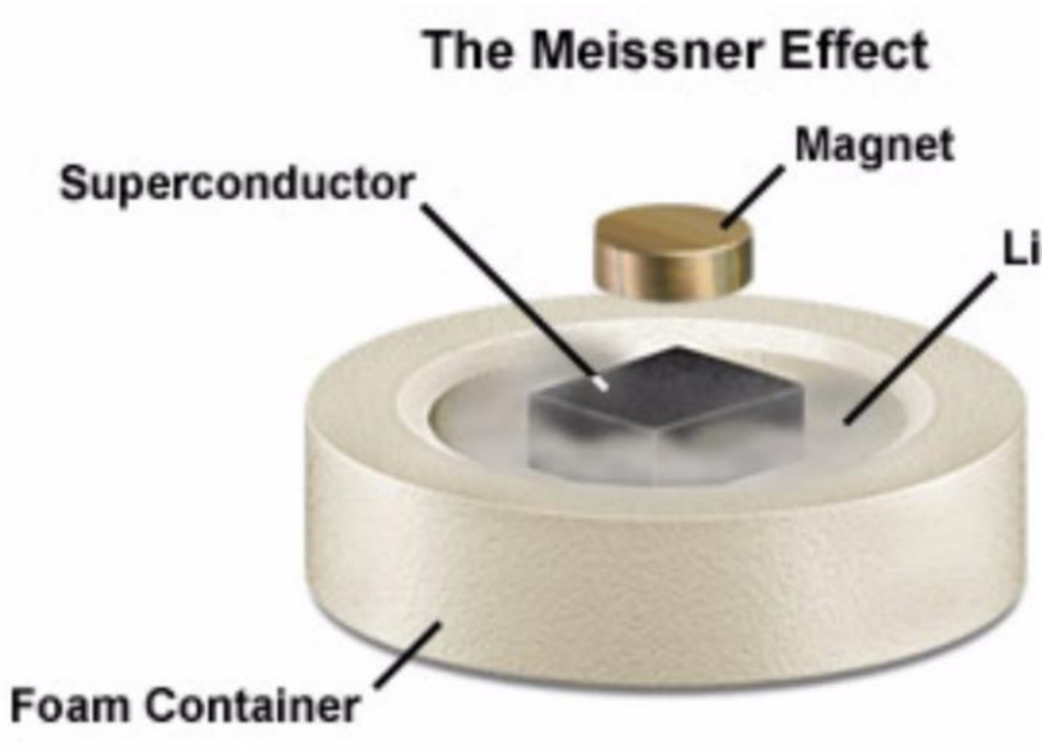
Theory of superconductivity:
“BCS” – due to Bardeen, Cooper and Schrieffer

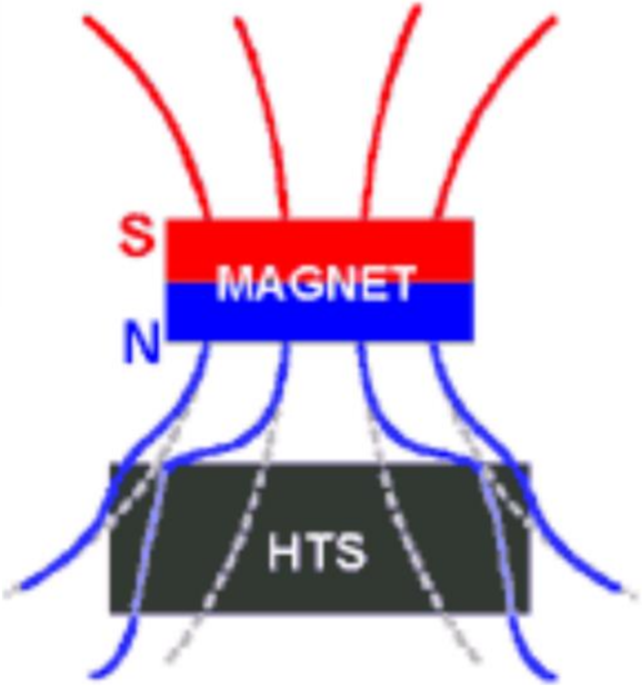


Formula	T_c (K)	H_c (T)	Type	BCS
Elements				
Al	1.20	0.01	I	yes
Cd	0.52	0.0028	I	yes
Diamond:B	11.4	4	II	yes
Ga	1.083	0.0058	I	yes
Hf	0.165		I	yes
α-Hg	4.15	0.04	I	yes
β-Hg	3.95	0.04	I	yes
In	3.4	0.03	I	yes
Ir	0.14	0.0016[7]	I	yes
α-La	4.9		I	yes
β-La	6.3		I	yes
Mo	0.92	0.0096	I	yes
Nb	9.26	0.82	II	yes
Os	0.65	0.007	I	yes

Importance of superconductivity: Qubits and modern quantum computers are made of superconductors

Interesting phenomenon: Magnetic levitation





Levitation is the process by which an object is held aloft, without mechanical support, in a stable position.

https://nationalmaglab.org/magnet-academy/read-science-stories/science-simplified/superconductivity-101/

Superconducting-magnet levitation train

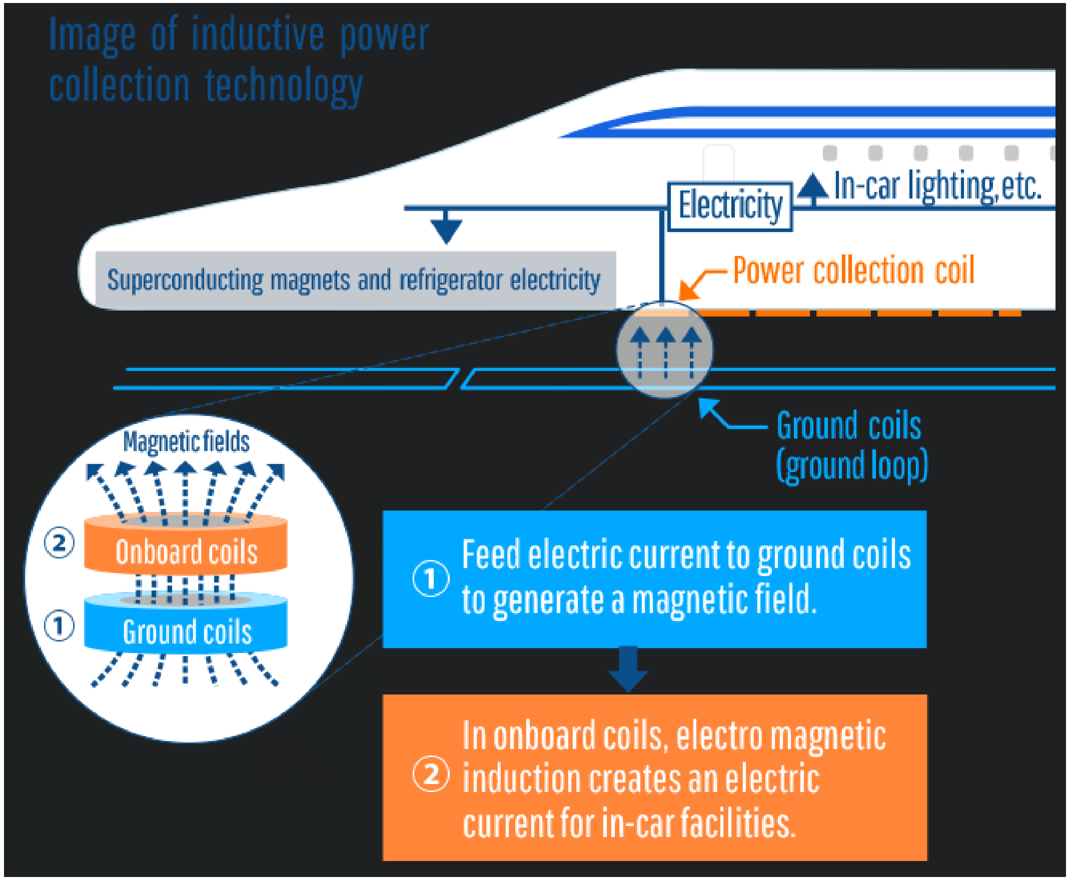


Image of inductive power collection technology

① Feed electric current to ground coils to generate a magnetic field.

② In onboard coils, electro magnetic induction creates an electric current for in-car facilities.

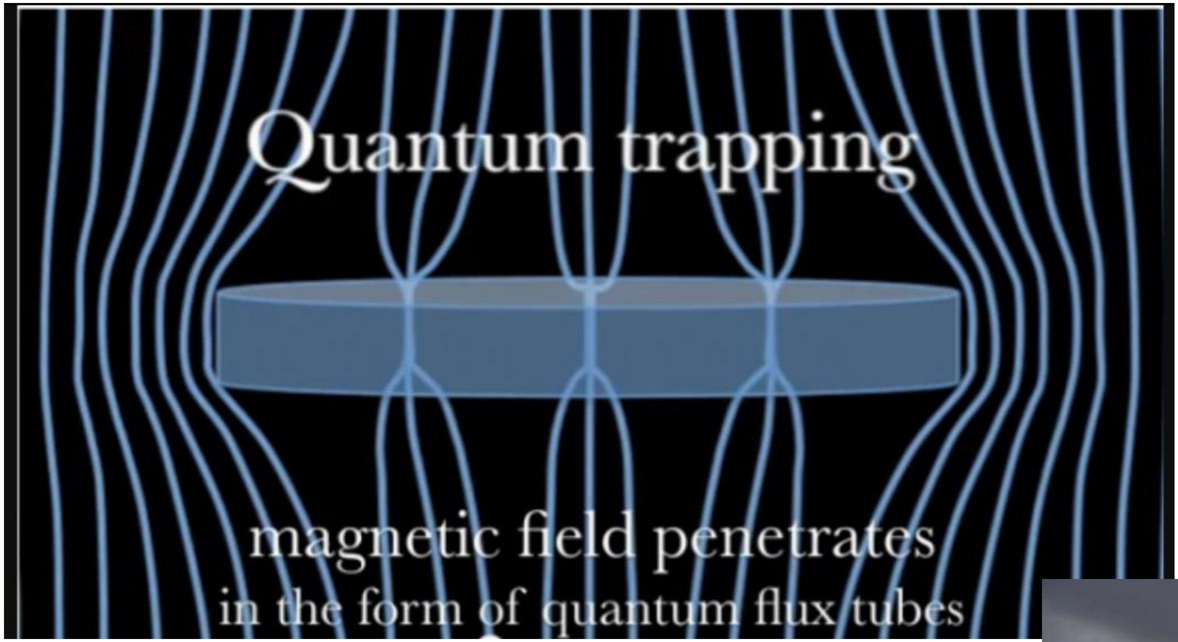
The L0 Series, a prototype vehicle based on SCMaglev technology, holds the record for fastest crewed rail vehicle with a record speed of 603 km/h (375 mph).
Time Urbana-Chicago: only 22 min

blogs.wsj.com

U.S. Transportation Secretary Foxx Rides on Japan's Maglev Train - Jap...

The SCMaglev system uses an electrodynamic suspension (EDS) system. The train's bogies have superconducting magnets installed, and the guideways contain two sets of metal coils.

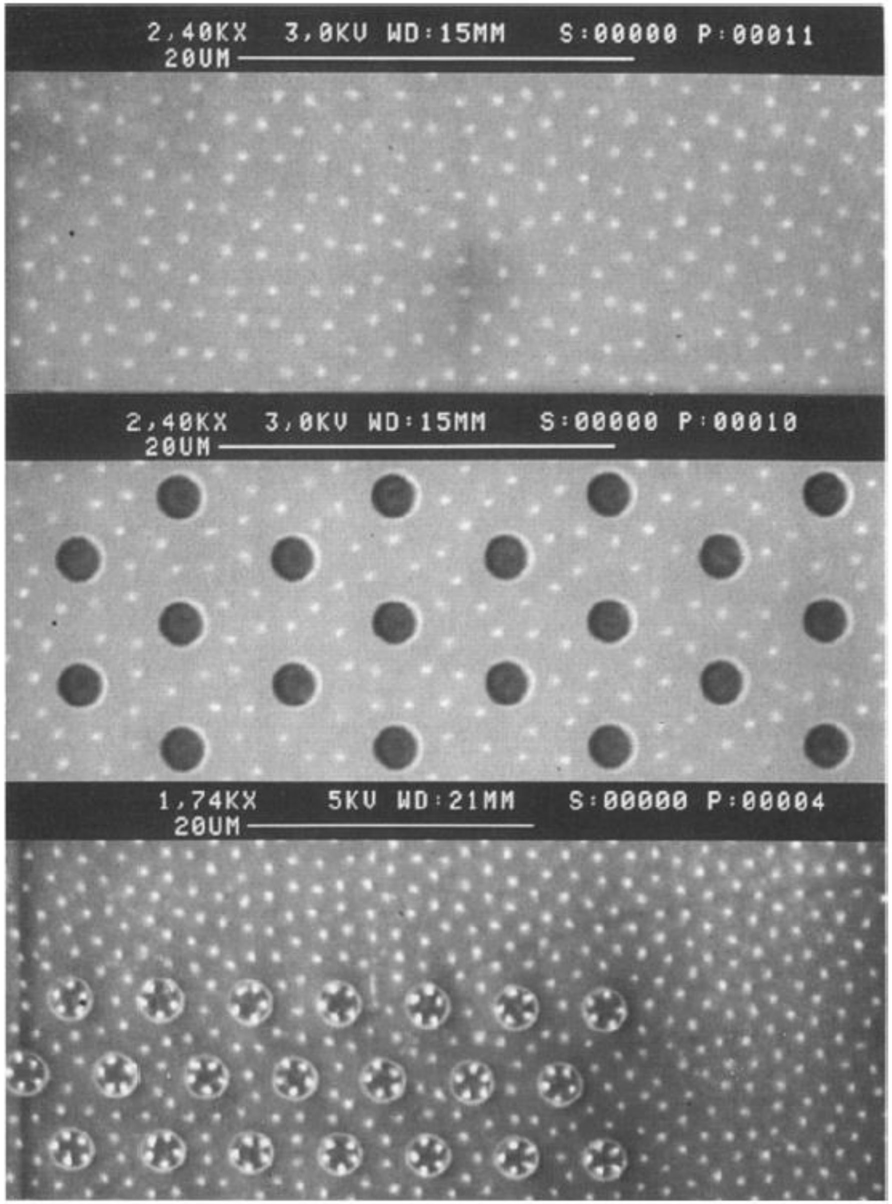
Magnetic field effect: Superconducting vortices



In superconductivity, a **fluxon** (also called an **Abrikosov vortex** or **quantum vortex**) is a vortex of supercurrent in a type-II superconductors

https://blog.tmcnet.com/blog/tom-keating/technology-and-science/quantum-levitation-back-to-the-future-hoverboard.asp

Vortices in superconducting films with "through" and "blind" holes ("antidots")



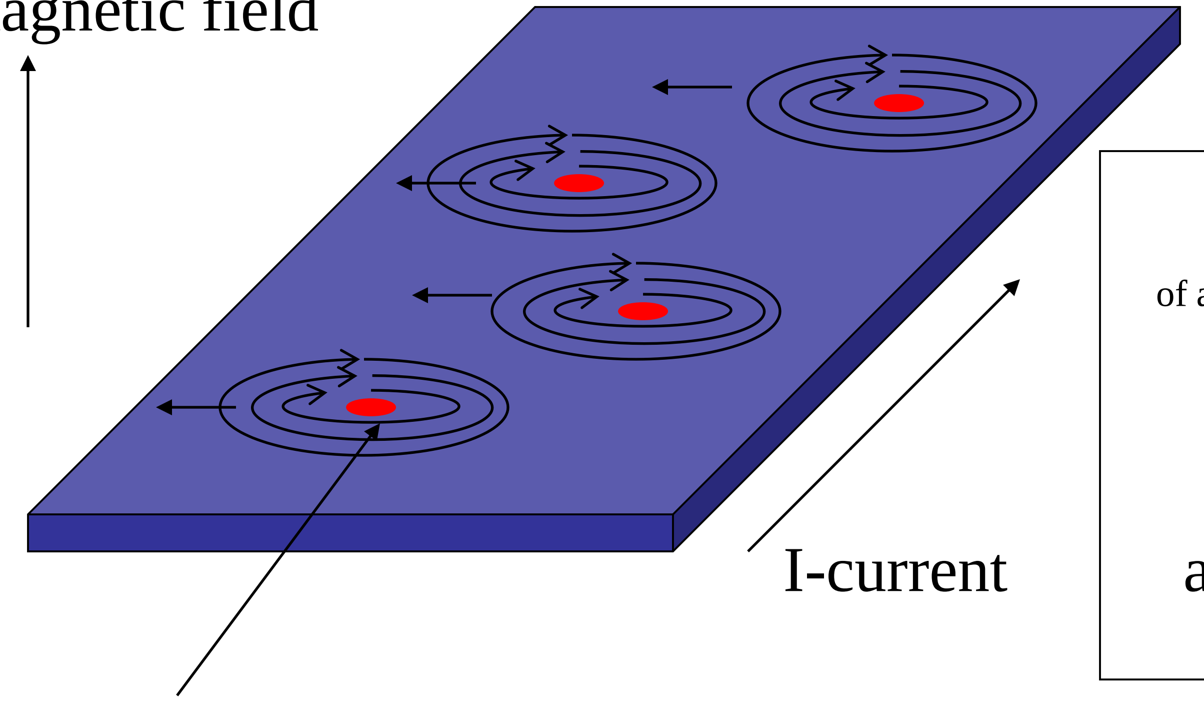
A. Bezryadin and B. Pannetier
"Role of Edge Superconducting States in Trapping of Multi-Quanta Vortices by Microholes. Application of the Bitter Decoration Technique",
J. Of Low Temp. Phys., V.102, p.73 (1996).

Vortices are quantized tubes carrying magnetic field into superconductor

Magnetic field creates vortices--

Vortices cause dissipation (i.e. a non-zero electrical resistance), if they move

B -magnetic field



I-current

Wave function
of all superconducting electrons:

$$\boxed{\Psi = |\Psi| \exp(-i\varphi)}$$

amplitude

phase

Vortex core (red): normal, not superconducting; diameter ξ~10 nm

The current is extended to a scale λ, which is larger than ξ in type II superconductors (such as thin films of any material)

Reminder: single electron in empty space

Wave function: $\Psi = |\Psi| \exp(ikx) = |\Psi| [\cos(kx) + i \sin(kx)]$

Wave number: $k = 2\pi/\lambda$

$i * i = -1$

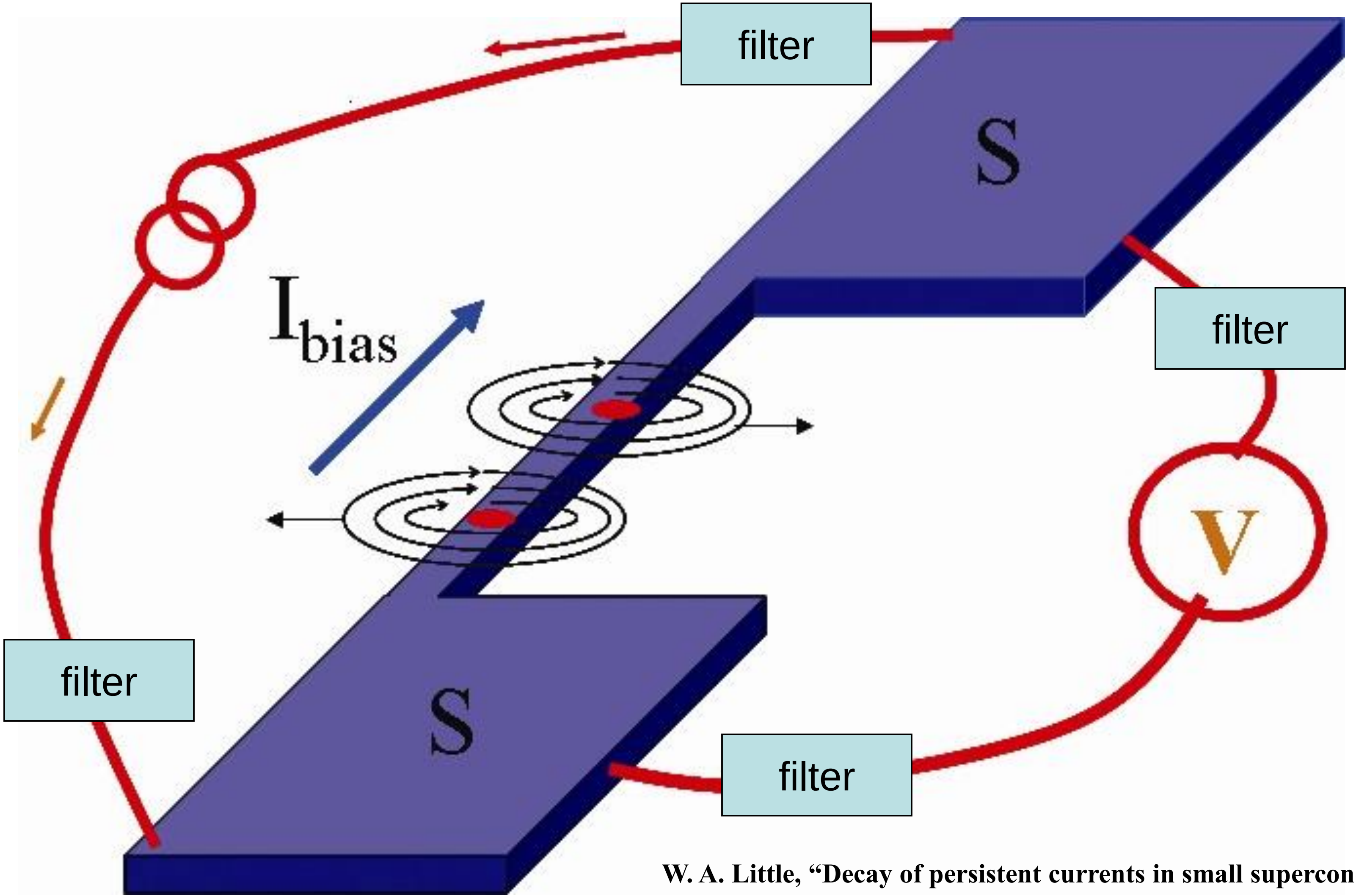
General form: $\Psi = |\Psi| \exp(-i\varphi)$

In this example of a plane wave, the phase is: $\varphi = kx$

DC transport measurement schematic to detect passing vortices

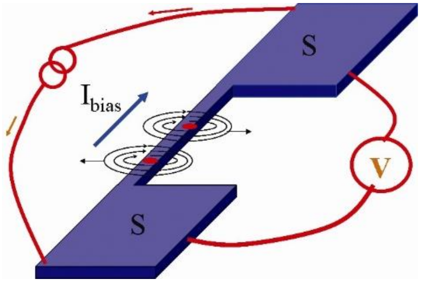
Bleu: superconducting film and wire

Red: Phase slip events or crossing vortices



W. A. Little, "Decay of persistent currents in small superconductors", Physical Review, V.156, pp.396-403 (1967).

How to use voltage to determine the rate of phase slips?



Key principle: every time a vortex crosses the wire the phase difference changes by 2π.

Phase evolution equation: $d\varphi/dt = 2eV/\hbar$

Simplified derivation:

1. Time-dependent Schrödinger equation with fixed energy:

________________$i\hbar(d\Psi/dt)=E\,\Psi$

2. The solution is: $\Psi=\exp(-iEt/\hbar)$ (here E is the energy)

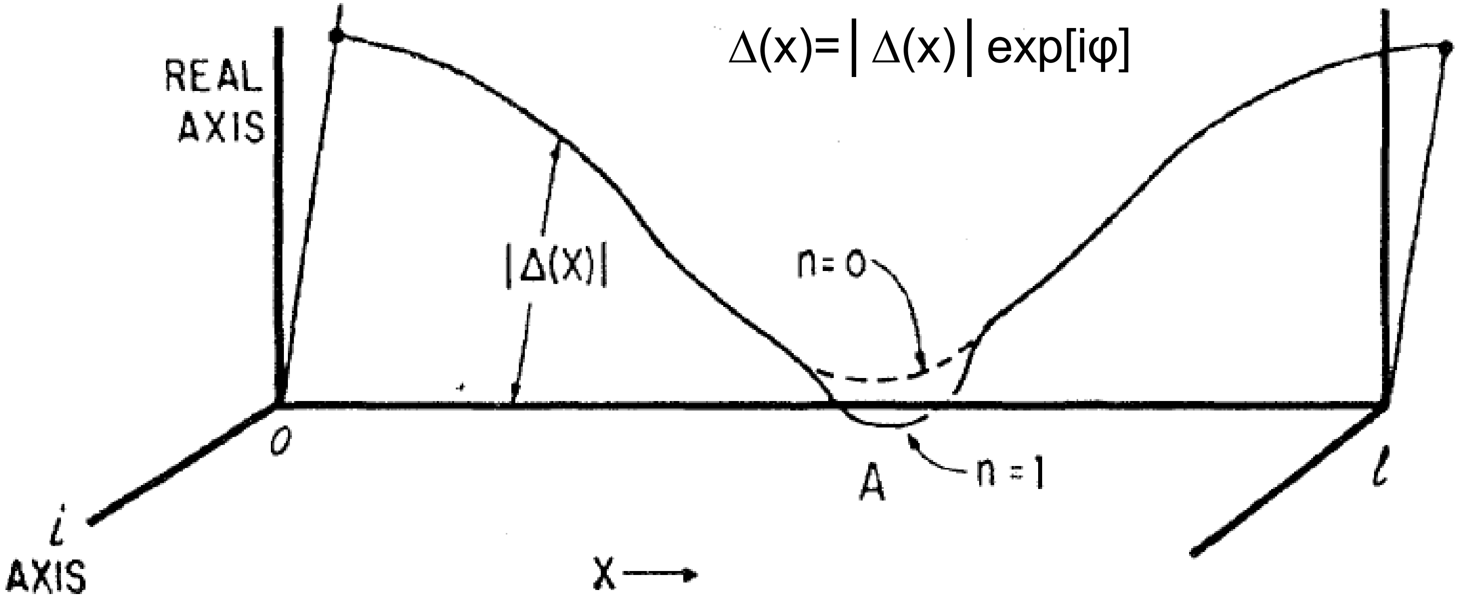
3. The phase of the wavefunction is $\varphi=Et/\hbar$

4. The energy is defined by the electric potential (voltage), V as follows: $E=2eV$. Note that the effective charge of superconducting electrons is 2e, where "e" is the charge of one electron. Such superconducting electron pairs are called Cooper pairs.

Thus, the resulting equation is: $d\varphi/dt = 2eV/\hbar$

Gor'kov, L.P. (1958) *Sov. Phys. JETP*, 7, 505.

Thin superconducting wire have some nonzero electrical resistance due to Little’s Phase Slips



$$\Delta(x)=|\Delta(x)|\exp[i\varphi]$$

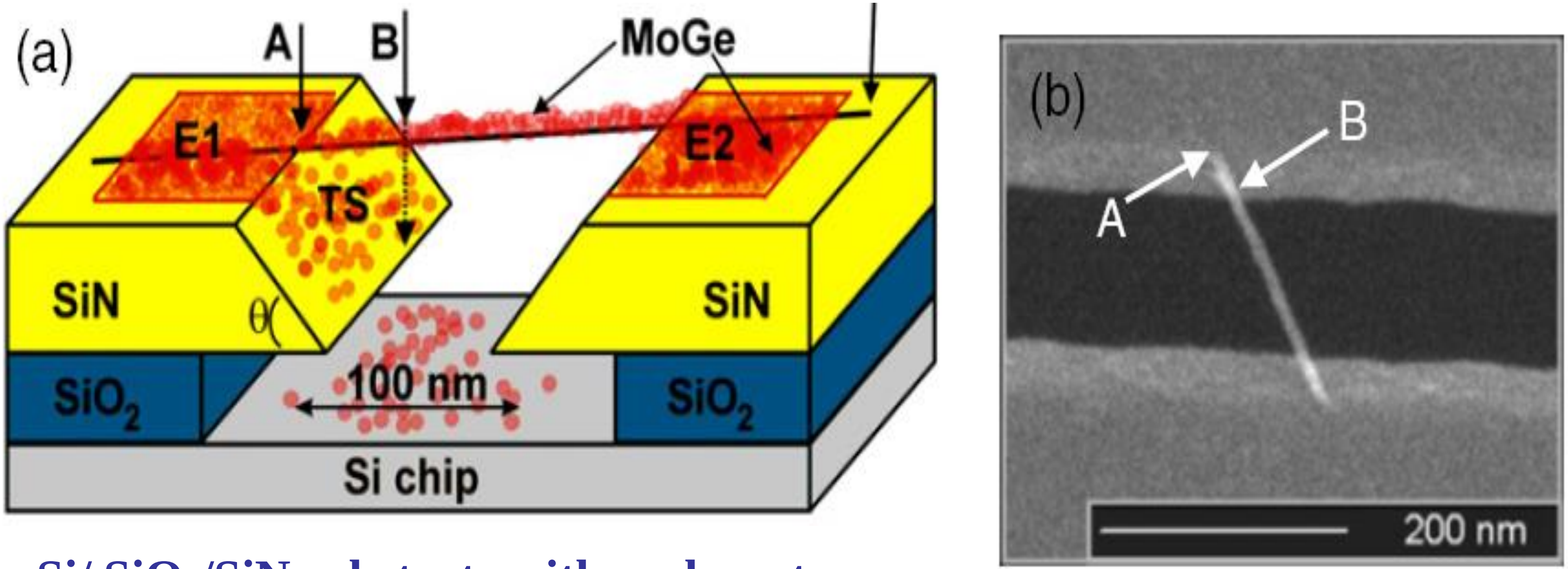
W. A. Little, “Decay of persistent currents in small superconductors”, Physical Review, V.156, pp.396-403 (1967).

Two types of phase slips (PS) can occur:

1. The usual, thermally activated PS (TAPS)
2. Quantum phase slip (QPS)

Fabrication of nanowires

Method of Molecular Templating



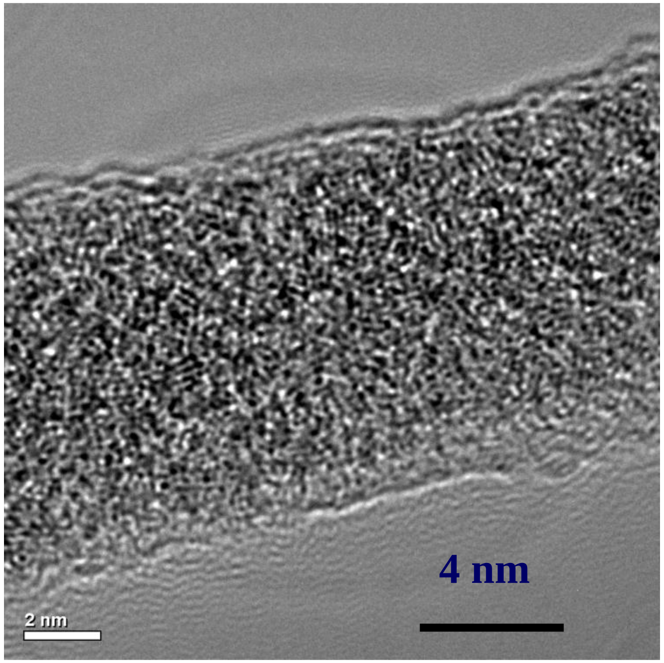
Si/ SiO_2/SiN substrate with undercut

~ 0.5 mm Si wafer
500 nm SiO_2
60 nm SiN
Width of the trenches ~ 50 - 500 nm

HF wet etch for ~10 seconds to form undercut

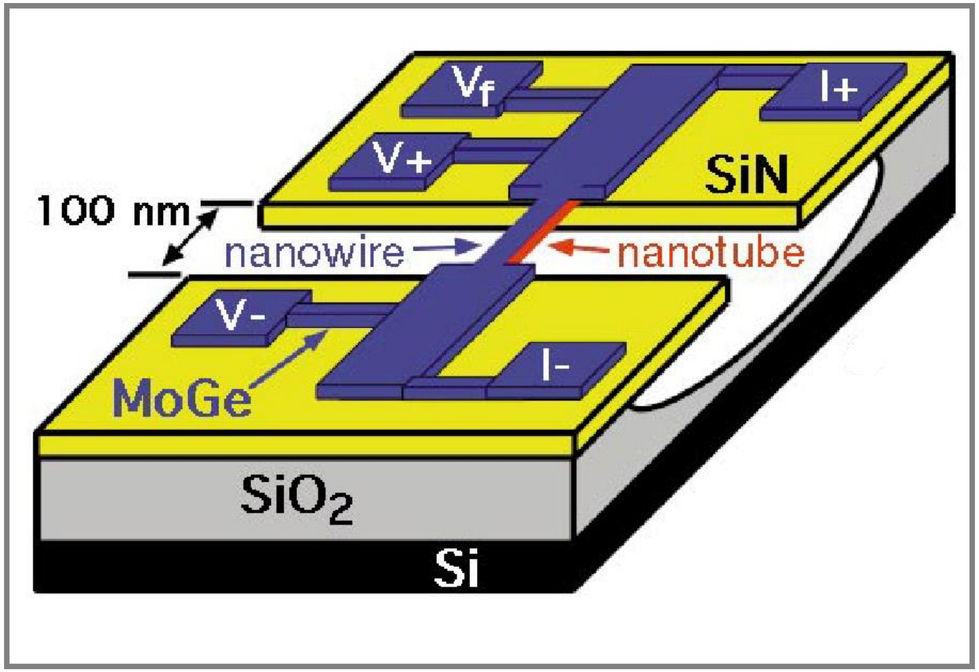
Bezryadin, Lau, Tinkham, *Nature* **404**, 971 (2000)

Sample Fabrication



TEM image of a wire shows amorphous wire morphology.

Nominal MoGe thickness = 3 nm



Schematic picture of the pattern Nanowire + Film Electrodes used in transport measurements

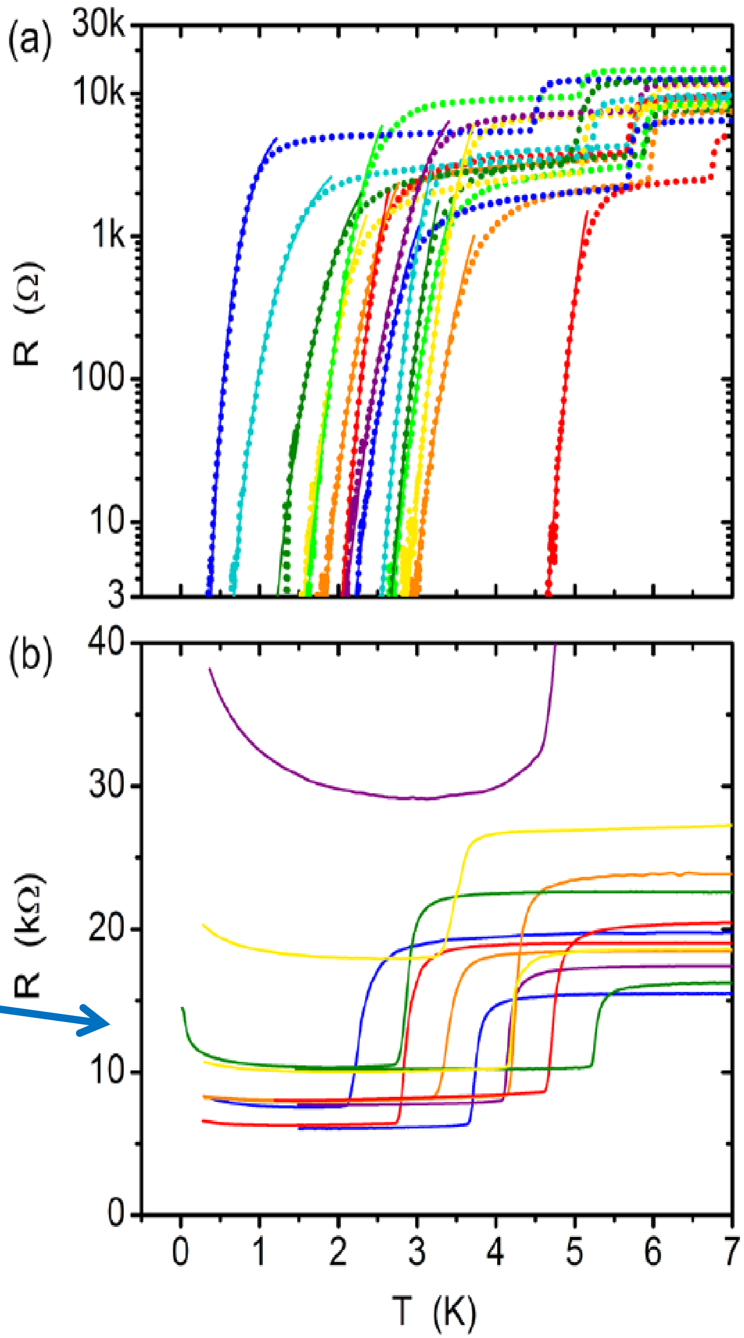
Dichotomy in nanowires: Evidence for superconductor-insulator transition (SIT)

R=V/I   I~3 nA

The difference between samples is the amount of the deposited Mo79Ge21.

Thin wires become insulating if their normal resistance is lager than resistance quantum h/4/e/e=6.5KOhm

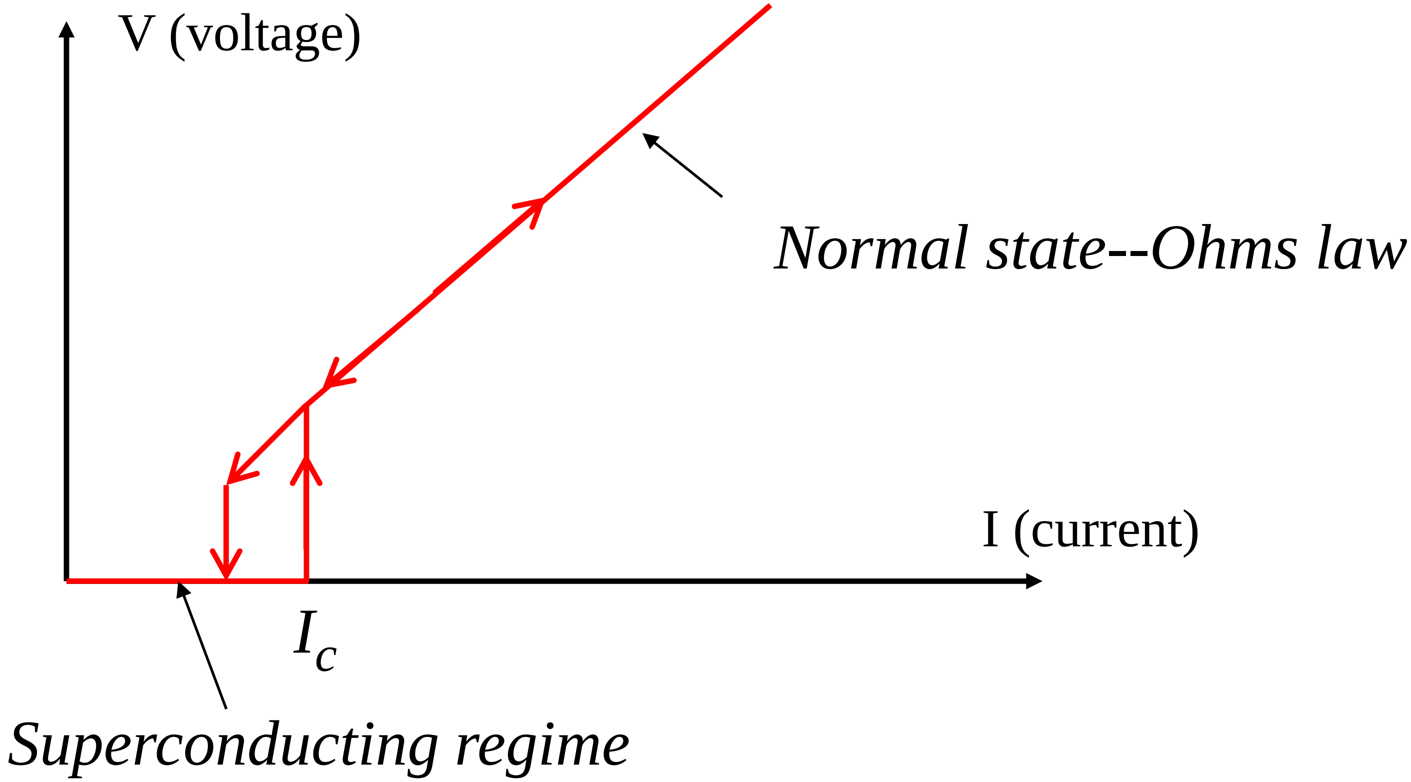
The insulating behavior is due to proliferation of quantum phase slips



Bollinger, Dinsmore, Rogachev, Bezryadin, Phys. Rev. Lett. **101**, 227003 (2008)

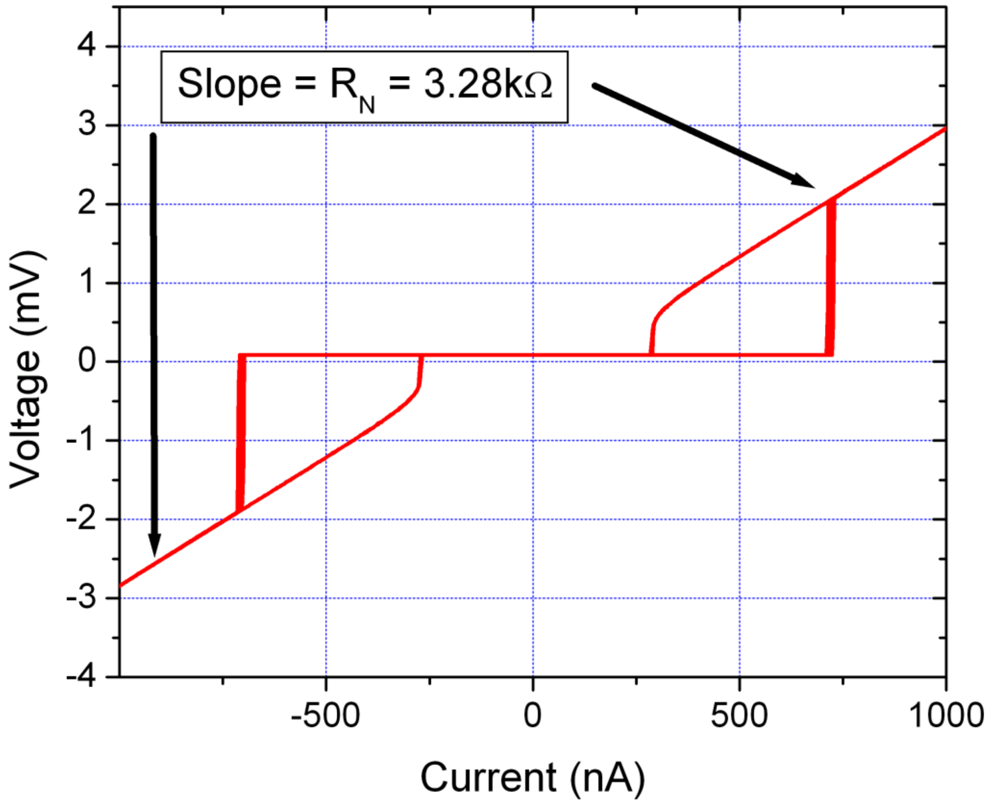
Expected voltage-current curve

Electrical resistance is zero only if current is not too strong

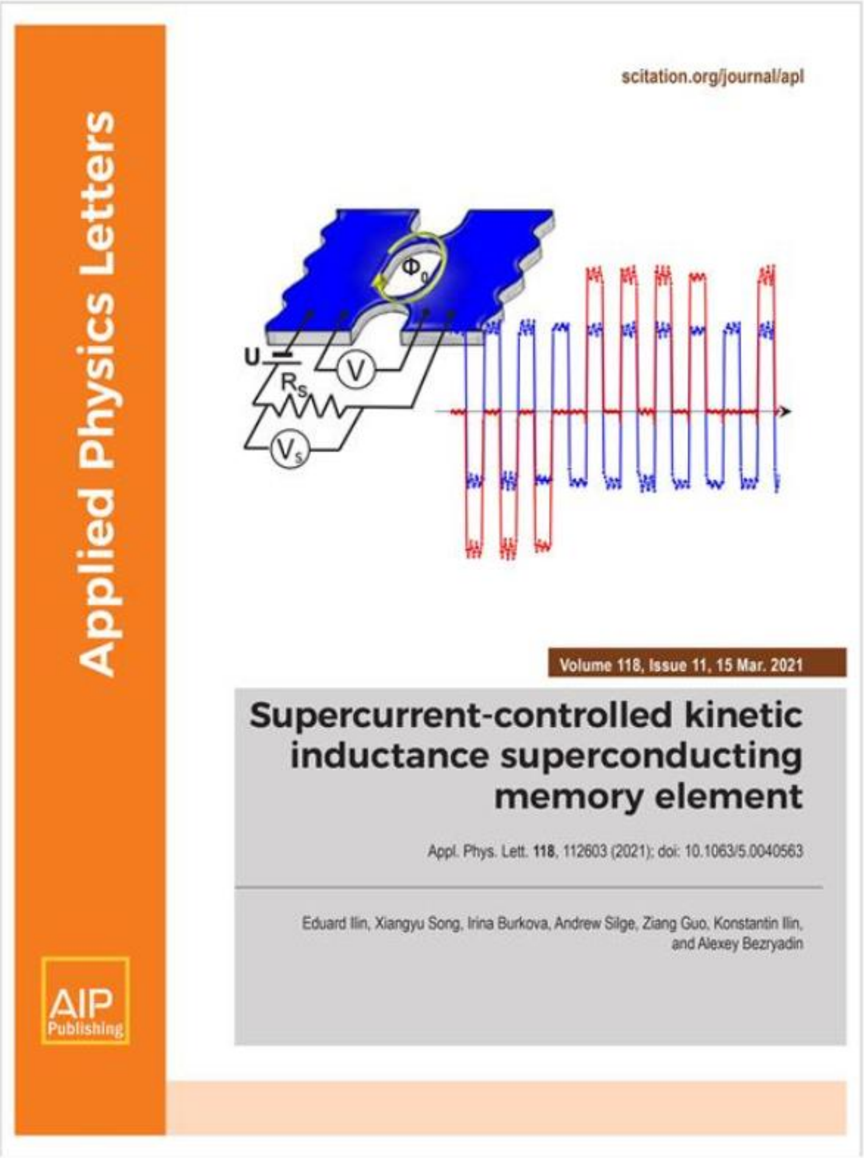


Experimental voltage-current curve.

Fluctuations of the switching current are due to Little's phase slips



Superconducting nanowire memory



scitation.org/journal/apl

Applied Physics Letters

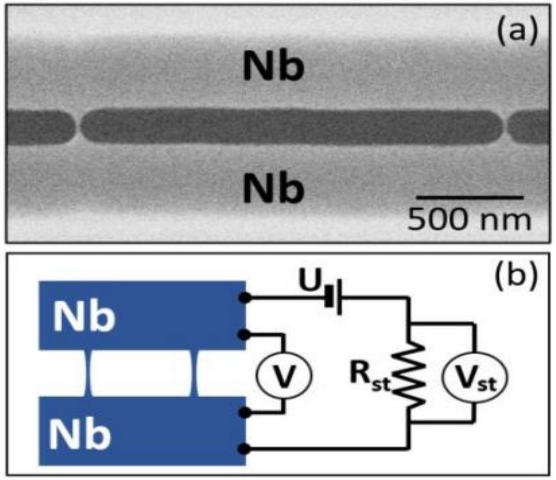
Volume 118, Issue 11, 15 Mar. 2021

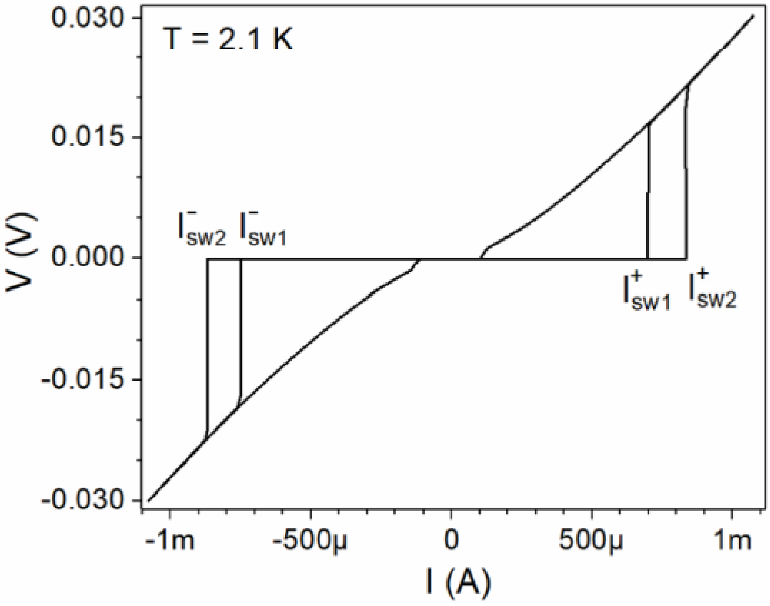
Supercurrent-controlled kinetic inductance superconducting memory element

Appl. Phys. Lett. **118**, 112603 (2021); doi: 10.1063/5.0040563

Eduard Ilin, Xiangyu Song, Irina Burkova, Andrew Silge, Ziang Guo, Konstantin Ilin,
and Alexey Bezryadin

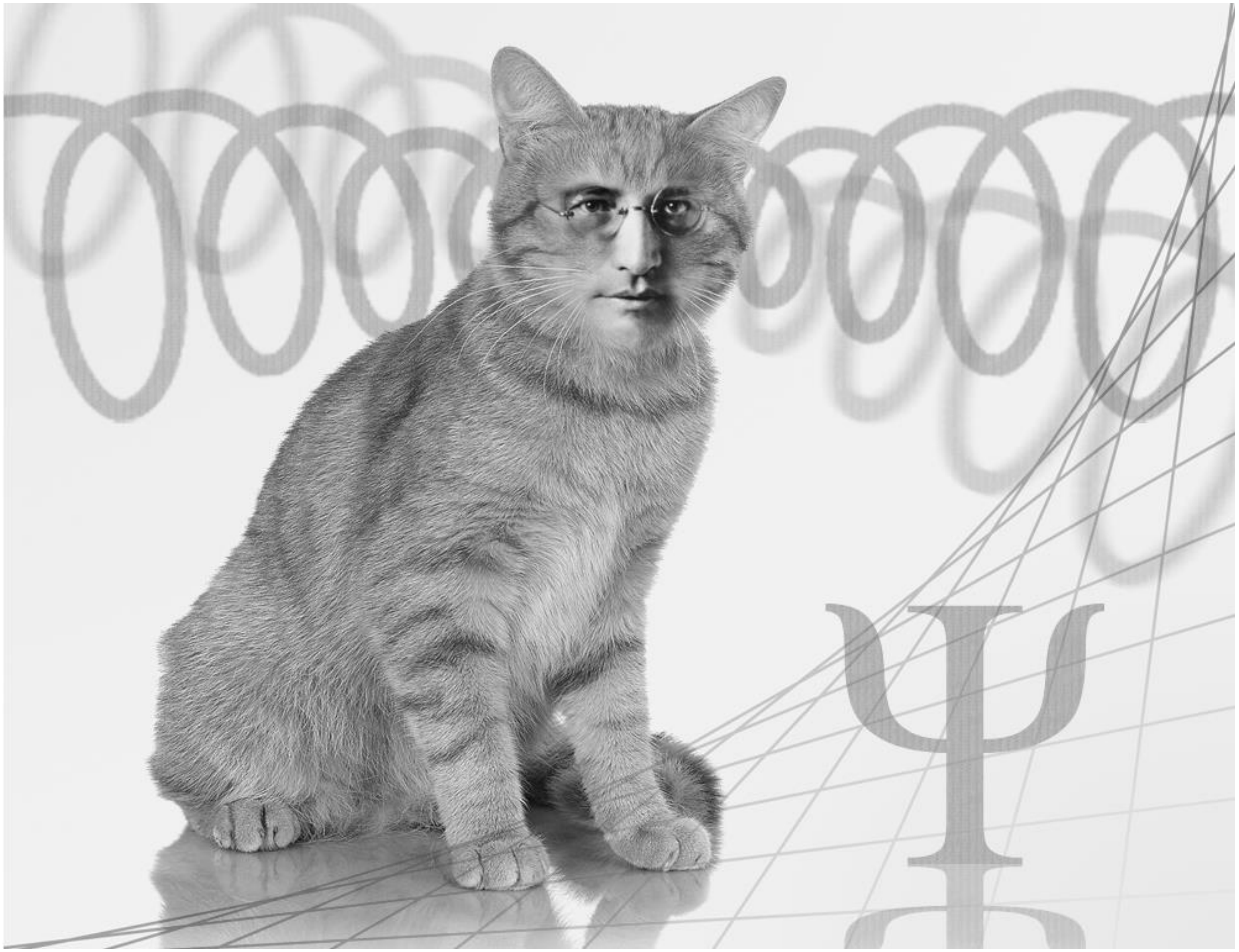
AIP Publishing



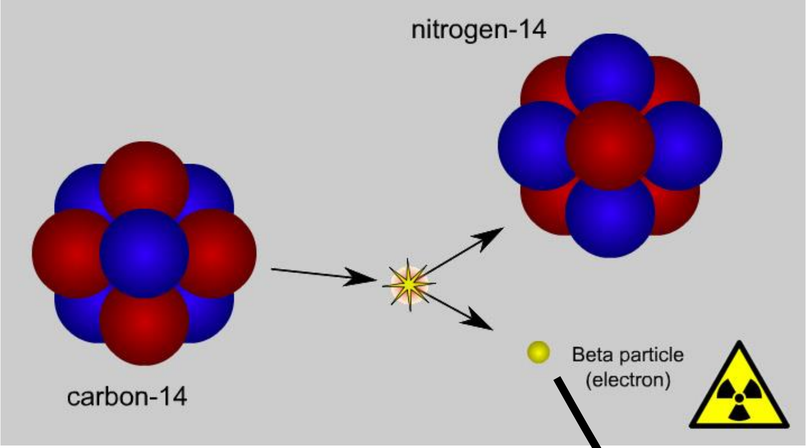


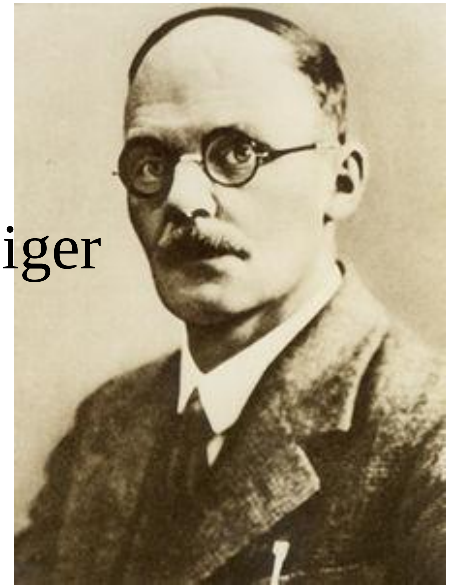
Schrödinger cat –
the ultimate macroscopic quantum phenomenon

E. Schrödinger, Naturwiss. **23** (1935), 807.



Schrödinger cat – thought experiment





Hans Geiger

Geiger counter

Linearity of the Schrödinger's equation

Suppose Ψ_1 is a valid solution of the Schrödinger equation:

$$i\hbar\frac{\partial\psi_1}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi_1}{\partial x^2} + U(x)\psi_1$$

And suppose that Ψ_2 is another valid solution of the Schrödinger equation:

$$i\hbar\frac{\partial\psi_2}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi_2}{\partial x^2} + U(x)\psi_2$$

Then $(\Psi_1 + \Psi_2)/\sqrt{2}$ is also a valid solution, because:

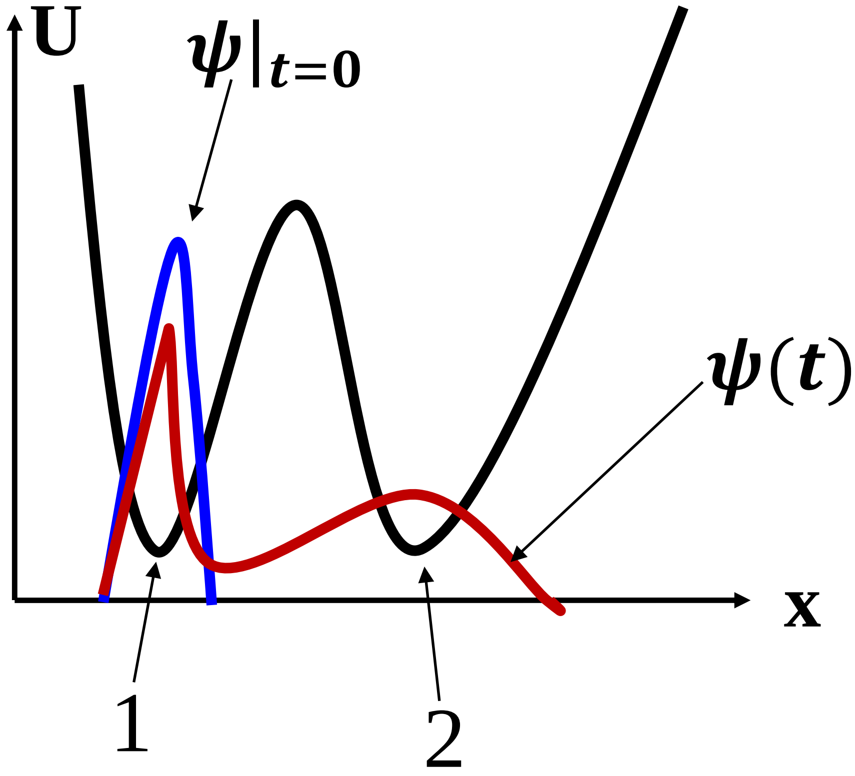
$$i\hbar\frac{\partial(\psi_1+\psi_2)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2(\psi_1+\psi_2)}{\partial x^2} + U(x)(\psi_1+\psi_2)$$

The state $(\Psi_1 + \Psi_2)/\sqrt{2}$ is a new combined state which is called "quantum superposition" of state (1) and (2)

George Gamow

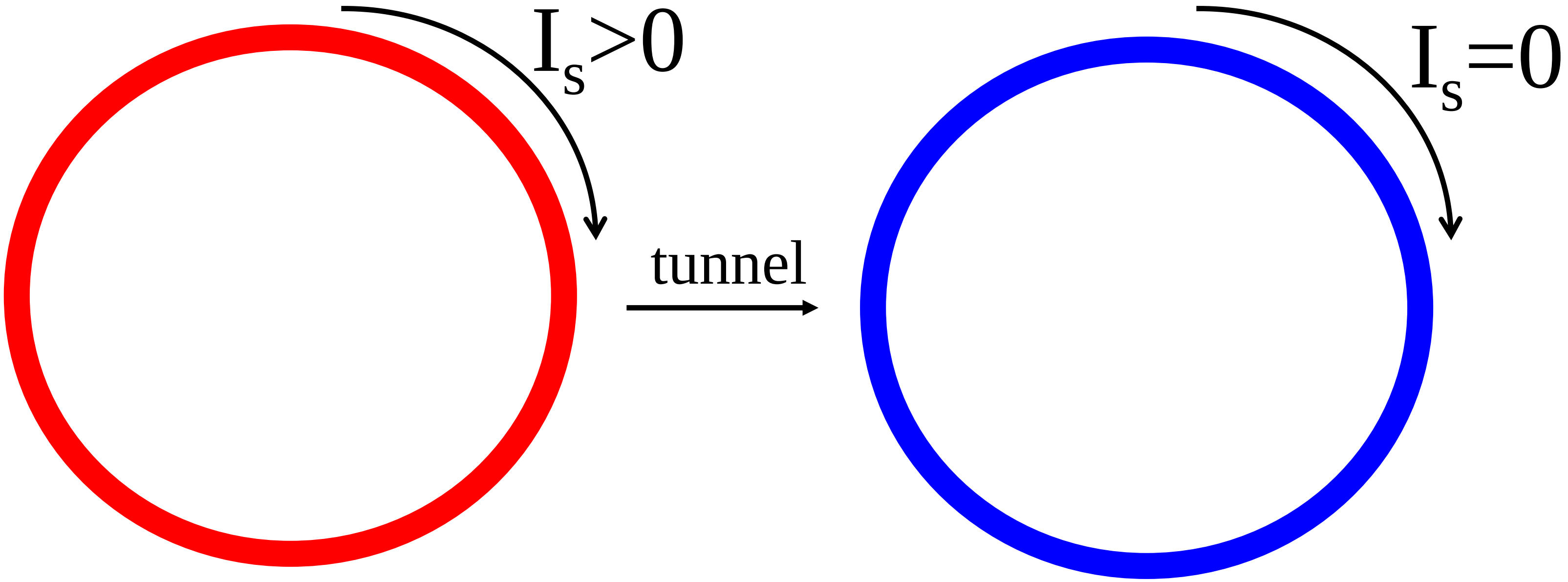
(He also helped to developed
Big Bang theory)

Quantum tunneling



Quantum tunneling is possible since quantum superpositions of states are possible.

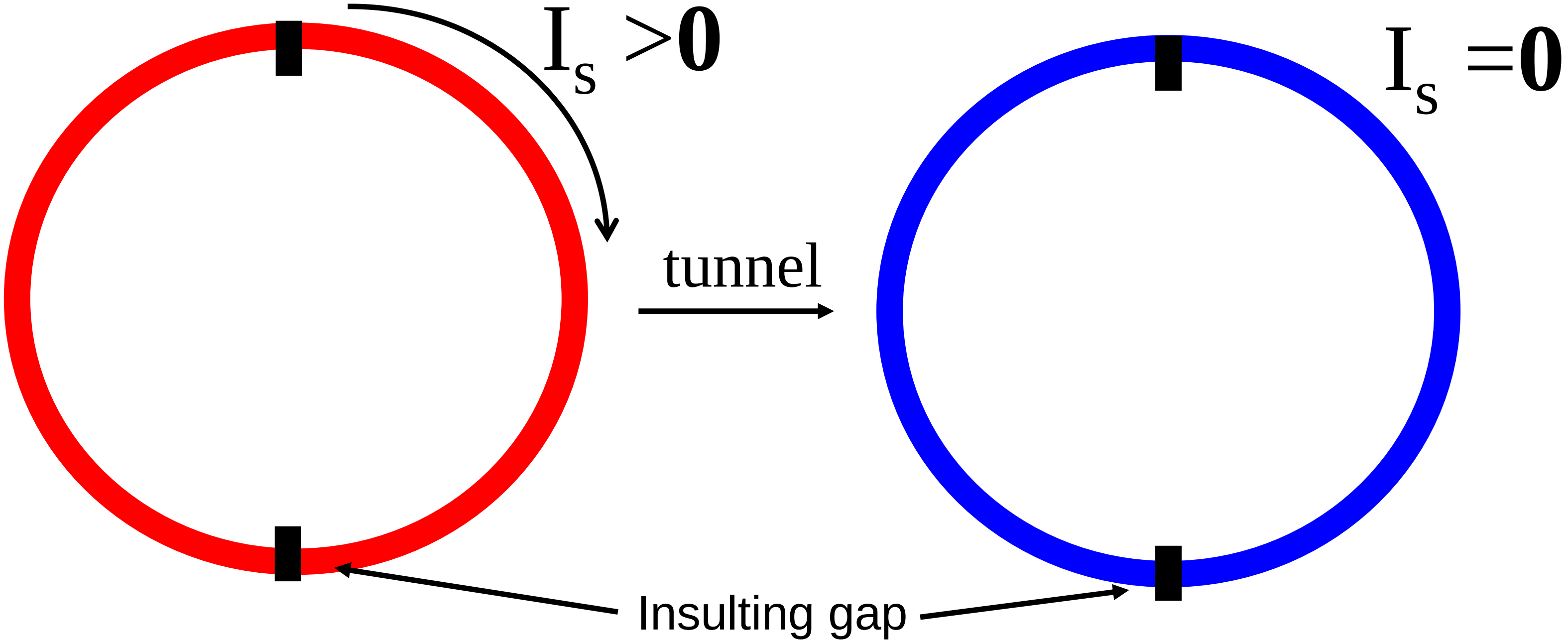
What sort of tunneling we will consider?



-Red color represents some strong current in the superconducting wire loop

-Blue color represents zero current in the loop

Previous results relate loops with insulating interruptions (SQUIDs)



$I_s > 0$

tunnel

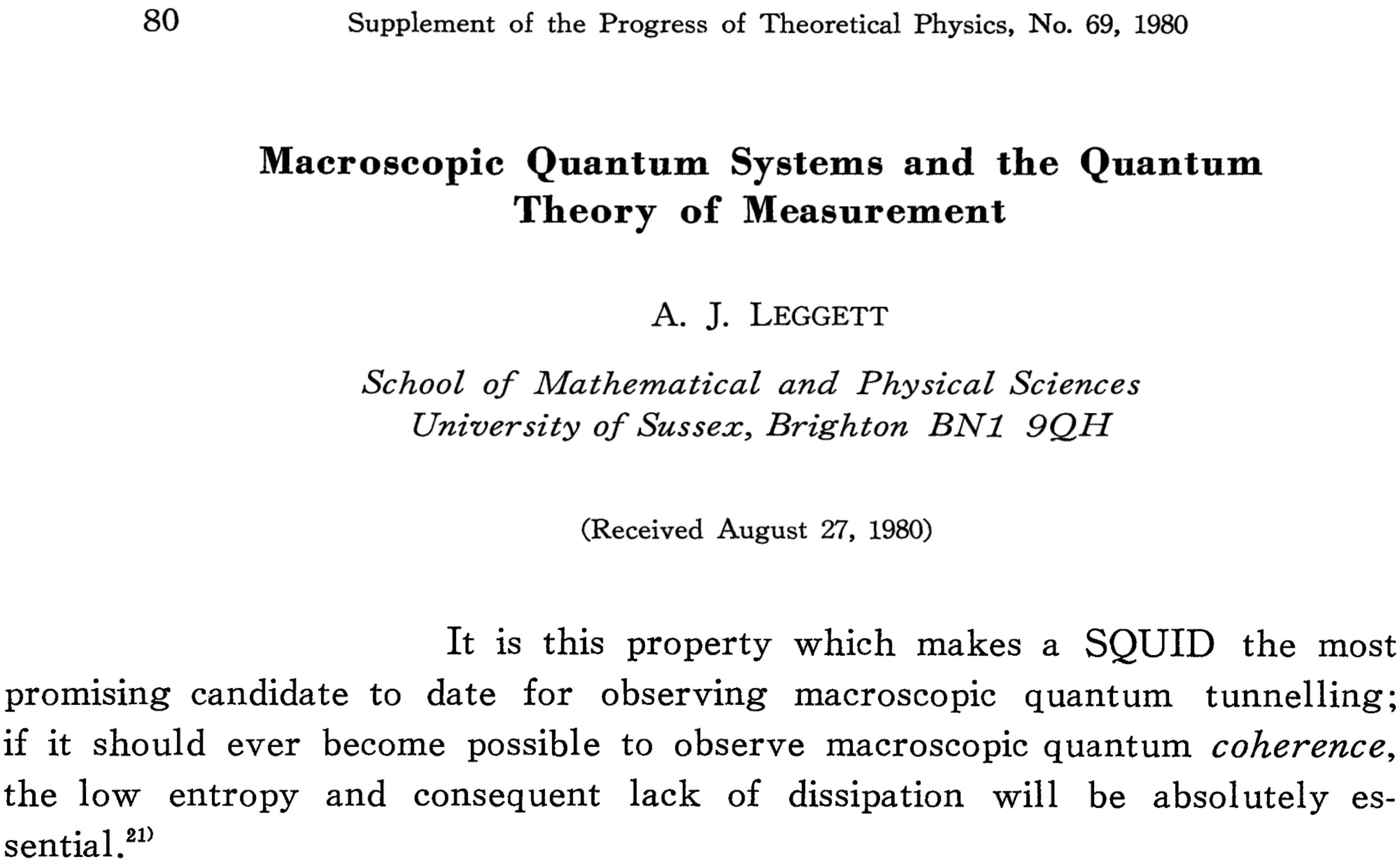
$I_s = 0$

Insulting gap

-Red color represents some strong current in the superconducting loop

-Blue color represents zero current in the superconducting loop

Leggett’s prediction for macroscopic quantum tunneling (MQT) in SQUIDs



80 Supplement of the Progress of Theoretical Physics, No. 69, 1980

Macroscopic Quantum Systems and the Quantum Theory of Measurement

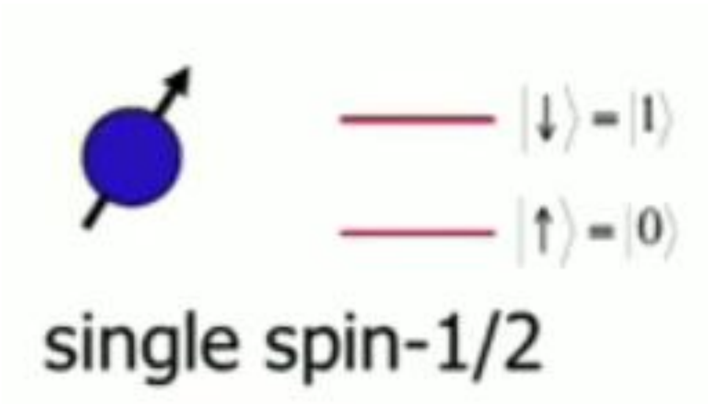
A. J. LEGGETT

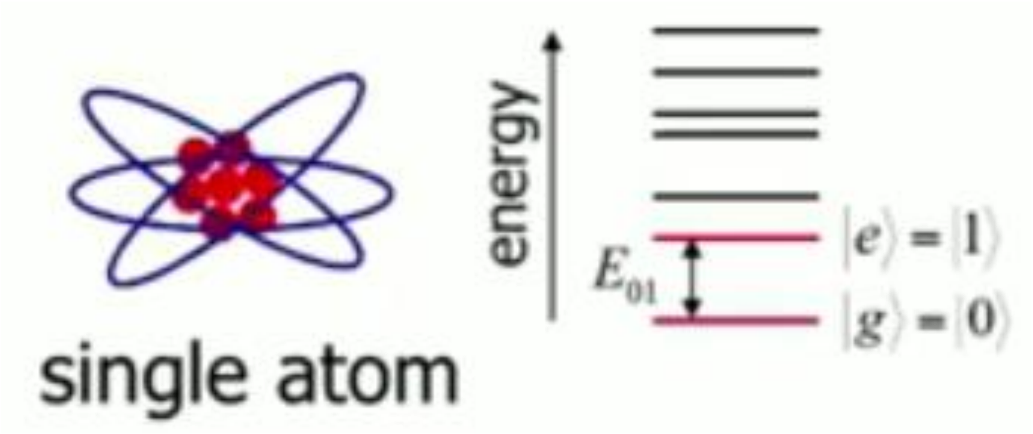
School of Mathematical and Physical Sciences
University of Sussex, Brighton BN1 9QH

(Received August 27, 1980)

It is this property which makes a SQUID the most promising candidate to date for observing macroscopic quantum tunnelling; if it should ever become possible to observe macroscopic quantum *coherence*, the low entropy and consequent lack of dissipation will be absolutely essential.21)

Types of Qubit



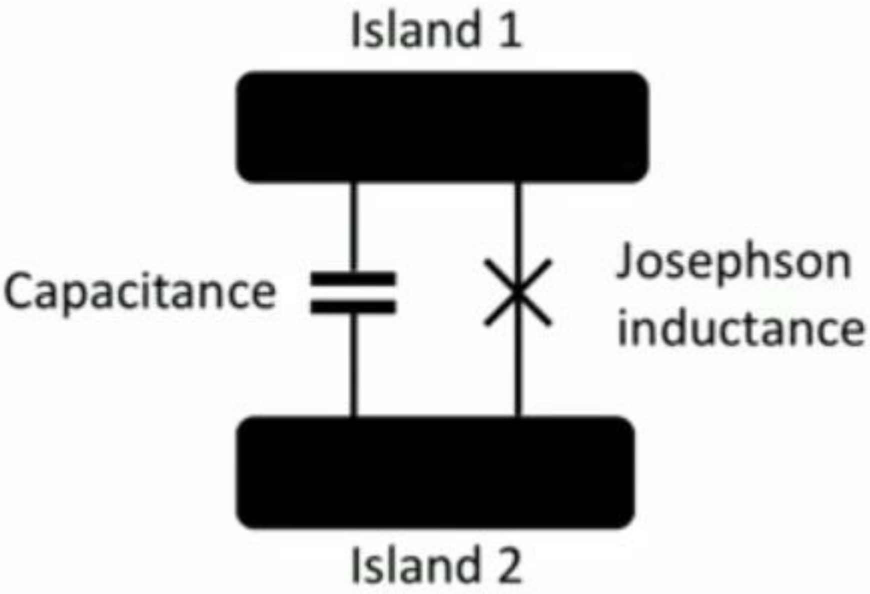


Quantum state:
$|\psi>=A*|0>+B*1>$

$A^2+B^2=1$

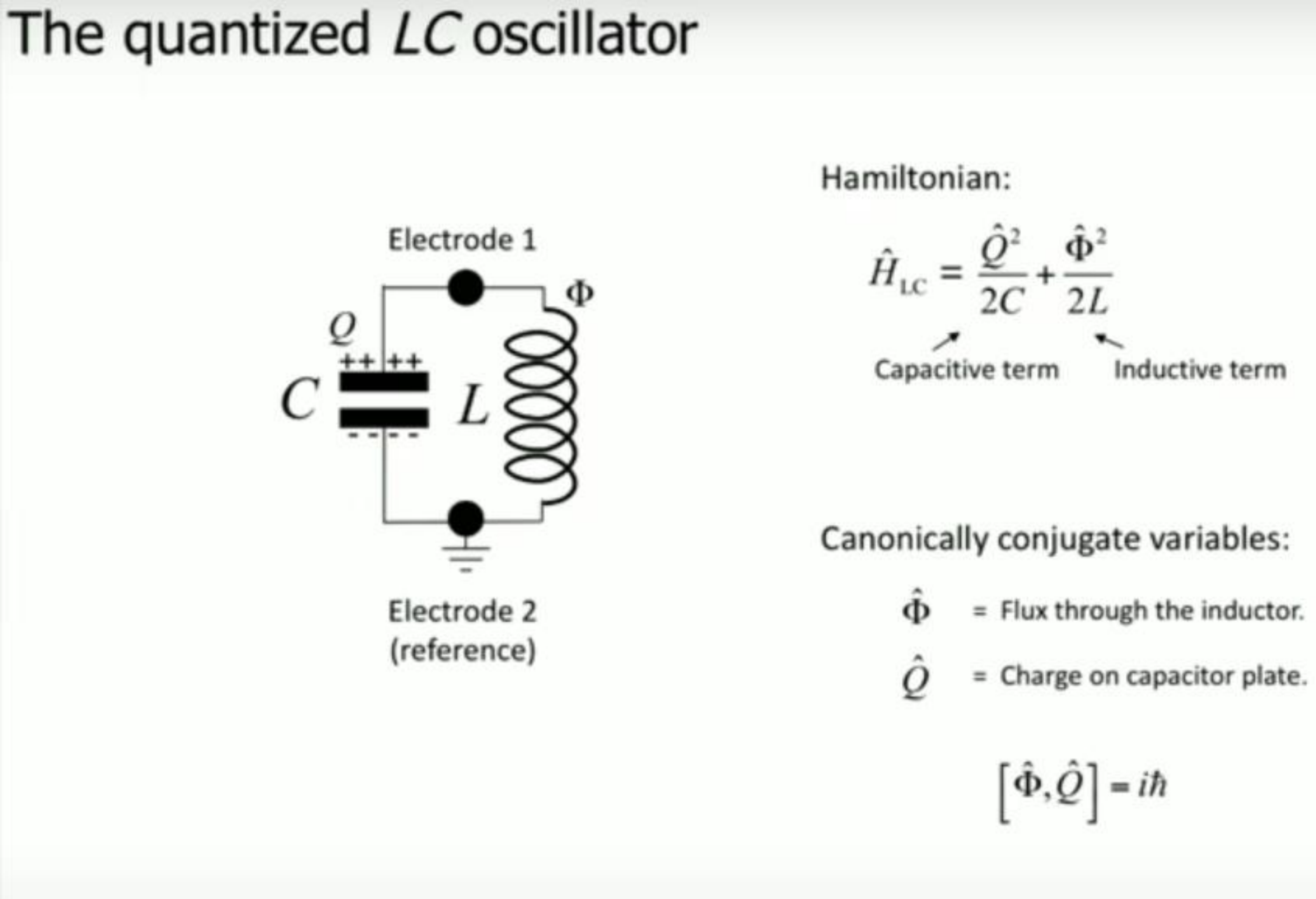
A and B are
complex numbers

Transmon Qubit



Theory of transmons: J. Koch et al., Phys. Rev. A **76**, 042319 (2007).

Quantization of electrical circuits



The quantized *LC* oscillator

Hamiltonian:

$$\hat{H}_{LC} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

Capacitive term

Inductive term

Canonically conjugate variables:

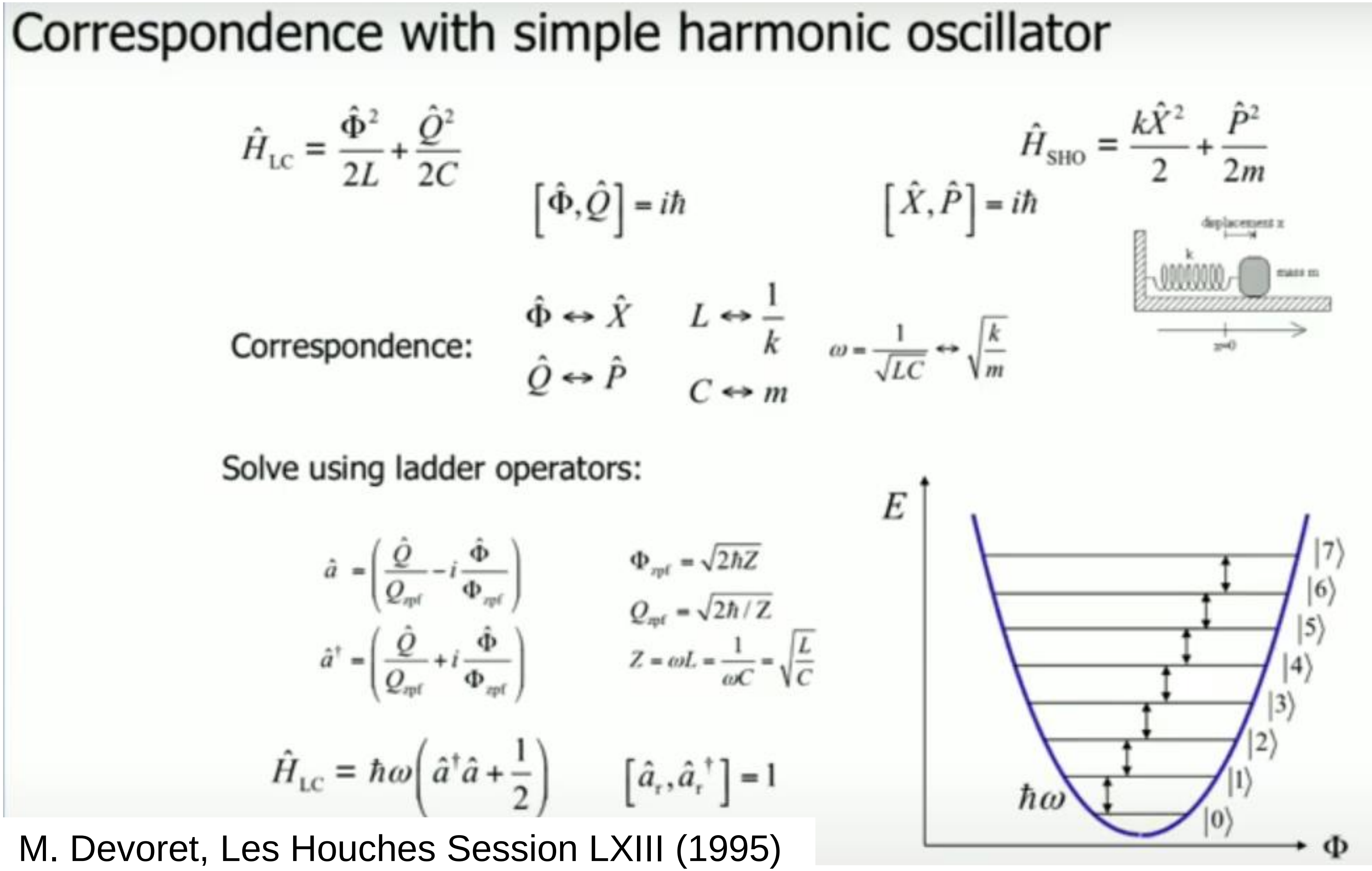
$\hat{\Phi}$ = Flux through the inductor.

$\hat{Q}$ = Charge on capacitor plate.

$$\left[\hat{\Phi}, \hat{Q}\right] = i\hbar$$

M. Devoret, Les Houches Session LXIII (1995)

Discrete energy spectrum of the LC-circuit



Correspondence with simple harmonic oscillator

$$\hat{H}_{\mathrm{LC}} = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C} \qquad \hat{H}_{\mathrm{SHO}} = \frac{k\hat{X}^2}{2} + \frac{\hat{P}^2}{2m}$$

$$\left[\hat{\Phi}, \hat{Q}\right] = i\hbar \qquad \left[\hat{X}, \hat{P}\right] = i\hbar$$

Correspondence:

$$\hat{\Phi} \leftrightarrow \hat{X} \qquad L \leftrightarrow \frac{1}{k} \qquad \omega = \frac{1}{\sqrt{LC}} \leftrightarrow \sqrt{\frac{k}{m}}$$

$$\hat{Q} \leftrightarrow \hat{P} \qquad C \leftrightarrow m$$

Solve using ladder operators:

$$\hat{a} = \left(\frac{\hat{Q}}{Q_{zpf}} - i\frac{\hat{\Phi}}{\Phi_{zpf}}\right) \qquad \Phi_{zpf} = \sqrt{2\hbar Z}$$

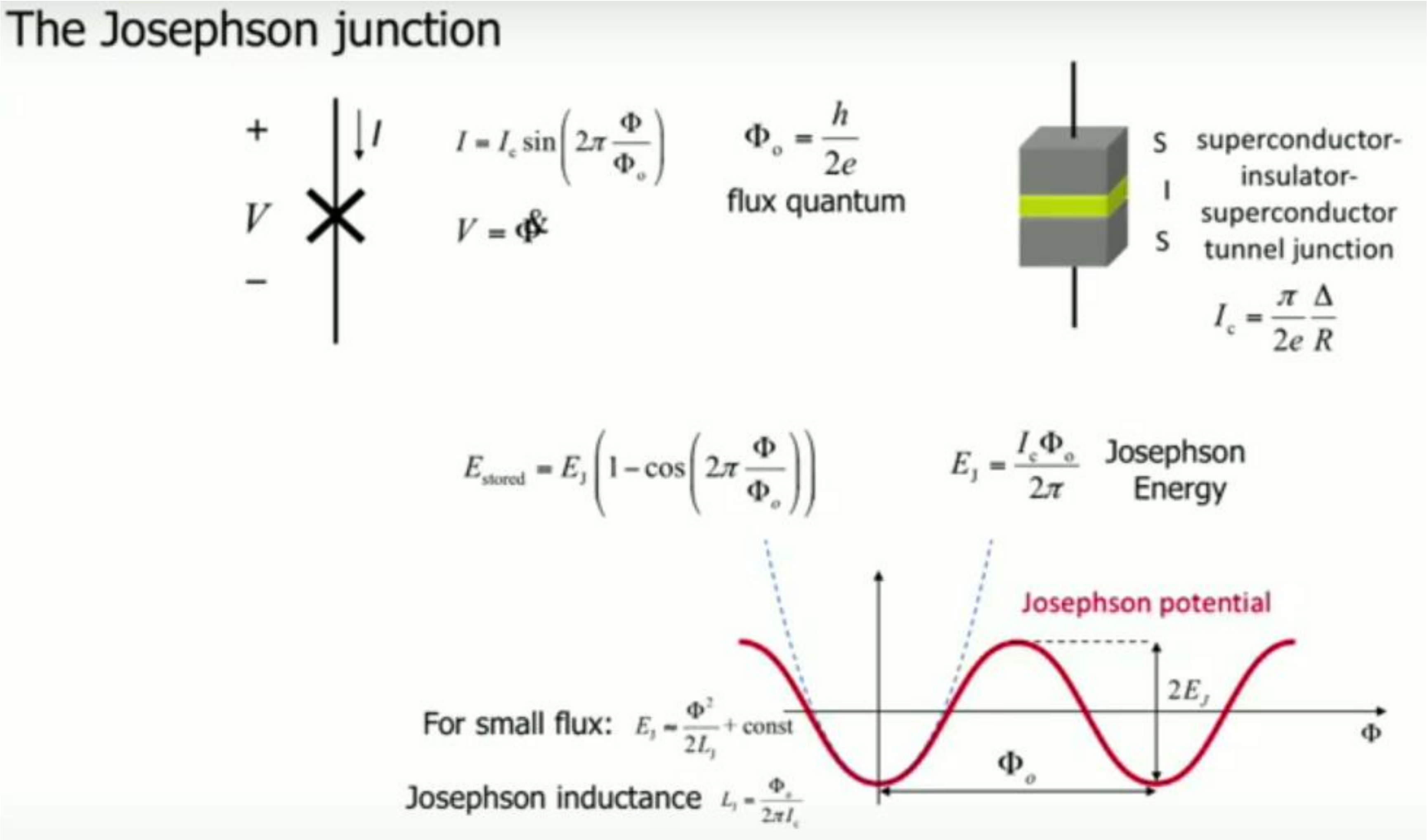
$$\hat{a}^\dagger = \left(\frac{\hat{Q}}{Q_{zpf}} + i\frac{\hat{\Phi}}{\Phi_{zpf}}\right) \qquad Q_{zpf} = \sqrt{2\hbar / Z}$$

$$Z = \omega L = \frac{1}{\omega C} = \sqrt{\frac{L}{C}}$$

$$\hat{H}_{\mathrm{LC}} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) \qquad \left[\hat{a}_r, \hat{a}_r^\dagger\right] = 1$$

M. Devoret, Les Houches Session LXIII (1995)

Non-harmonicity is the key factor



The Josephson junction

$$I = I_c \sin\left(2\pi \frac{\Phi}{\Phi_o}\right)$$

$$V = \dot{\Phi}$$

$$\Phi_o = \frac{h}{2e}$$

flux quantum

S superconductor-
I insulator-
S superconductor
tunnel junction

$$I_c = \frac{\pi}{2e}\frac{\Delta}{R}$$

$$E_{stored} = E_J\left(1-\cos\left(2\pi\frac{\Phi}{\Phi_o}\right)\right)$$

$$E_J = \frac{I_c\Phi_o}{2\pi}$$ Josephson Energy

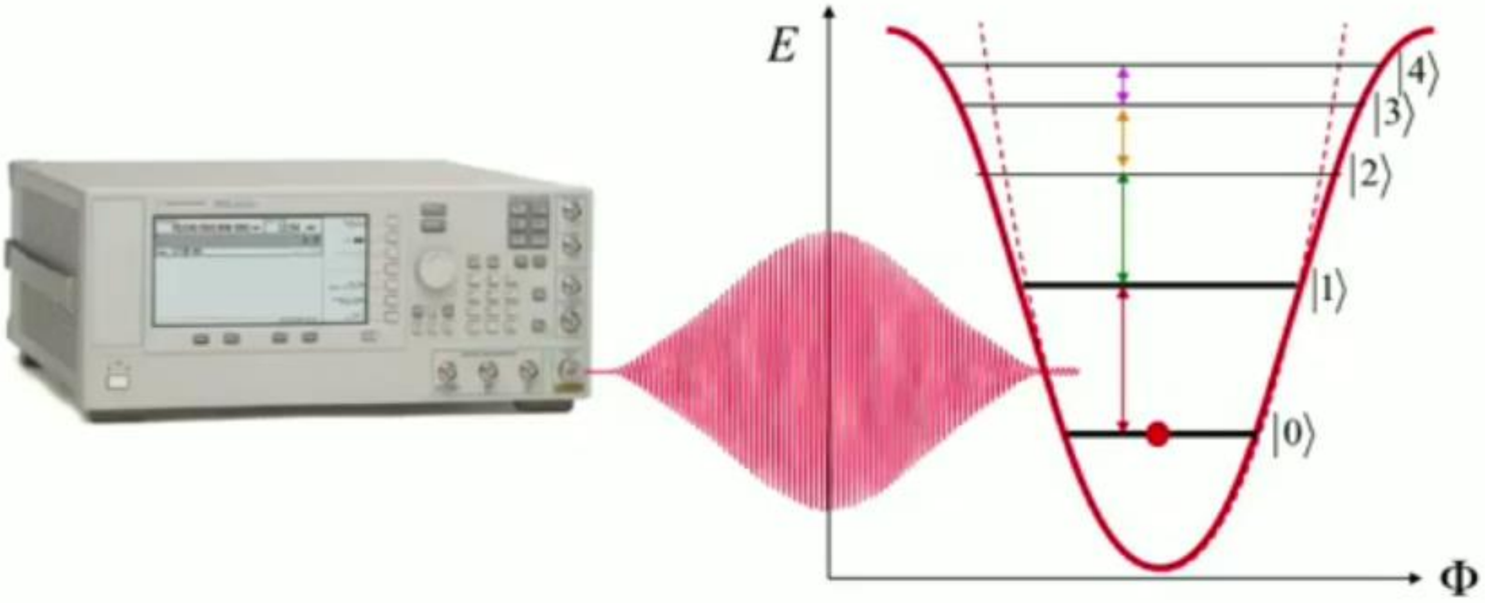
For small flux: $E_J = \frac{\Phi^2}{2L_J} + \text{const}$

Josephson inductance $L_J = \frac{\Phi_o}{2\pi I_c}$

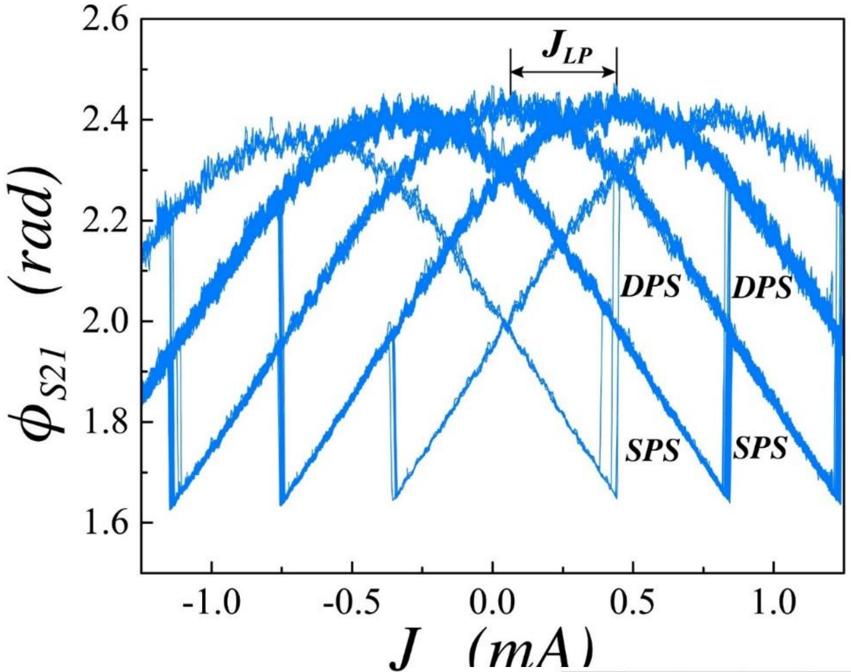
M. Devoret, Les Houches Session LXIII (1995)

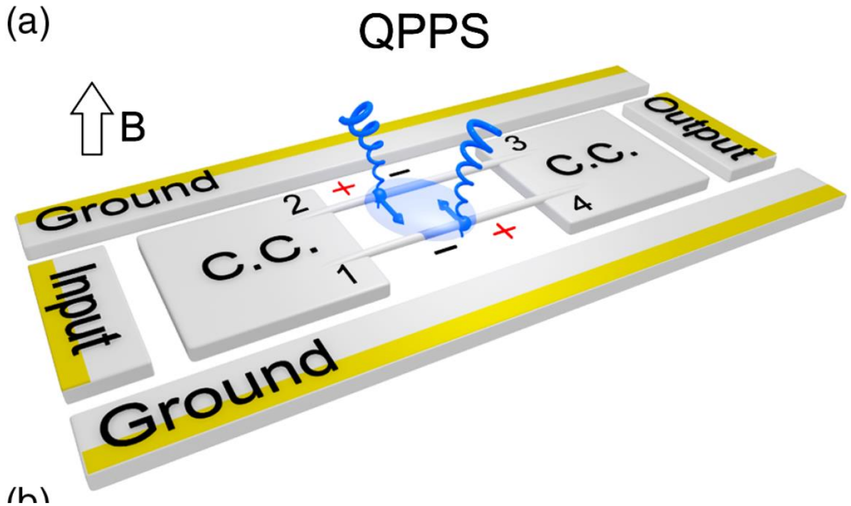
Non-harmonicity is the key factor

Transmon energy spectrum

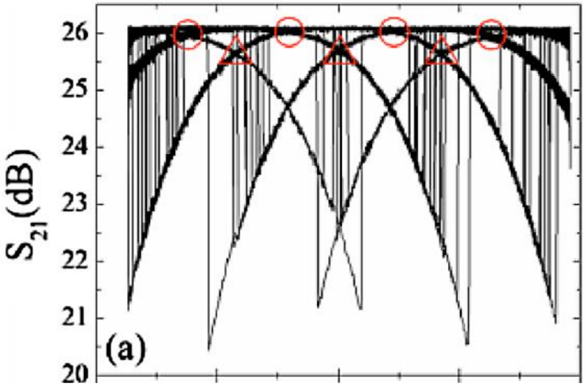


Superconducting nanowire memory: microwave readout





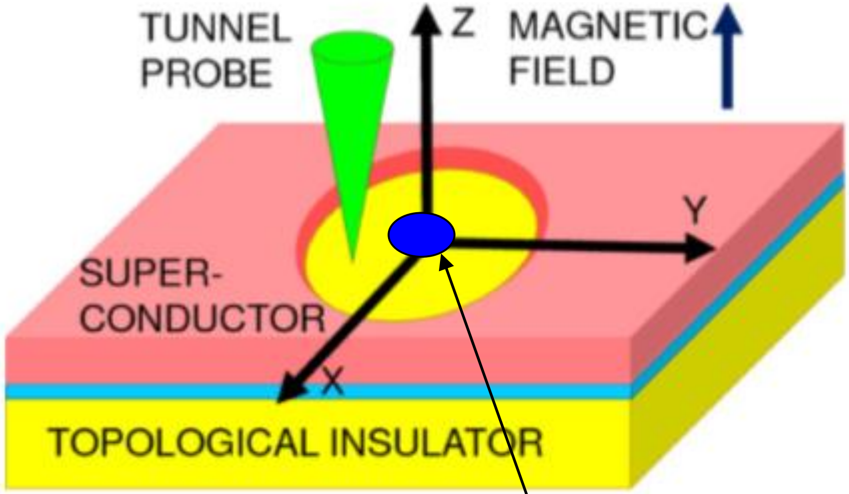
A. Belkin et al, PRX **5**, 021023 (2015)



Andrey Belkin,[a] Matthew Brenner, Thomas Aref, Jaseung Ku, and Alexey Bezryadin,
PPLIED PHYSICS LETTERS **98**, 242504 (2011)

T = 360 mK
$f = f_0(H=0)$

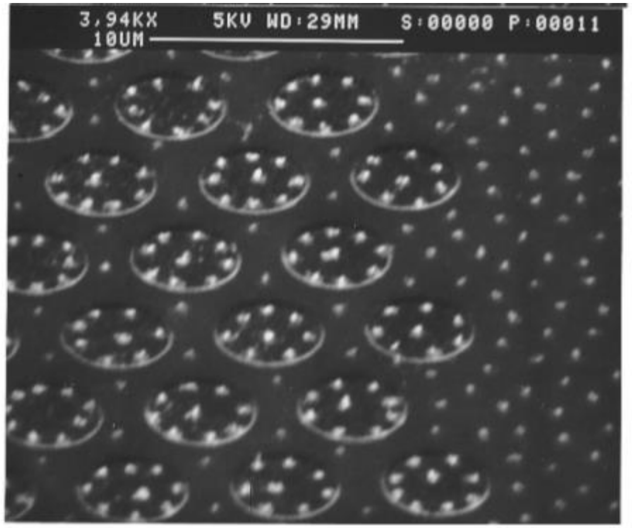
Majorana modes in a vortex



Theory: Vortex in the nano-hole contains Majorana states (gap is about Tc)

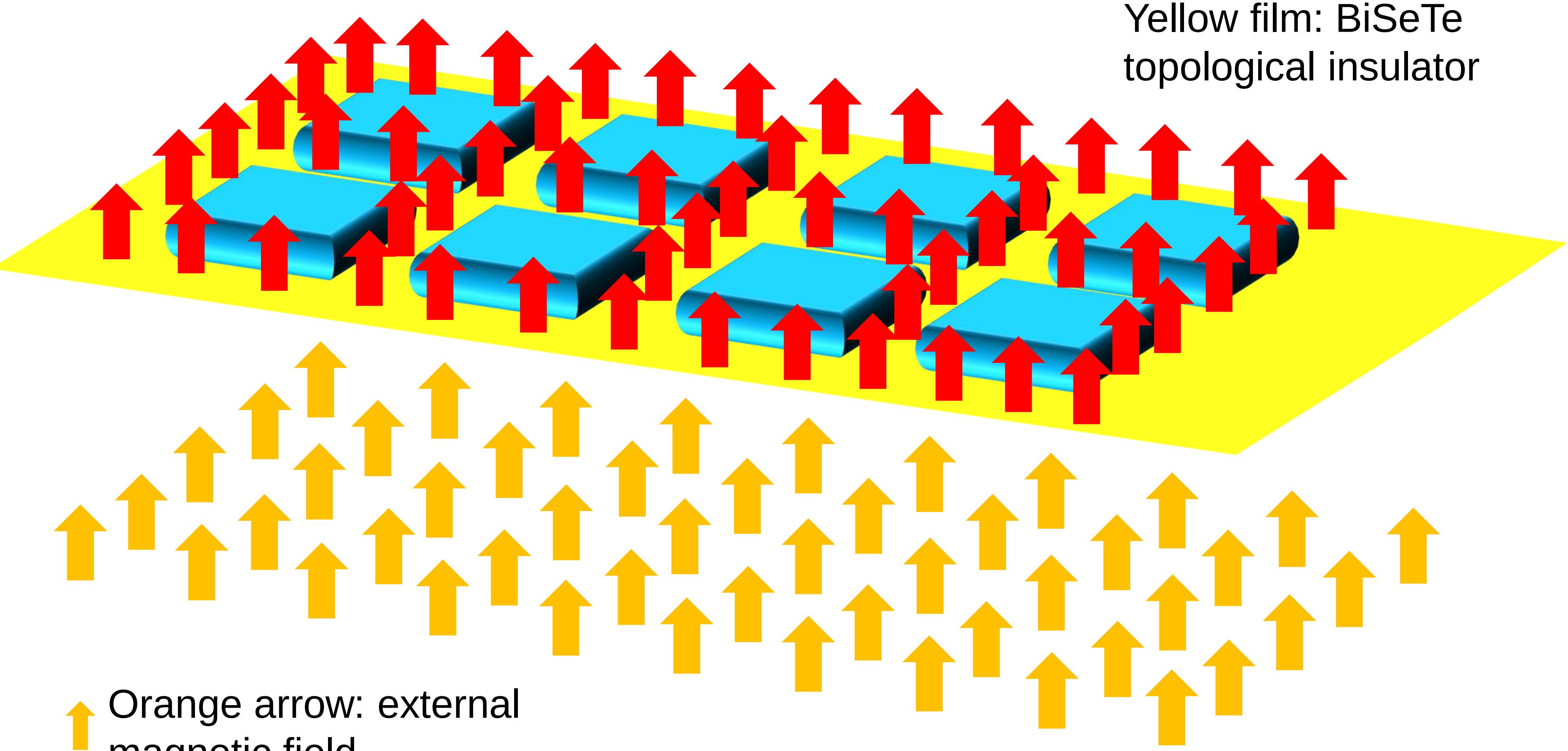
R.S. Akzyanov, A.V. Rozhkov, A.L.Rakhmanov, and F. Nori, PRB 89, 085409 (2014)

PHYSICAL REVIEW B 84, 075141 (2011)



A. Bezryadin, Yu. Ovchinnikov, B. Pannetier, PRB 53, 8553 (1996)

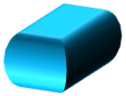
Superconducting array based on topological insulator- BST



Yellow film: BiSeTe
topological insulator

⬆ Orange arrow: external
magnetic field

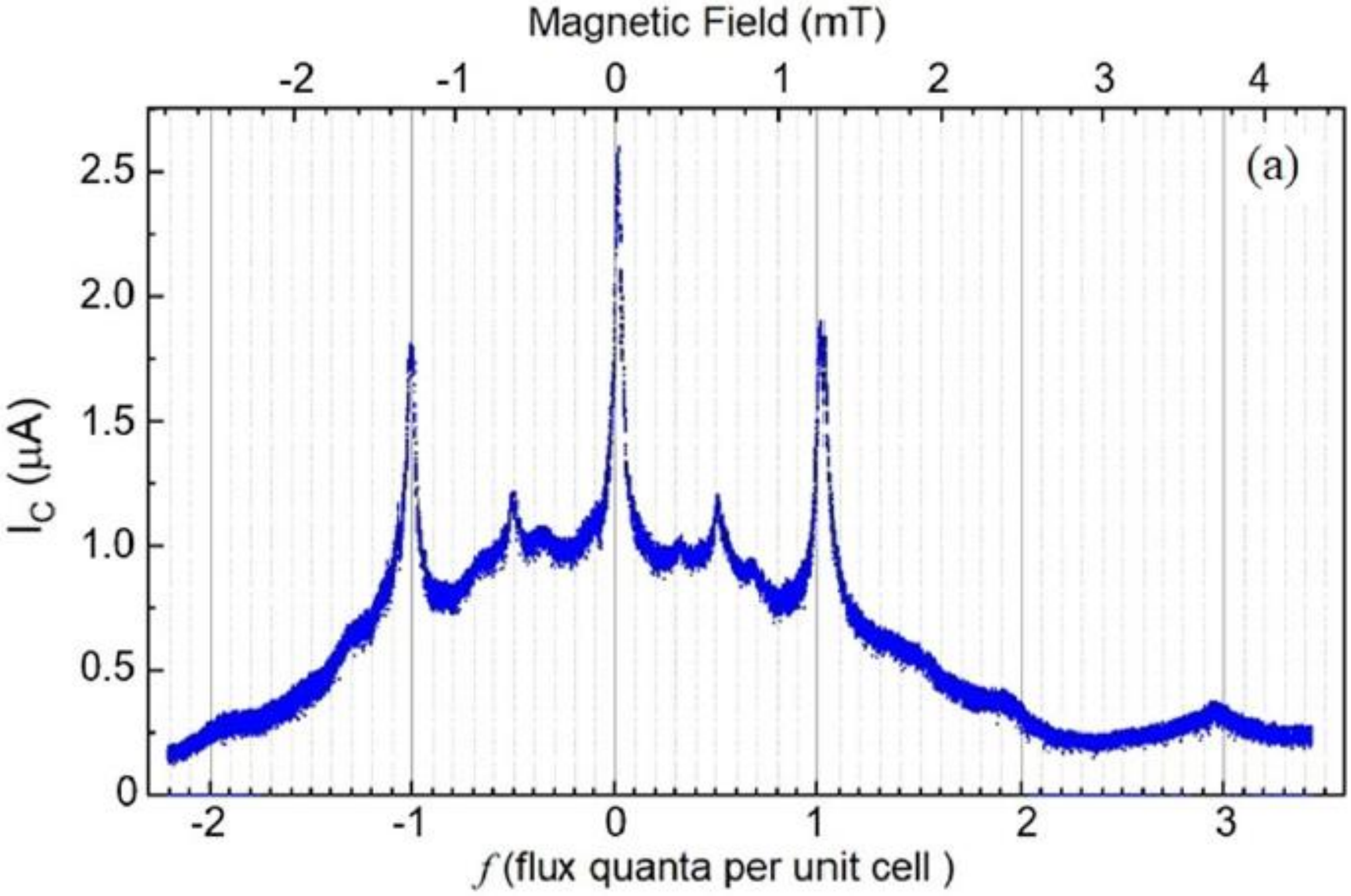
⬆ Red arrow: magnetic field that
penetrates the array



Blue block: Niobium
square island

Xiangyu Song, Soorya Suresh Babu, Yang Bai, Dmitry S. Golubev, Irina Burkova, Alexander Romanov, Eduard Ilin,
James N. Eckstein & Alexey Bezryadin
“Interference, diffraction, and diode effects in superconducting array based on bismuth antimony telluride topological insulator”
COMMUNICATIONS PHYSICS | (2023) 6:177 | https://doi.org/10.1038/s42005-023-01288-9 |www.nature.com/commsphys

Diffraction grating analogy with superconducting arrays

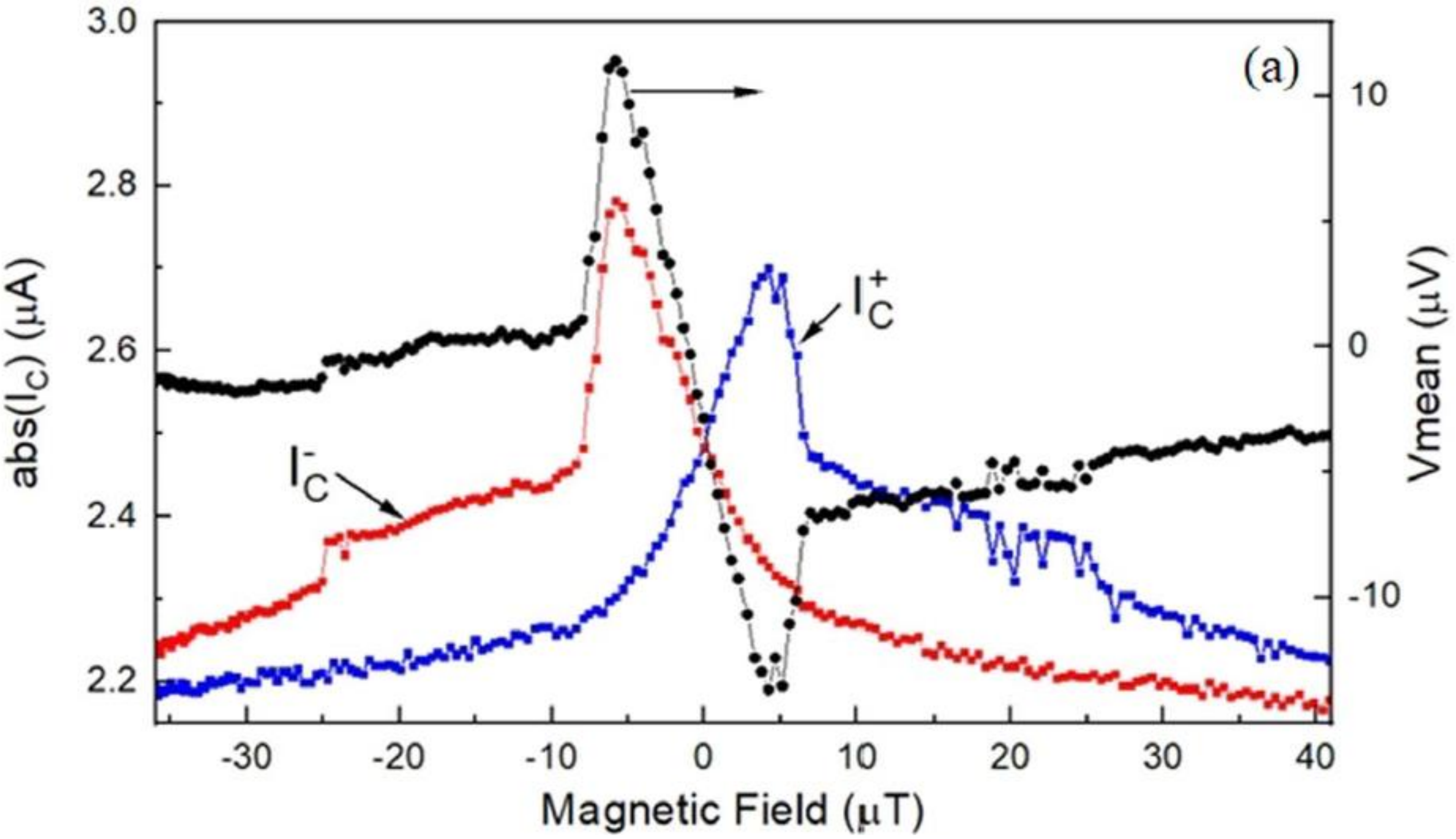


Xiangyu Song, Soorya Suresh Babu, Yang Bai, Dmitry S. Golubev, Irina Burkova, Alexander Romanov, Eduard Ilin, James N. Eckstein & Alexey Bezryadin

“Interference, diffraction, and diode effects in superconducting array based on bismuth antimony telluride topological insulator”

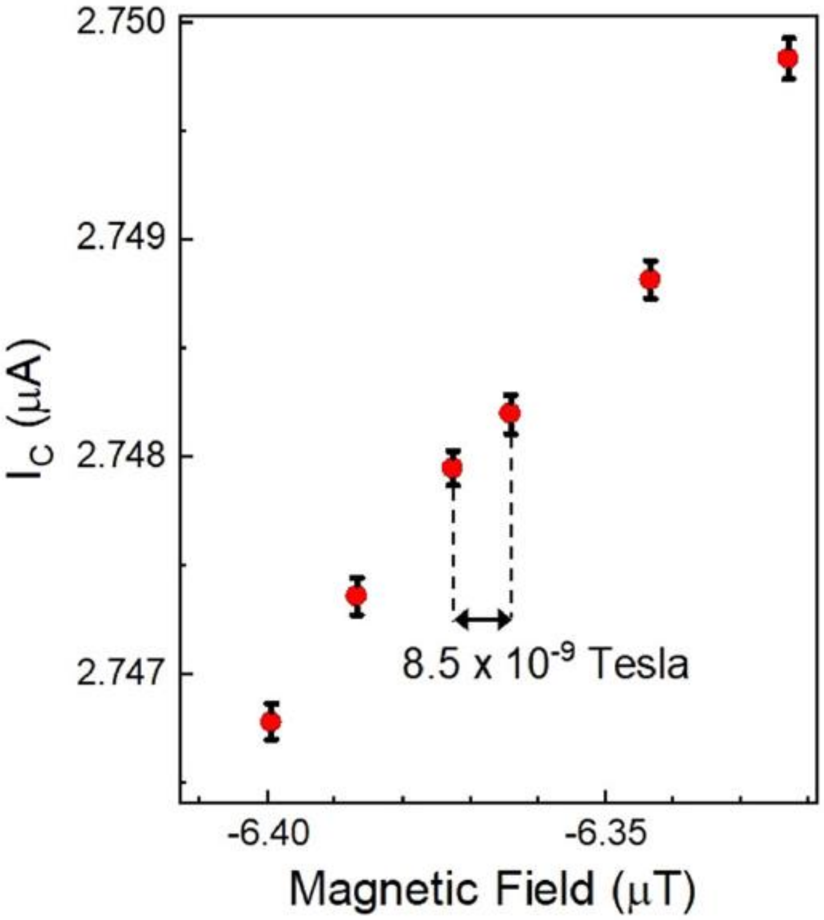
COMMUNICATIONS PHYSICS | (2023) 6:177 | https://doi.org/10.1038/s42005-023-01288-9 |www.nature.com/commsphys

Diode effect with superconducting arrays



Xiangyu Song, Soorya Suresh Babu, Yang Bai, Dmitry S. Golubev, Irina Burkova, Alexander Romanov, Eduard Ilin,
James N. Eckstein & Alexey Bezryadin
“Interference, diffraction, and diode effects in superconducting array based on bismuth antimony telluride topological insulator”
COMMUNICATIONS PHYSICS | (2023) 6:177 | https://doi.org/10.1038/s42005-023-01288-9 |www.nature.com/commsphys

Absolute magnetic field sensor based on superconducting array



Xiangyu Song, Soorya Suresh Babu, Yang Bai, Dmitry S. Golubev, Irina Burkova, Alexander Romanov, Eduard Ilin,
James N. Eckstein & Alexey Bezryadin
“Interference, diffraction, and diode effects in superconducting array based on bismuth antimony telluride topological insulator”
COMMUNICATIONS PHYSICS | (2023) 6:177 | https://doi.org/10.1038/s42005-023-01288-9 |www.nature.com/commsphys

Conclusions

- Superconductivity is related to fundamental quantum phenomena. We have revied some of them. They will be discussed in more details in the future lectures.

- Superconductors have been used to create strong and stable magnetic fields, in levitating trains for example.

- Superconducting quantum interference devices enabled researches to measure very small magnetic fields, such as those produced by human brain.

- Superconductors are used to build qubits, which are the building blocks of quantum computers.