Physics 489 S 04 Lecture 3
Drude theory of transport in metals (Ashcroft and Mermin, chapter 1,3)
Non-Equilibrium Properties

1. Drude theory of transport in metals (Ashcroft and Mermin, chapter 1)
   Model: Z electrons detach from each atom and flow freely through solid.
   Assumptions for classical theory of transport:
   (1) Between collisions, electrons are free and independent.
   (2) Collision randomizes direction of velocity.
   (3) Relaxation time assumption. Mean free time = τ.
   (4) Collision thermalizes magnitude of velocity to local temperature
2. DC electrical conductivity.
   $V = IR$ or $\mathbf{j} = \sigma \mathbf{E}$ with $\sigma = \frac{ne^2\tau}{m}$
   $\tau = \frac{\ell}{v}$; T-dependence through properties of τ or ℓ.
   Classical mean free paths $\ell = \tau v_{gas} \approx 10A$ seemed reasonable to Drude.
3. Semiclassical Theory
   Quantum (Fermi) mean free path $\ell = \tau v_{Fermi} \approx 1000A$ at room T - remarkably long.
   Observed experimentally. ℓ can be very long at low T.
   What about uncertainty principle in using a classical description?
   Valid *only* for $\ell p_f > \hbar/2$.
4. Thermal conductivity.
   $\mathbf{j}_Q = -\kappa \nabla T$ with $\kappa = \frac{1}{3} v^2 \tau c_V$
   Experimental Wiedemann-Franz law $\frac{\kappa}{\sigma T}$ = constant = Lorentz number.
   Drude gives $\frac{\kappa}{\sigma T} = \frac{3}{2}(\frac{k_B}{e^2})^2$ with fortuitous quantitative agreement.
   Classical $c_V = \frac{3}{2} k_B$ becomes Quantum $c_V \propto T$ (100 times smaller).
   Classical $\frac{1}{2} m v^2 = \frac{3}{2} k_B T$ becomes Quantum $\frac{1}{2} m v^2 = E_F$ (100 times larger).
   Semiclassical theory gives $\frac{\kappa}{\sigma T} = \frac{\pi^2}{3}(\frac{k_B}{e^2})^2$ with even better agreement.
5. Thermopower and Seebeck Effect – Discuss briefly
   Must have electric field to prevent current flow in presence of temperature gradient.
   $E = Q \nabla T$, $Q = -\frac{1}{3ne} c_V$

6. Dynamical response: $\frac{d\mathbf{p}}{dt} = \mathbf{f} - \frac{\mathbf{p}}{\tau}$
   AC conductivity: $\sigma(\omega) = \sigma_0 \frac{1}{1-i\omega\tau}$ with $\sigma_0 = \frac{ne^2\tau}{m}$.

7. Optical Properties - response in electromagnetic field.
   Dielectric function $\epsilon(\omega) = 1 + 4\pi i\sigma(\omega) = n(\omega)^2$, where n is the refractive index.
   Wave velocity $= \frac{\omega}{k} = \frac{c}{n(\omega)}$
   Examples:

   - When $\omega\tau << 1$, $\sigma(\omega)$ is mainly real $[\frac{1}{\tau} \approx 10^{13} Hz]$.
     Imaginary part of ϵ gives absorption; large at low ω.
     Real part of n gives wave velocity.
   - When $\omega\tau >> 1$, $\sigma(\omega)$ is mainly imaginary
     $\epsilon(\omega) = 1 - (\frac{\omega_P}{\omega})^2$ is real, with ω_P = plasma frequency
     When $\omega < \omega_P$, n is imaginary, reflectivity = 1, metal is opaque and shiny.
     When $\omega > \omega_P$, n is real and the metal becomes transparent (in ultraviolet).
     When $\omega = \omega_P$, plasma oscillation occurs.

8. Hall effect.
   Must have electric field to prevent transverse current flow in presence of magnetic field.
   $E_y = R_H H_z j_x$ with $R_H = \frac{-1}{nec}$
   Depends on density and sign of charges only.

9. Major failures of free, independent electron theory (Ashcroft and Mermin, chapt. 3)

   - Solid have an enormous range of behaviors:
   - Some solids are insulators; Conductivity = 0 at T=0); No linear T term in specific heat
   - Semiconductors
   - Sometimes hall constant of real metals sometimes have opposite sign.
   - Magnetism, superconductivity, ...