Physics 489 S 04 Lecture 19
Measuring the Fermi Surface; A & M Chpt. 14; Kittel, Chpt. 9

1. Quantization of orbits in H field leads to experimentally observable effects related to only fundamental constants and geometrical properties of the orbits

   Most direct experimental demonstration that Fermi Surface exists

   Most direct measurement of the shape of the Fermi Surface in k-space

   Purely quantum effect ("Bohr - van Leeuwen theorem" that no equilibrium property of a classical system can depend upon magnetic field H; N. Bohr, Studier over Metallernes Elektrontheori (thesis), 1911; H. J. van Leeuwen, Vraagstukken uit de Electrontheorie van het Magnetisme (thesis), 1911; H. J. van Leeuwen, Problemes de la Theorie Electronique du Magnetisme, J. Phys. Radium 6:361, 1921.)

2. The same principles as the famous Aharonov-Bohm effect (Phys. Rev. 115, 485 (1959); See review in Rev. Mod. Phys. 57, 339 (1985)): an electron path encircling a magnetic flux interferes constructively if the flux is an integral multiple of Φ_0 = flux quantum = hc/e.

   Hand drawn figure on notes passed out in class

3. In two dimensions electron energies in a free gas $\epsilon = (\hbar^2/2m)(k_x^2+k_y^2)$ become discrete Landau levels in H field with energy $\epsilon = (n+1/2)\hbar\omega_c$,where $\hbar\omega_c = eH/mc$.

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   The number of states per unit area in a Landau level is $N/A = g^0\hbar\omega_c$, where g = density of state in absence of H field $g^0 = m/(2\pi\hbar^2)$.
   Result: There is one orbit per flux quantum $\Phi_0 = hc/e$, i.e., $N/A = H/\Phi_0$.

4. In three dimensions there is still motion parallel to the field (the z direction) so that the electron energies are $\epsilon = (n+1/2)\hbar\omega_c+(\hbar^2/2m)k_z^2$. The energies form a continuum with a one-dimensional density of states for each Landau level, i.e., $g(\epsilon) \propto \epsilon^{-1/2}$, which diverges at each Landau energy $\epsilon = (n+1/2)\hbar\omega_c$.

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5. All properties of the system change as the peaks in $g(\epsilon)$ move through the Fermi energy: specific heat, resistance, magnetic susceptibility, ... all vary as a function of H

   Leads to oscillations with period in 1/H

   $Delta(1/H) = (A_r/\Phi_0) = (1/\Phi_0)(4\pi^2/A_k)$ where Φ_0= flux quantum = hc/e; A_r is the area of an orbit in real space; and A_k is the area of an orbit in k space

   In 2d A_k is the area of the orbit in k space

   In 3d oscillations as function of 1/H occur where A_k is area of extremal orbit. (Non-extremal areas add together to give smooth background.)

6. Typical magnitudes:
   The characteristic temperature below which the effect can be seen is $T \approx \frac{\hbar\omega_c}{k_B} = \frac{e\hbar}{mck_B}H = (1.3 \times 10^{-4}\frac{K}{G})H$
   Typically the experiments can be done at $H \approx 10 - 100KG$ and low temperature is required, $T \approx 1 - 10K$