Physics 489 S 04 Lecture 20
Quantized Hall Effect; Notes, Kittel, 6th or 7th ed, Ch. 19

1. In 2 dimensions it is appropriate to define the Hall resistance $\rho = V_y/I_x$, i.e., the total voltage divided by the total current. The dimensions of the sample cancel out. The simple Hall expression is:
   $\rho = V_y/I_x = \pm H/\nu ec$ where ν is the density of carriers per unit area

2. In 2 dimensions the states are quantized in Landau levels with number of states per unit area for each Landau level $eH/mc = H/\Phi_0$. Thus if the Fermi energy is between Landau level i and level i+1, the density of electrons is the number per unit area in the filled states below the Fermi energy, which is $\nu = iH/\Phi_0$.

3. This is observed! (Nobel prize! – Von Klitzing, Dorda and Pepper, Phys. Rev. Lett. 45, 494 (1980). The Hall resistance jumps between plateaus given by:
   $\rho = V_y/I_x = \pm H/\nu ec = (1/i)(h/e^2) = (1/i)(2\pi/c\alpha)$
   Now the world standard for resistance, the best measurement of the fine structure constant α! $(1/\alpha = \hbar c/e^2)$

4. Perhaps even more important for understanding, the normal effective resistance vanishes ($V_x/I_x = 0$) when the Hall resistance is a quantized value.

   (a) This is related to the effects described before in 3 d. The de Haas Shubnikov effect is the variation of the resistance R with H field. In a three dimensional system as we discussed in Lecture 19, this gives small oscillations in R(H) periodic in 1/H.

   (b) In a two dimensional electron system, the oscillation is dramatic. Paradoxically, both the conductivity σ_{xx} and the effective resistance V_x/I_x both vanish! This can be understood from the form of the resistance tensor in a magnetic field: the 2x2 matrix ρ_{ij} is the matrix inverse of the conductivity matrix σ_{ij}. This is related to analysis of the Hall effect we have done in Chapter 12.

5. Laughlin (Phys. Rev. 23, 5632 (1981) and discussed in Kittel) has given the simple, elegant proof that the Hall effect is indeed quantized exactly even in real samples with impurities. The argument is based upon the behavior of the wave function in a magnetic field and the fact that states are localized (non-conducting) in some energy range between the Landau levels. Thus one arrives at the astounding result that a "dirty" sample with impurities, etc., has an integer Hall effect that is exactly obeyed!

6. Laughlin went on the propose the "fractional quantum Hall effect" when $\nu = 1/$(odd integer) - later observed - Nobel prize in 1998. This results from electron-electron interactions.