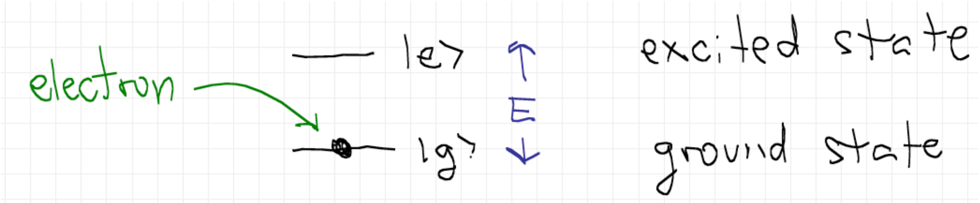
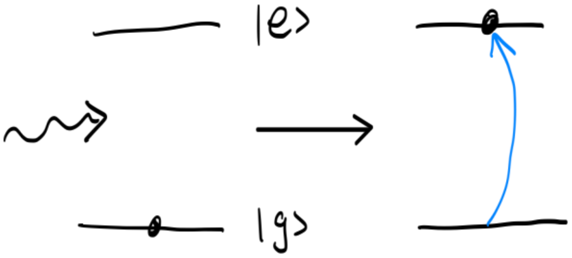
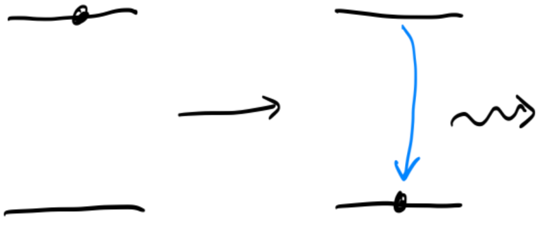
Lasers

As described by Einstein in 1916 and later by Dirac in 1927, the interaction of light with matter can be described in terms of three fundamental processes. Consider two discrete energy levels of an atom or molecule

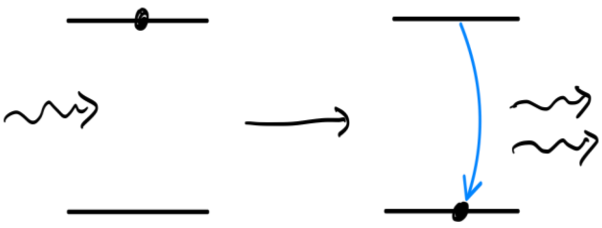




Absorption is the process of promoting the electron from $|g\rangle$ to $|e\rangle$ by annihilating a photon



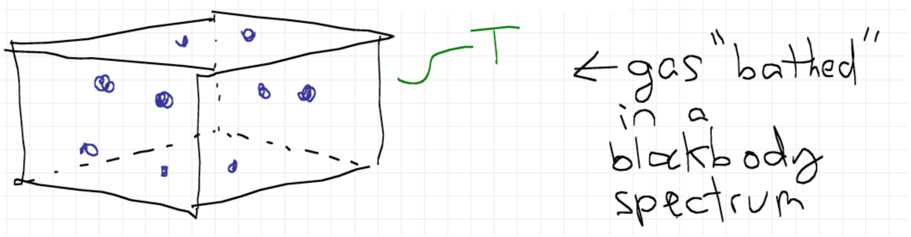
Spontaneous emission is the process of creating a photon by the system decaying from $|e\rangle$ to $|g\rangle$ without any driver from the outside world (the origin of this process is the fact that the electromagnetic field in vacuum is not perfectly zero but actually fluctuates – see e.g. Cohen-Tannoudji, Quantum Mechanics, p. 618).



Stimulated emission is the forced de-excitation of an atom by a photon. Like spontaneous emission, a photon is created, but it has exactly the same frequency, polarization, travel direction and phase as the incident photon.

Population of energy levels

To determine what proportion of atoms or molecules are in the ground and excited states, consider a gas of atoms or molecules contained in a blackbody cavity and held at temperature T:



In 1900 Planck derived the spectral energy density of the electromagnetic field in the cavity to be

$$\rho(\omega) \equiv \left(\frac{dE}{d\omega}\right)\frac{1}{V} = \frac{\hbar}{c^3\pi^2}\frac{\omega^3}{e^{\hbar\omega/k_BT}-1}$$

For a single molecule in the gas, the probability of being in the ground and excited states are given by the Maxwell-Boltzmann distribution (first described in 1860):

$$P(E_g) = \frac{e^{-E_g/k_BT}}{Z}, \qquad P(E_e) = \frac{e^{-E_e/k_BT}}{Z}$$

where Z is a normalization constant. Note that, generally speaking, the molecule has additional states beyond $|g\rangle$ and $|e\rangle$, and we may find the molecule in neither $|g\rangle$ nor $|e\rangle$. Nonetheless, it is clear that

$$\frac{P(E_e)}{P(E_g)} = e^{-(E_e-E_g)/k_BT} = e^{-\hbar\omega/k_BT} \qquad \omega \equiv \frac{E_e - E_g}{\hbar}$$

If there are N molecules in the box, the number in ground and excited states are (at relatively low temperatures)

$$N_g = NP(E_g), \qquad N_e = NP(E_e)$$

Hence the ratio of populations in the excited and ground states is

$$\frac{N_e}{N_g} = e^{-\hbar\omega/k_BT}$$

Rate equations

Now we can consider the role of absorption, spontaneous emission and stimulated emission in the distribution of population.

Absorption increases the population of the $|e\rangle$ state and decrease the population of the ground state. Similarly, emission (either spontaneous or stimulated) decreases N_e and increases N_g. In addition, we expect both absorption and stimulated emission to be proportional to the number of photons passing by the atom, which is proportional to the energy density of the EM field.

Under these assumptions, the evolution of $N_g(t), N_e(t)$ is given by

$$\frac{dN_g}{dt} = \underbrace{A_{eg}N_e}_{\text{rate of spontaneous emission}} - \underbrace{B_{ge}\rho(\omega)N_g}_{\text{absorption rate}} + \underbrace{B_{eg}\rho(\omega)N_e}_{\text{rate of stimulated emission}}$$

$$\frac{dN_e}{dt} = -A_{eg}N_e + \overbrace{B_{ge}\rho(\omega)N_g} - \overbrace{B_{eg}\rho(\omega)N_e}$$

Note that since we only consider here the transitions between $|g\rangle$ and $|e\rangle$

$$\frac{d}{dt}\left(N_g + N_e\right) = 0 \Longrightarrow \frac{dN_g}{dt} = -\frac{dN_e}{dt}$$

For a system in a steady-state

$$\frac{dN_g}{dt} = -\frac{dN_e}{dt} = 0$$

Of course, we still have absorption and emission, but those processes cancel each other. Thus

$$A_{eg}N_e - B_{ge}\rho(\omega)N_g + B_{eg}\rho(\omega)N_e = 0$$

We may now solve for $\rho(\omega)$

$$\rho(\omega) = \frac{\left(\frac{A_{eg}}{B_{eg}}\right)}{\left(\frac{B_{ge}}{B_{eg}}\right)\left(\frac{N_g}{N_e}\right) - 1}$$

In the special case of thermal equilibrium

$$\left(\frac{N_e}{N_g}\right) = e^{-\hbar\omega/k_B T}$$

we obtain:

at thermal equilibrium $\rho(\omega) = \frac{\left(\frac{A_{eg}}{B_{eg}}\right)}{\left(\frac{B_{ge}}{B_{eg}}\right)e^{\hbar\omega/k_B T} - 1}$

The above formula holds in thermal equilibrium (which is a special case of steady-state), and must, therefore, match the Planck's spectral density formula for any temperature:

$$\frac{\left(\frac{A_{eg}}{B_{eg}}\right)}{\left(\frac{B_{ge}}{B_{eg}}\right)e^{\hbar\omega/k_B T} - 1} = \frac{\hbar\omega^3}{c^3\pi^2}\frac{1}{e^{\hbar\omega/k_B T} - 1}$$

The only way this can happen is if

$$B_{ge} = B_{eg} \equiv B \quad \text{and} \quad A_{eg} = \frac{\hbar\omega^3}{\pi^2 c^3}B \equiv A$$

These are called the Einstein A and B coefficients.

The rate equation then becomes:

$$\frac{dN_g}{dt} = AN_e - B\rho(\omega)N_g + B\rho(\omega)N_e$$

The laser

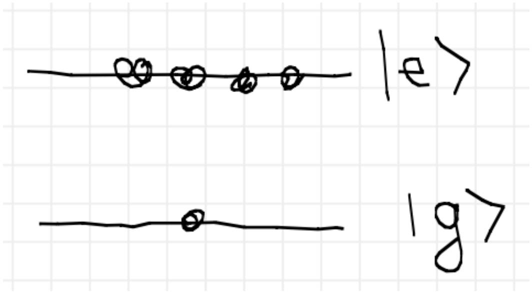
Consider many molecules, some of them, $N_g(t)$, are in the ground state and some, $N_e(t)$, are in the excited state; if we send light into the gas, the excited molecules may emit photons (either through stimulated or spontaneous emission), and ones in the ground state may absorb photons. In order to obtain gain, we need more photons to be emitted than absorbed. The total change in the number of created photons is the same as the change in N_g; therefore, to experience gain we must have

$$\frac{dN_g}{dt} > 0 \Longrightarrow AN_e + B\rho(\omega)\left(N_e - N_g\right) > 0$$

However, that would include photons from spontaneous emission – these photons do not add up coherently and therefore are of no interest. We conclude, then, that in order to increase the number of coherent photons, we need

$$B\rho(\omega)\left(N_e - N_g\right) > 0$$

$$\Longrightarrow N_e > N_g$$



This condition is called population inversion.
Under this condition, existing light, e.g. a photon that originates from spontaneous emission, is coherently amplified in a process called "light amplification by stimulated emission of radiation" – LASER.

The medium that is responsible for the amplification is called the gain medium.

How do we achieve population inversion in the gain medium?

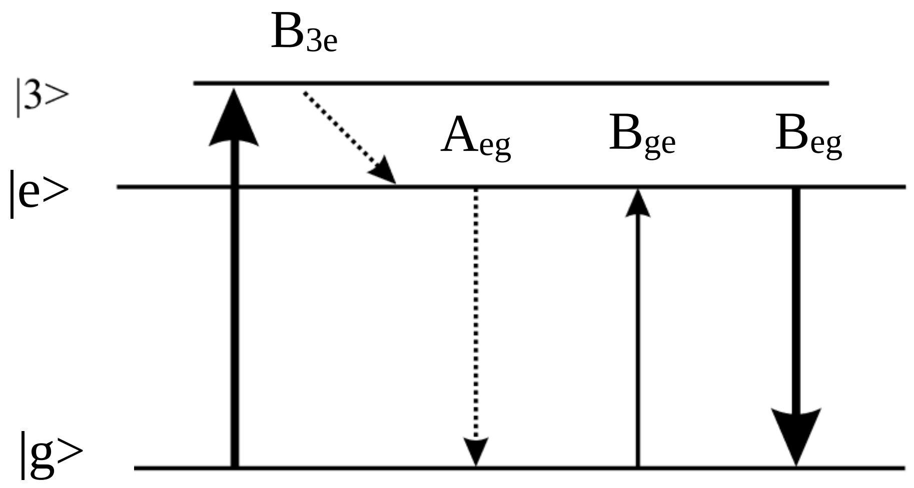
Clearly, this cannot be done by warming up the gas since

$$\frac{N_e}{N_g} = e^{-\hbar\omega/k_BT} < 1$$

It is clear, then, that population inversion cannot occur at thermal equilibrium.

We need somehow to excite the molecule. There are various ways to excite the electrons in the gain medium. The most common ones are done electronically or optically. The process of exciting the matter is called pumping.

In addition, in a two-level system, the symmetry of the absorption and emission process is only broken by spontaneous emission, resulting in $N_e < N_g$. In order to circumvent this problem and to create an inversion, we need to avoid stimulated emission into the pump wave. This is fulfilled if the stimulated emission towards the lower level occurs at a different wavelength from that of the (pump) absorption, and therefore, at minimum three energy levels are needed for lasing.

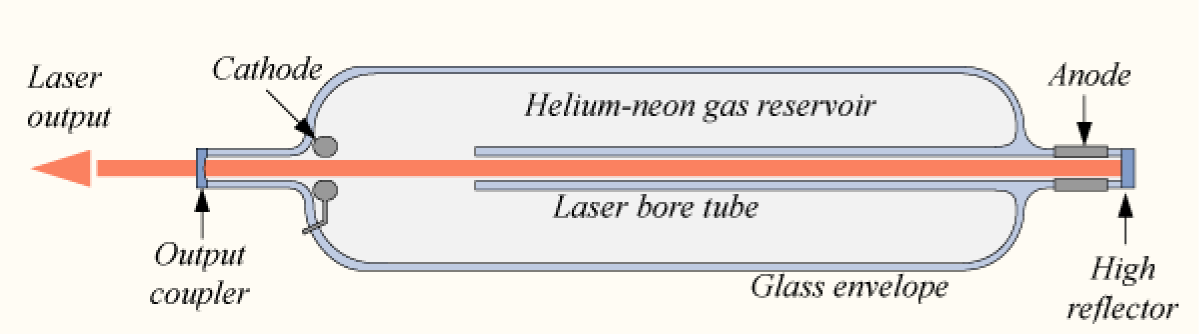


For fast relaxation from 3 → e, when level 3 is pumped, no back-emission into the pump can happen, since the population of level 3 stays low. Then a population inversion can be produced in level e.

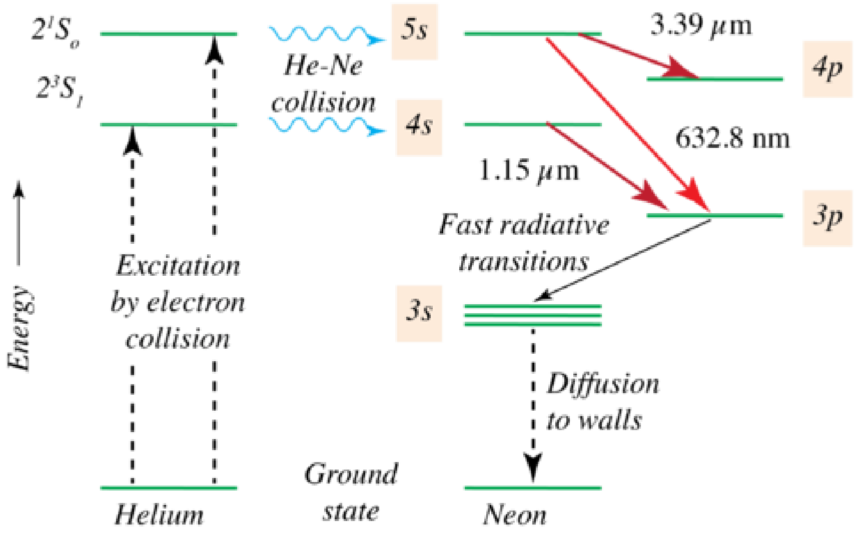
However, at least half the population needs to be inverted to provide gain, resulting in a high lasing threshold (pump power required to initiate laser oscillation), which from the relation between the Einstein coefficients, happens at an energy density

$$\rho_{threshold}(\omega) \gg \frac{A}{B} = \frac{\hbar\omega^3}{\pi^2 c^3}$$

Example: Helium-Neon (He-Ne) laser



A common laser is based on a gas containing Helium and Neon. Pumping is achieved through electrical discharge with high voltage across the gas chamber. The free electrons collide with the gas atoms, exciting them and freeing more electrons that are then accelerated by the applied voltage, creating more excitations. To harness the optical amplification, an optical oscillator is created using a high reflector (~99.9% reflectivity) and an output coupler (~1% transmission). In this way more power in each pass is gained than that lost due to transmission and diffraction. The primary lasing wavelength is at 632.8 nm.



https://en.wikipedia.org/wiki/Helium%E2%80%93neon_laser

Lasers, lecture 2

Review

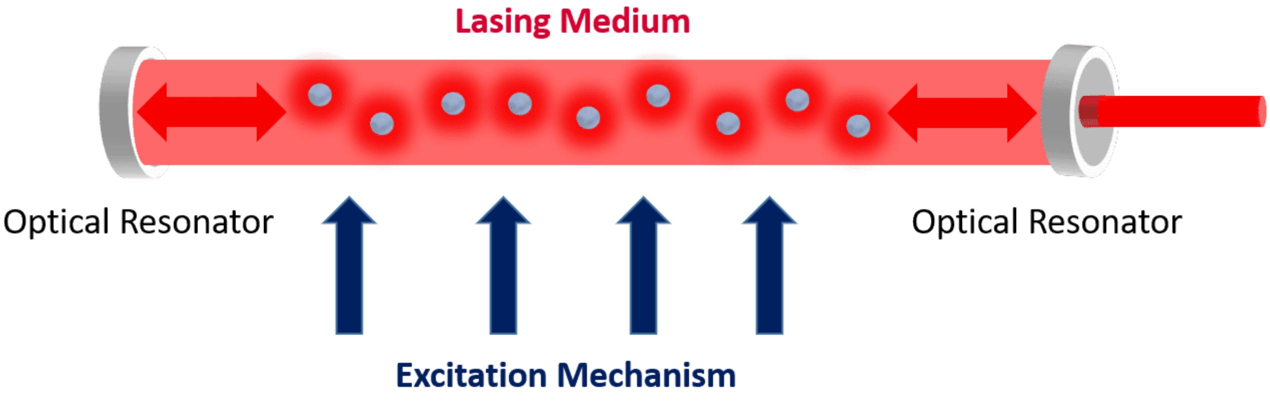
Properties of Laser Light

Light
Amplification by
Stimulated
Emission of
Radiation

Laser light is *coherent*, which means the EM oscillations are in phase. Compared to spontaneously emitted light it can be highly collimated, where the photons travel in the same direction (high directivity, small divergence). Laser light can be monochromatic (one wavelength), but there do exist broadband lasers (with both high and low coherence).

Optical resonant cavity

To generate light by stimulated emission, 3 main components are required:
lasing medium, excitation mechanism, optical resonator



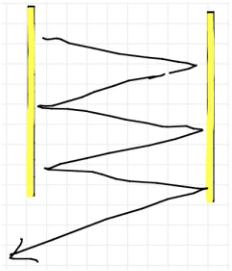
The lasing medium contains atoms that can be stimulated to emit photons. Can be gases, liquids, solids. An excitation mechanism (optical, electrical, chemical) provides the energy to excite the atoms.

While a pumped gain medium can give rise to coherent amplification, in the absence of the initial coherent light to be amplified, there is no mechanism to determine the properties of the generated light and only randomly (amplified) spontaneous emission will emerge.

In order to build a laser, we need <u>optical feedback</u> – a mechanism to take the emitted light that we want to keep and feed it back to the system so that it is amplified.

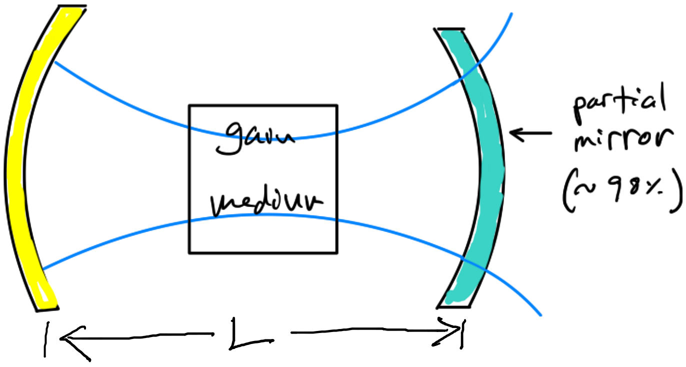
This is most commonly achieved using a resonant cavity, which, in its simplest form, is constructed with a pair of mirrors – a Fabry-Perot etalon.

We could build such a resonator with flat mirrors; however, such a system is leaky: a beam traveling off axis is drifting out of the resonator.



A better design can be achieved with curved mirrors: the cavity defines an optical mode that oscillates back and forth between the mirrors.

The gain medium is put inside the cavity. Due to the feedback imposed by the mirrors, most of the emission is in the mode of the cavity.



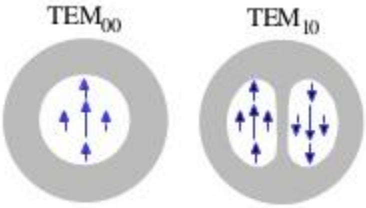
One of the mirrors allows a small portion of the light to leak out of the cavity – this is the light from the laser.

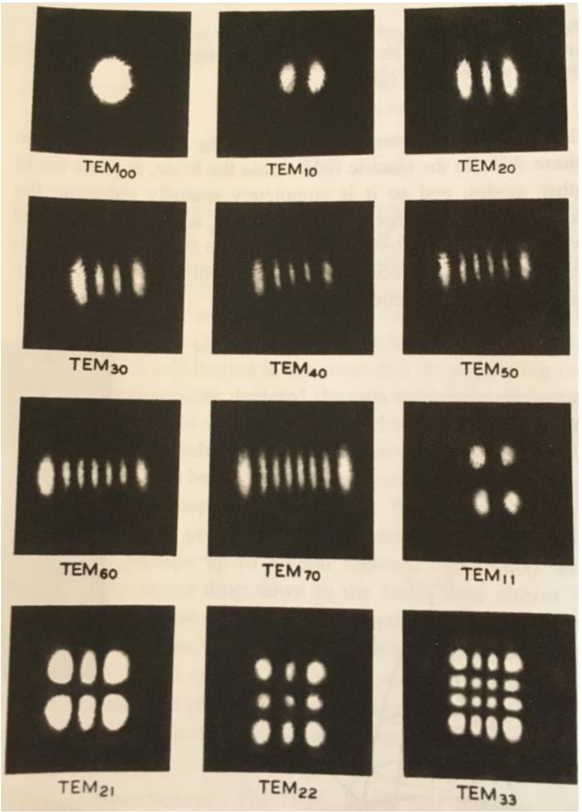
Remember that the cavity supports only wavelengths that interfere constructively after a round trip inside the cavity, that is

$$\text{round trip} = 2L = m\lambda, \qquad \text{or} \quad \lambda_m = \frac{2L}{m}$$

These wavelength/frequency modes are called longitudinal cavity modes. The design needs to make sure that the desired wavelength (or wavelengths), and only that wavelength, is both supported by the cavity and has the associated transition energy $|g\rangle$ to $|e\rangle$.

In addition to longitudinal modes, there are transverse modes that are associated with the cross-sectional profile of the field; they are called transverse electromagnetic, or TEM, modes. The name comes from the fact that the electric and magnetic fields oscillate in a direction transverse to the propagation direction (as in the case of plane-waves modes).



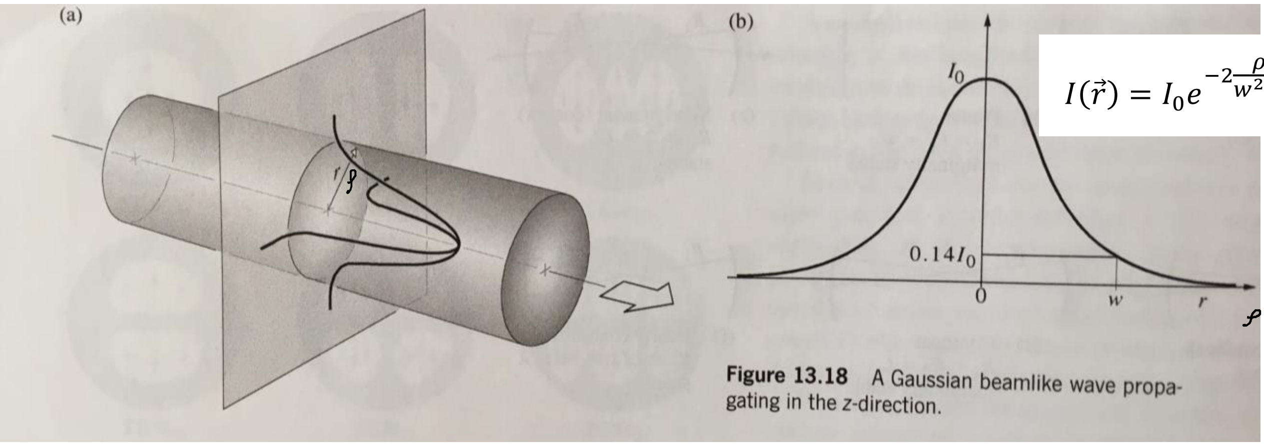


The *intensity* of a laser beam can be given in power per unit area: I [W/cm^2] = Power [W] / Area [cm^2]

In almost all cases, a well-designed laser will only excite the TEM_{00} mode, which has a Gaussian intensity profile:

$$I(\vec{r}) = I_0 e^{-2\frac{\rho^2}{w^2(z)}}$$

where $\rho = \sqrt{x^2 + y^2}$ is the displacement from the center of the propagation axis z.



$$I(\vec{r}) = I_0 e^{-2\frac{\rho^2}{w^2(z)}}$$

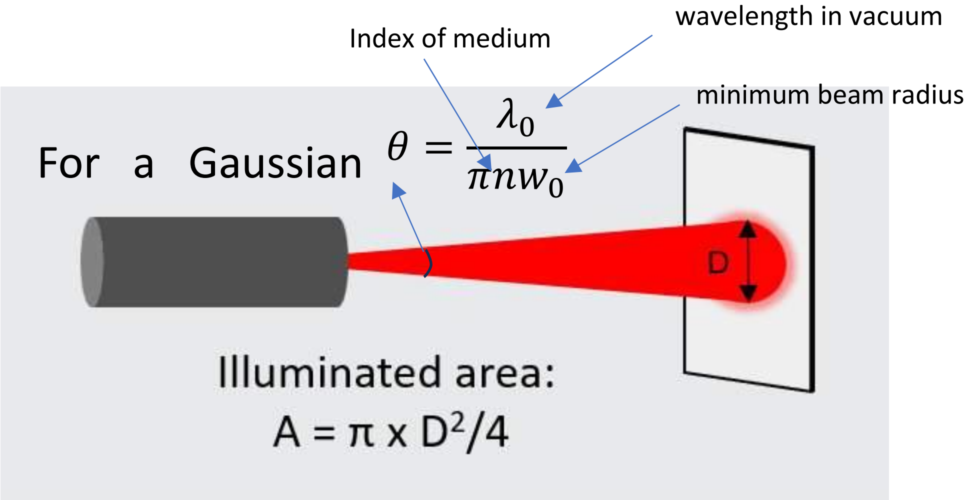
Figure 13.18 A Gaussian beamlike wave propagating in the z-direction.

At any given plane z along the propagation direction, the intensity is a Gaussian function of the distance ρ from the optical axis. The width of the Gaussian profile,

$$w(z) = w_0\left[1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2\right]^{1/2}$$

is changing along the propagation direction. The minimal width of the beam, w_0 (obtained at $z = 0$), is called the beam waist. This minimal width is, in most designs, obtained inside the cavity; however, we can use a lens to refocus the laser beam and create a new waist.

All beams diverge. The larger the beam and the more it diverges, the lower the irradiance or radiant exposure and the less hazardous it becomes.



Index of medium

wavelength in vacuum

minimum beam radius

For a Gaussian $\theta = \frac{\lambda_0}{\pi n w_0}$

Illuminated area:
$A = \pi \times D^2/4$

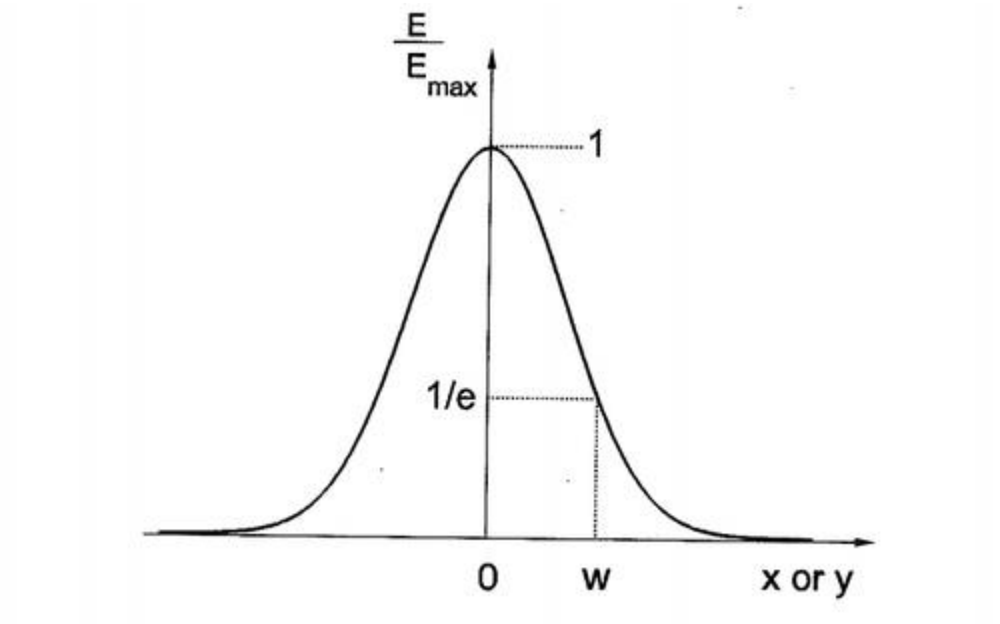
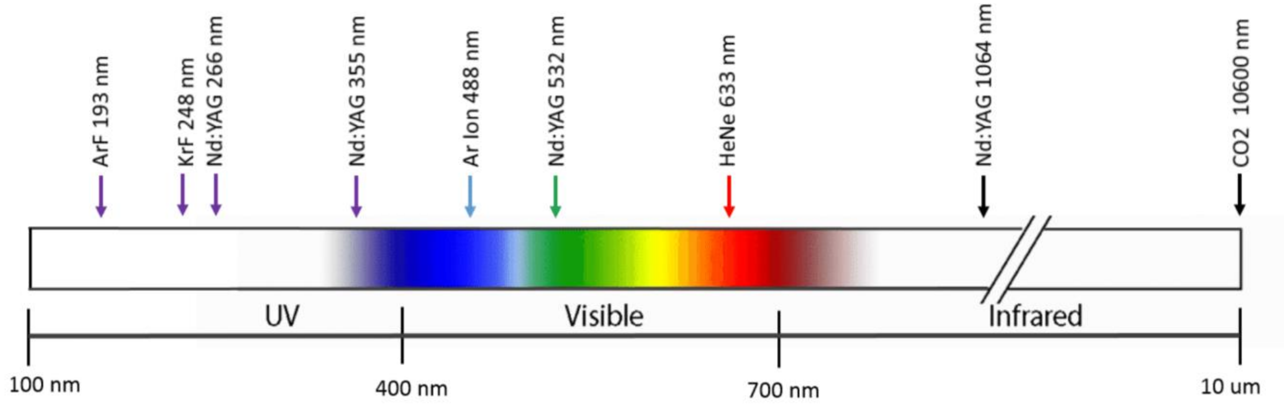


Figure 17-2 Variation of electric field in the radial direction for a Gaussian beam.

For intensity, which is proportional to the electric field squared, the 1/e point is w_0/√2

<u>Spectral output</u>

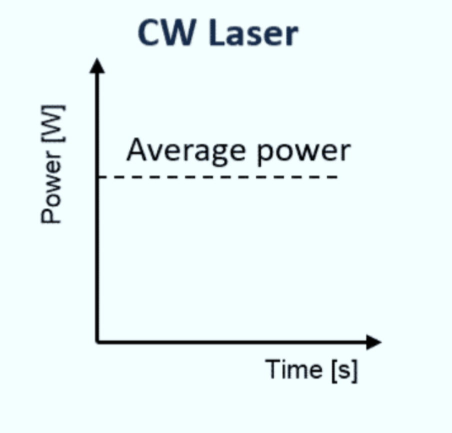
Lasers can emit wavelengths from the ultraviolet (100 nm) to the far infrared (1 mm) The first device to amplify stimulated emission was in the microwave region (12.5 mm) in 1953. It was called a maser, for microwave amplification by stimulated emission, and was used as amplifiers in radio telescopes and space communication. It inspired the work that led to the invention of the laser in 1960.



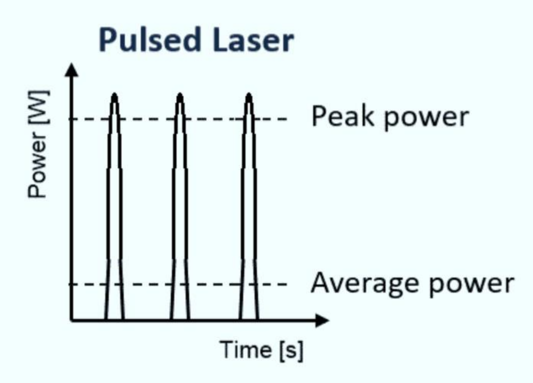
Temporal Output

Lasers can emit continuous wave (CW) light or pulses of light.

Continuous wave lasers have a constant output power. The manufacturer usually provides this power in the unit of Watts (W).



Pulsed lasers emit pulses of light at a regular frequency (repetition rate).



The energy of the pulses and pulse frequency determine the average power of a pulsed laser. However, the power is concentrated within the pulse duration and therefore the peak power is higher.

The output for pulsed lasers is provided as average power in Watts, energy per pulse in Joules, or peak power in Watts. The units can be converted for a given frequency and pulse duration:

Average Power [W] = Energy per pulse [J] x pulses per second [-1]

Peak Power [W] = Energy per pulse [J] / pulse duration [s]

Laser Safety

All lasers sold in the United States are classified based on their output characteristics and their risk of causing eye injuries. The *class* of a laser depends on all its spectral, temporal, and spatial characteristics.

Class 1: weak or fully enclosed (laser printer)

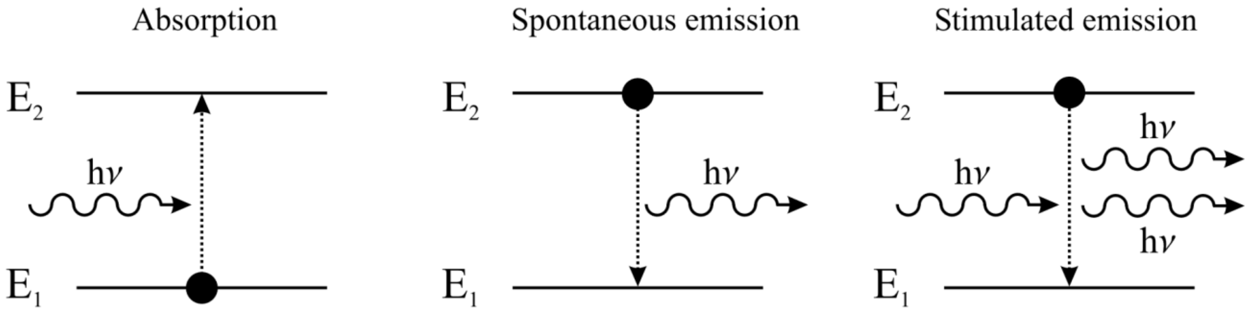
Class 2: weak, visible (barcode scanner)

Class 3: visible, hazardous for direct exposure (e.g. CW diode laser at 405 nm, 250 mW)

Class 4: highest class. Hazards from viewing of the beam directly, diffuse reflections; skin and fire hazard.

PHET demonstration

Quick review: 2-level system with photon interactions



Absorption of a photon of energy hν = E2 − E1 , causing a transition from level 1 to level 2.

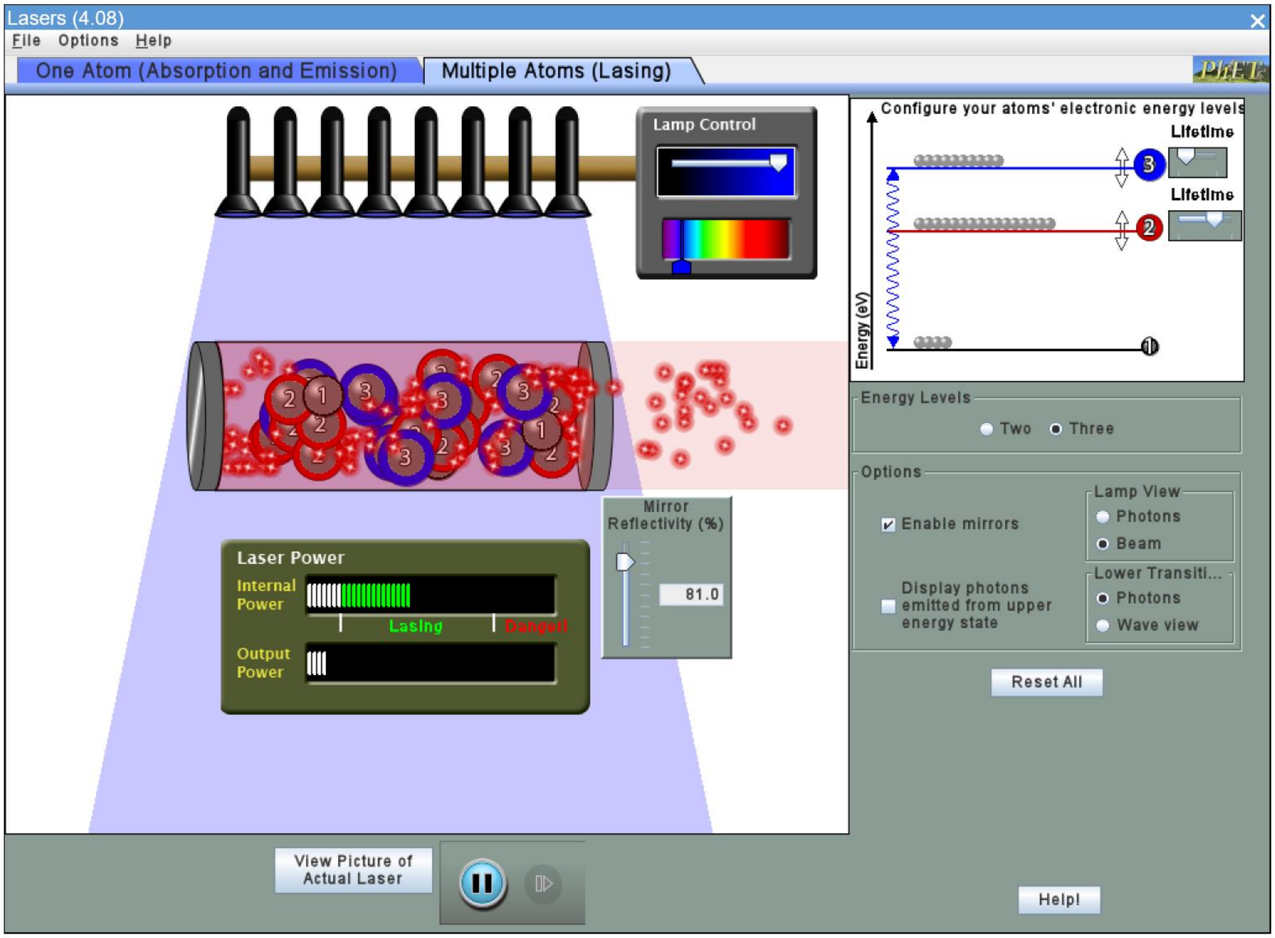
Spontaneous emission of a photon of energy hν by the system returning from level 2 to level 1. The phase, polarization, and direction of the radiation is random. Thus spontaneous emission causes incoherent radiation and is responsible for the fluorescence of excited media.

Stimulated emission: an incoming photon induces a resonant transition from the excited level 2 to level 1, emitting a second photon of energy hν. As photons are Bosons, i.e. they are allowed to be in the same quantum-mechanical state, and as stimulated emission is a resonant process, both photons are identical in all their properties. This effect, therefore, allows the amplification of light, the fundamental process of any laser.

Absorption and stimulated emission are completely equivalent processes.

A 2-level system cannot lase.

https://phet.colorado.edu/sims/cheerpj/lasers/latest/lasers.html?simulation=lasers



Mirror Reflectivity (%) 81.0

Energy Levels: ○ Two ● Three

☑ Enable mirrors

☐ Display photons emitted from upper energy state

Lamp View: ○ Photons ● Beam

Lower Transiti...: ● Photons ○ Wave view