Physics 489 S 04 Lecture 18
Dynamics of Electrons in Crystals - Metals; A & M Chpt. 12

1. In a metal electron states near Fermi energy can respond to applied electric and magnetic fields. For the present discussion we assume fields are slowly varying in time (compared to characteristic electronic time scales).
   Also assume fields are weak (compared to crystal potential U(r)) and slowly varying (compared to atomic dimensions)

2. Dynamics of electrons in metals: equation of motion of a wave packet in the presence of external fields
   Equation of motion (most direct discussion in Kittel; See also A&M, App. E; Kittel, App. E; Madelung, Sec. 2.2)
   $\mathbf{v} = d\mathbf{r}/dt = \nabla_{\mathbf{k}}\omega(\mathbf{k}) = (1/\hbar)\nabla_{\mathbf{k}}\epsilon(\mathbf{k})$
   $\hbar d\mathbf{k}/dt = \mathbf{F} = -|e|(\mathbf{E} + (1/c)\mathbf{v} \times \mathbf{H})$
   Key points:
   In a constant **E** field, **k** changes uniformly for every electron state
   In a constant **H** field, the electrons move on constant energy surfaces

3. Filled bands correspond to insulators:
   A static applied electric field does not lead to a net current - the D.C. conductivity vanishes at temperature =0.
   There can be time dependent "polarization currents" as described in the section on optical properties.
   This leads to the idea of conduction due to "holes" in a band that is not completely filled.

4. Holes are as real as electrons in crystals - a partially filled band can be considered to contain either electrons or holes (but not both)
   Electrons near bottom of band have effective mass $m^* = d^2\epsilon/dk^2$
   Most convenient to use holes for states near the top of bands
   Holes act as particles with positive charge and effective mass $m^* = -d^2\epsilon/dk^2$, which is positive for states near the top of bands

5. Effective mass tensor $m*_{\alpha,\beta} = \pm d^2\epsilon/dk_\alpha dk_\beta$
   Isotropic only in special cases like cubic crystals at k=0

6. Semiclassical motion in H field
   Orbits in **k** space at constant energy perpendicular to **v** and **H**
   $\hbar d\mathbf{k}/dt = -(|e|/c)(d\mathbf{r}/dt \times \mathbf{H}) \;\rightarrow\; \mathbf{r}_\perp(t) - \mathbf{r}_\perp(0) = (-\hbar c/|e|H)(\hat{\mathbf{H}} \times (\mathbf{k}(t) - \mathbf{k}(0))$
   Electron orbits enclose filled states, hole orbits enclose empty states
   Cyclotron frequency $\omega_c = eH/m^*c$ (defines effective mass m^* - full expression in terms of $\epsilon(\mathbf{k})$ given by $m^* \rightarrow (h^2/2\pi)dA(\epsilon, k_z)/d\epsilon$, see A&M p. 231-3)

7. Hall effect - motion in perpendicular E and H fields
   If there are carriers in only one band results are simple: $R_H = -1/nec$ for electrons, $R_H = 1/nec$ for holes
   Can be very complex for many bands (Problem 12.4)
   Examples: Na, Al, Bi

8. Open Orbits give special effects in magnetic field.
   Usual Hall analysis not valid. Can be detected experimentally by magnetoresistance

9. Conductivity of metals [Chapt. 13 of A&M]
   Generalizes ideas already discussed for free electrons - not covered further in this course