Electron Dynamics (of Semiconductors)

Basic assumption: independent electron approximation (single-particle picture)

Question: what happens to an electron under an external field (transport properties)?

Bloch state $\psi_{nk} = e^{ik \cdot r} u_k (r)$

$\mathbf{p} \psi_{nk} = -i \hbar \nabla \psi_{nk} = e^{ik \cdot r} (\mathbf{p} + \hbar \mathbf{k}) u_k (r)$

$\psi_{nk}$ is not a momentum eigenfunction.

**k · p method**: calculating $\varepsilon_n (\mathbf{k}_0 + \mathbf{k})$ from $\varepsilon_n (\mathbf{k}_0)$ by perturbation

$$H_k u_{nk} = \varepsilon_n (\mathbf{k}) u_{nk}$$

$$H_k = \frac{p^2}{2m} + \frac{\hbar}{m} \mathbf{k} \cdot \mathbf{p} + \frac{\hbar^2 k^2}{2m} + U(r)$$

$$H_{k_0 + \mathbf{k}} = H_{k_0} + \frac{\hbar}{m} \mathbf{k} \cdot (\mathbf{p} + \hbar \mathbf{k}_0) + \frac{\hbar^2 k^2}{2m} \equiv H_{k_0} + H' + \frac{\hbar^2 k^2}{2m}$$

$H' =$ perturbation

$$\varepsilon_n (\mathbf{k}_0 + \mathbf{k}) = \varepsilon_n (\mathbf{k}_0) + \langle u_{nk_0} | H' | u_{nk_0} \rangle + \frac{\hbar^2 k^2}{2m} + \sum_{n' \neq n} \frac{\left| \langle u_{nk_0} | H' | u_{nk_0} \rangle \right|^2}{\varepsilon_n (\mathbf{k}_0) - \varepsilon_{n'} (\mathbf{k}_0)} + O(k^3)$$

Taylor expansion
\( \varepsilon_n (k_0 + k) = \varepsilon_n (k_0) + k \cdot \nabla \varepsilon_n (k_0) + \frac{1}{2} (k \cdot \nabla)^2 \varepsilon_n (k_0) + O(k^3) \)

\( = \varepsilon_n (k_0) + \sum_i k_i \frac{\partial}{\partial k_i} \varepsilon_n (k_0) + \frac{1}{2} \sum_{ij} k_i k_j \frac{\partial^2}{\partial k_i \partial k_j} \varepsilon_n (k_0) + O(k^3) \)

Comparing terms \( O(k) \), \( \nabla \varepsilon_n (k_0) = \frac{\hbar}{m} \langle u_{nk_0} | p + \hbar k_0 | u_{nk_0} \rangle = \frac{\hbar}{m} \langle \psi_{nk_0} | p | \psi_{nk_0} \rangle = \hbar \mathbf{v}_n (k_0) \).

Let \( k_0 \to k \) to simplify notation:

Expectation value of \( \mathbf{v} = \text{group velocity} \equiv \mathbf{v}_n (k) = \frac{1}{\hbar} \nabla \varepsilon_n (k) = \nabla \omega_n (k) \)

Comparing terms \( O(k^2) \),

\[ \frac{\hbar^2 k^2}{2m} + \sum_{n \neq n'} \left| \left\langle u_{n'k_0} | \frac{\hbar}{m} (p + \hbar k_0) | u_{nk_0} \right\rangle \right|^2 = \frac{1}{2} \sum_{ij} k_i k_j \frac{\partial^2}{\partial k_i \partial k_j} \varepsilon_n (k_0) \]

\[ \frac{\partial^2 \varepsilon_n (k)}{\partial k_i \partial k_j} = \frac{\hbar^2}{m} \delta_{ij} + \frac{\hbar^2}{m^2} \sum_{n \neq n'} \left[ \left\langle u_{n'k} | p_i + \hbar k_i | u_{nk} \right\rangle \left\langle u_{nk} | p_j + \hbar k_j | u_{n'k} \right\rangle \right] + (i \leftrightarrow j) \]

**Theorem:** if \( \sum_{ij} k_i k_j f_{ij} = \sum_{ij} k_i k_j g_{ij} \) then \( f_{ij}^S = g_{ij}^S \) (symmetric part).

\( f_{ij} = f_{ij}^S + f_{ij}^A \), where \( f_{ij}^S = f_{ji}^S \) (symmetric part) and \( f_{ij}^A = -f_{ji}^A \) (antisymmetric part).
Definition: \((M_n^{-1})_{ij} \equiv \frac{1}{\hbar^2} \frac{\partial^2 \varepsilon_n(k)}{\partial k_i \partial k_j}\) = inverse effective mass tensor for band \(n\) at \(k\)

Making use of \(\langle u_{nk_0} | p + \hbar k_0 | u_{nk_0} \rangle = \langle \psi_{nk_0} | p | \psi_{nk_0} \rangle\)

\[
(M_n^{-1})_{ij} = \frac{1}{m} \delta_{ij} + \frac{1}{m^2} \sum_{n'} \frac{\langle \psi_{n'k} | p_i | \psi_{nk} \rangle \langle \psi_{nk} | p_j | \psi_{n'k} \rangle}{\varepsilon_n(k) - \varepsilon_{n'}(k)} + (i \rightleftharpoons j)
\]

For a free electron gas: \(\varepsilon_n(k) = \frac{\hbar^2 k^2}{2m}\) and all matrix elements vanish.

\[
(M_n^{-1}) = \begin{pmatrix}
\frac{1}{m} & 0 & 0 \\
0 & \frac{1}{m} & 0 \\
0 & 0 & \frac{1}{m}
\end{pmatrix}
\]

, as expected.

In general, \(M_n^{-1}\) = real & symmetric; diagonalizable with a real rotation of axes (principal axes transformation).

\[
(M^{-1})_{ij} = \begin{pmatrix}
\frac{1}{m_1} & \frac{1}{m_2} & \circ \\
\circ & \frac{1}{m_2} & \frac{1}{m_3}
\end{pmatrix}; \quad m_{1,2,3} = \text{effective masses}
\]
\[ \varepsilon_n (k_0 + k) = \varepsilon_n (k_0) + \hbar \mathbf{k} \cdot \mathbf{v}_n (k_0) + \sum_i \frac{\hbar^2 k_i^2}{2m_i} \]

At **critical points** (van Hove singularities), such as **band edges** (CBM and VBM),

\[ \mathbf{v}_n (k_0) = \frac{1}{\hbar} \nabla \varepsilon_n (k_0) = 0 \]

The system is like a free electron gas, but with different masses along different directions.

Referring to the principal axes,

\[ \frac{m}{m_{ni}} = 1 + 2 \sum_{n' \neq n} \frac{2}{m} \left| \left\langle \psi_{n'k_0} | \mathbf{p}_i | \psi_{nk_0} \right\rangle \right|^2 \]

**Semiclassical Model for Electron Dynamics**

Semiclassical regime: wavepackets with \((k)_{\text{ave}}\) and \((r)_{\text{ave}}\).

\[ \Delta k \Delta r \sim 1 \quad \text{(uncertainty principle).} \]

\[ \Delta k \ll BZ \sim \frac{1}{a} \quad \text{(reasonably well defined } k \text{ in BZ).} \]

\[ \Delta r \gg a \quad \text{(large size relative to individual atoms).} \]
\( \lambda \) (wavelength of applied field) \( \gg \Delta r \), and so the wave packet is particle-like.

Summary: semiclassical regime \( \lambda \gg \Delta r \sim \frac{1}{\Delta k} \gg a \)

Assuming a weak external force; no interband transitions; adiabatic approximation; band index \( n \) a good quantum number -- **single-band approximation**. Band index \( n \) is omitted in the following for simplicity.

\[
v = \frac{1}{m} \langle \psi_k | p | \psi_k \rangle = \frac{1}{\hbar} \nabla \varepsilon (k) = \text{average (group) velocity of wavepacket}
\]

\[
\psi_k (r) = e^{i k \cdot r} u_k (r) = e^{i k \cdot r} \sum_k c_{k-K} e^{-iK \cdot r}
\]

Average momentum of wave packet =

\[
\langle \psi_k | p | \psi_k \rangle = \hbar k + \langle u_k | p | u_k \rangle = \hbar k + \left( \sum_{k-K} c_{k-k}^* e^{iK \cdot r} | p | \sum_k c_{k-k} e^{-iK \cdot r} \right) = \hbar k - \sum_K' |c_{k-K}|^2 \hbar K
\]

\[
\sum_K' |c_{k-K}|^2 \hbar K = \Sigma \text{(probability) x (momentum transfer to crystal)} = \text{(momentum of crystal)}
\]

\( \hbar k = \text{total momentum of system (electron + interaction with crystal)} \)

Under an applied field (or force),

\[
\hbar \dot{k} = F = -eE - \frac{e}{c} \mathbf{v} \times \mathbf{H}
\]
\[ adt = d\mathbf{v} = d \left[ \frac{1}{\hbar} \nabla \varepsilon (\mathbf{k}) \right] = d\mathbf{k} \cdot \nabla \left[ \frac{1}{\hbar} \nabla \varepsilon (\mathbf{k}) \right] ; \text{ time dependence is determined by } \mathbf{k}(t). \]

\[ a = \frac{1}{\hbar} \mathbf{k} \cdot \nabla \left[ \nabla \varepsilon (\mathbf{k}) \right] = \frac{\mathbf{F}}{\hbar^2} \cdot \nabla \left[ \nabla \varepsilon (\mathbf{k}) \right]. \]

With \( (M^{-1})_{ij} \equiv \frac{1}{\hbar^2} \frac{\partial^2}{\partial k_i \partial k_j} \varepsilon (\mathbf{k}), \quad a = M^{-1} \cdot \mathbf{F}. \)

Along the principal axes, \( F_i = m_i a_i \) (Newton's law for the wavepacket).

Another way to look at the problem:

\[ d\varepsilon = \mathbf{F} \cdot \mathbf{v} \ dt, \quad \varepsilon = \text{ave. energy of wavepacket}, \quad \mathbf{v} = \text{group velocity} \]

With a weak \( \mathbf{F} \), \( \varepsilon (\mathbf{k}) = \varepsilon (\mathbf{k}(t)); \mathbf{k}(t) \) evolves continuously.

\[ d\varepsilon (\mathbf{k}(t)) = \nabla \varepsilon \cdot \dot{\mathbf{k}} \ dt = \hbar \mathbf{v} \cdot \dot{\mathbf{k}} \ dt = \mathbf{F} \cdot \mathbf{v} \ dt \]

If \( \mathbf{F} \parallel \mathbf{v} \parallel \dot{\mathbf{k}}, \) then \( \mathbf{F} = \hbar \dot{\mathbf{k}}. \) (This derivation does not work for a magnetic field.)

See A&M, Problem 6, Chapter 12 (page 241).
Semiconductors

$0 < E_g$ (fundamental gap) $< \text{a few eV}$; larger $E_g \Rightarrow$ insulators.

**Undoped = intrinsic** (i-type).

Usually doped to be n or p type, with electrons and holes as the majority carriers (differentiated by Hall effect).

Examples: Si and Ge (group IV), GaAs (III-V), CdS (II-VI), GaSe (III-VI, layered), SiC (IV-IV), etc.

CBM = conduction band minimum; VBM = valence band maximum

Difference $= \text{fundamental gap}$

**Direct gap**: CBM and VBM at the same point in $k$ space (usually zone center, $\Gamma$ point)

**Indirect gap**: different places
Si
Indirect gap, 1.1 eV
CBM $\sim$0.8 $\Gamma X$ [100]
CBM, $m_L = 0.98$ $m$, $m_T = 0.19$ $m$
VBM, heavy hole band $m^* = 0.49$ $m$; light hole band $0.16$ $m$; split-off-hole band $0.24$ $m$
Spin-orbit splitting = 0.044 eV

Ge
Indirect gap, 0.66 eV
CBM at $L$, [111] direction
CBM, $m_L = 1.6$ $m$, $m_T = 0.082$ $m$
VBM, heavy hole band $m^* = 0.33$ $m$; light hole band $0.043$ $m$; split-off-hole band $0.084$ $m$
Spin-orbit splitting = 0.29 eV (larger than Si; more relativistic)
GaAs: direct gap 1.4 eV (good for optoelectronic applications)

Typically: $m > 0$ at CBM and $m < 0$ at VBM.

With $k$ measured from CBM and VBM,

$$\varepsilon(k) = \varepsilon_c + \sum_\alpha \frac{\hbar^2 k^2}{2m^c_{e\alpha}}, \text{ where } m^c_{e\alpha} > 0$$

CBM

$$\varepsilon(k) = \varepsilon_v + \sum_\alpha \frac{\hbar^2 k^2}{2m^v_{e\alpha}} = \varepsilon_v - \sum_\alpha \frac{\hbar^2 k^2}{2m^v_{h\alpha}}, \text{ where } m^v_{h\alpha} = -m^v_{e\alpha} > 0$$

VBM

$m_h = \text{hole effective mass}$

**Hole** = one $e^-$ missing in the valence band

Motion of hole = motion of the missing electron (as if it were there), or motion of the nearby electrons

Motion of $e^-$: 

$$F_\alpha = m_{e\alpha} \dot{v}_\alpha = -eE_\alpha - \frac{e}{c}(v \times H)_\alpha$$

Motion of hole: 

$$m_{h\alpha} \dot{v}_\alpha = e E_\alpha + \frac{e}{c}(v \times H)_\alpha$$
• Charge of hole = $e > 0$

• $v_h = v_e$

• Energy of a hole = (total energy of valence band with one $e^-$ missing) – (total energy of full VB; no hole)
  $$\varepsilon_v (k) = \varepsilon_v (0) + \sum_{\alpha} \frac{\hbar^2 k^2_{\alpha}}{2m_{\alpha}^v}$$
  positive dispersion
  That is, larger $k_{\alpha} \rightarrow$ farther away from $\Gamma \rightarrow$ larger energy

• Current density of a hole
  $$\mathbf{J} = \sum_{all \, e^-} (-e) \, \mathbf{v}$$
  with $V = 1$

  $\mathbf{J}$ (one hole in VB) = $\mathbf{J}$ (filled VB) – $\mathbf{J}$ (one electron at $\mathbf{k}$)
\[ 0 - (-e) \mathbf{v}_e = e\mathbf{v}_e = e\mathbf{v}_h \]

= current density of a particle with charge +e and velocity \( \mathbf{v}_h = \mathbf{v}_e \)

Summary for holes:

\[ m^h_{\alpha} \equiv -m^e_{\alpha} > 0 \quad \text{charge} = +e \quad \varepsilon_h = -\varepsilon_e \]

\( \mathbf{v}_h = \mathbf{v}_e \) (hole motion is the same as the \( e^- \) as if it were there).

**Doped Semiconductors**

**n-type doping:**

```
          Ge          Ge            Ge
          ..          ..          ..
Ge : Ge : Ge : As : Ge : Ge
          ..          ..          ..
          Ge          Ge
```
At $T = 0$, the extra electron does not fit into the tetrahedral bonding; it orbits around the As by Coulomb force, and so the As impurity is electrically neutral. The bonding is weak.

At $T > 0$, the extra electron can easily ionize because of the weak bonding. After ionization, it has to go into the conduction band (the valence band is full). This gives rise to n-type conduction.

The As impurity = donor (it donates one electron)

Near CBM, effective mass approx:  
$$
\varepsilon(k) = \varepsilon_c + \sum_{\alpha} \frac{\hbar^2 k^2}{2m_{\alpha}} = \varepsilon_c + \frac{\hbar^2 k^2}{2m^*_e}
$$

where $m^*_e$ = average effective mass of electron (ignoring anisotropy).

Impurity bare Coulomb potential $\phi_{ext} = \frac{e}{r}$. 
Total potential (after screening by dielectric constant) \( \phi = \frac{e}{\varepsilon r} \), where \( \varepsilon = 10-20 \) typically (can be determined from capacitance measurements).

\[
F = m_e^* a = -eE = -e(-\nabla \phi) = \nabla \left( \frac{e^2}{\varepsilon r} \right)
\]

Like the hydrogen atom problem: \( m_e \rightarrow m_e^*; \quad e \rightarrow \frac{e}{\sqrt{\varepsilon}} \).

\[
E_n = -\frac{m_e^* e^4}{2\varepsilon^2 \hbar^2} \frac{1}{n^2} = -\left( \frac{m_e^*}{m_e} \right) \frac{1}{\varepsilon^2} \frac{13.6}{n^2} \text{eV} \quad \text{for bound states; } E \geq 0 \quad \text{for unbound states}
\]
(continuum \( \equiv \) conduction band).

\[
\frac{m_e^*}{m_e} \sim 0.1; \quad \varepsilon \sim 10. \quad |E_1| \equiv E_d \equiv \text{binding energy of donor level } \sim 0.01 \text{ eV}
\]

For comparison, \( \sim kT \sim 0.025 \text{ eV} \) at room temperature; gap \( \sim 1 \text{ eV} \).

So, most impurities are ionized. Intrinsic excitation across the gap is negligible.

Radius of ground state \( r_i = \varepsilon \frac{m_e^*}{m_e} a_0 \sim 10^2 a_0 \sim 50 \text{ Å} \) (effective mass approx OK).
p-type doping:

\[
\begin{align*}
Ge & \quad \vdots \\
Ge & : \quad Ga & : \quad Ge \\
\vdots & \quad Ge
\end{align*}
\]

\( T = 0 \quad \text{Room temperature} \)

Mathematics similar. The majority carriers are holes (+). The Ga impurity = acceptor.
Thermal Excitation of Semiconductor Carriers

$p = \text{hole density in valence band}; \ n = \text{electron density in conduction band}$

**i-type** $(p = n)$  
**p-type** $(p > n)$  
**n-type** $(n > p)$

Carrier generation probability = $f(T) = \text{carrier recombination probability} \ \propto np$

**Law of mass action:** $np = \text{function of } T$

$$n(T) = \int_{\varepsilon_c}^{\infty} D_C(\varepsilon) \frac{1}{\exp \left( \frac{\varepsilon - \mu}{kT} \right) + 1} \, d\varepsilon,$$

where $D_C(\varepsilon) = \frac{1}{2\pi^2} \left( \frac{2m_e}{\hbar^2} \right)^{3/2} (\varepsilon - \varepsilon_C)^{1/2} \Theta(\varepsilon - \varepsilon_C) = \text{CB}$

DOS per unit volume (from free electron gas results, with $m$ replaced by effective mass).

Likewise, $D_V(\varepsilon) = \frac{1}{2\pi^2} \left( \frac{2m_h}{\hbar^2} \right)^{3/2} (\varepsilon_V - \varepsilon)^{1/2} \Theta(\varepsilon_V - \varepsilon)$
\[
p(T) = \int_{-\infty}^{\bar{\varepsilon}_V} D_V(\varepsilon) \left[ 1 - \frac{1}{\exp\left(\frac{\varepsilon - \mu}{kT}\right) + 1} \right] d\varepsilon = \int_{-\infty}^{\bar{\varepsilon}_V} D_V(\varepsilon) \frac{1}{\exp\left(\frac{\mu - \varepsilon}{kT}\right) + 1} d\varepsilon
\]

Because \( \varepsilon - \mu \gg kT \) typically for \( \varepsilon \) within the CB (\( \mu \) is mostly confined within the gap; see below); FD distribution can be approximated by the Boltzmann distribution.

\[
n(T) = \int_{\varepsilon_c}^{\infty} D_c(\varepsilon) \frac{1}{\exp\left(\frac{\varepsilon - \mu}{kT}\right) + 1} d\varepsilon \approx \int_{\varepsilon_c}^{\infty} D_c(\varepsilon) \exp\left(\frac{\mu - \varepsilon}{kT}\right) d\varepsilon
\]

\[
= \exp\left(-\frac{\varepsilon_c - \mu}{kT}\right) \int_{\varepsilon_c}^{\infty} D_c(\varepsilon) \exp\left(-\frac{\varepsilon - \varepsilon_c}{kT}\right) d\varepsilon \equiv \exp\left(-\frac{\varepsilon_c - \mu}{kT}\right) N(T)
\]

Likewise, \( \mu - \varepsilon \gg kT \) typically for \( \varepsilon \) within the VB.

\[
p(T) = \int_{-\infty}^{\tilde{\varepsilon}_V} D_V(\varepsilon) \frac{1}{\exp\left(\frac{\mu - \varepsilon}{kT}\right) + 1} d\varepsilon
\]

\[
= \exp\left(-\frac{\mu - \tilde{\varepsilon}_V}{kT}\right) \int_{-\infty}^{\tilde{\varepsilon}_V} D_V(\varepsilon) \exp\left(-\frac{\varepsilon_V - \varepsilon}{kT}\right) d\varepsilon \equiv \exp\left(-\frac{\mu - \tilde{\varepsilon}_V}{kT}\right) P(T)
\]
$N$ and $P = "quantum concentrations" = integrated DOS weighted by the Boltzmann factor = "effective number of states at the band edge per unit volume"

Within the effective mass approximation:

$$N(T) \equiv \int_{\varepsilon_c}^{\infty} D_c(\varepsilon) \exp\left(-\frac{\varepsilon - \varepsilon_c}{kT}\right) d\varepsilon = 2\left(\frac{m_e kT}{2\pi \hbar^2}\right)^{3/2}$$

$$P(T) \equiv \int_{-\infty}^{\varepsilon_v} D_v(\varepsilon) \exp\left(-\frac{\varepsilon_v - \varepsilon}{kT}\right) = 2\left(\frac{m_h kT}{2\pi \hbar^2}\right)^{3/2}$$

At RT, $N(T) = \left(\frac{m_e}{m_0}\right)^{3/2} \times 2.5 \times 10^{19}$/cm$^3$; $P(T) = \left(\frac{m_h}{m_0}\right)^{3/2} \times 2.5 \times 10^{19}$/cm$^3$

Quantum concentrations are $\sim 10^{19}$/cm$^3$ at RT.

$$np = NP \exp\left(\frac{-\varepsilon_g}{kT}\right) \text{ law of mass action}$$

**Intrinsic case:**

$$n = p = \sqrt{NP} \exp\left(\frac{-\varepsilon_g}{2kT}\right) \equiv n_i$$
Since $\varepsilon_g \gg kT$, the intrinsic carrier density $n_i$ is very small. $n_i = 2\times10^{13}/\text{cm}^3$ for Ge, $1\times10^{10}/\text{cm}^3$ for Si, $2\times10^6/\text{cm}^3$ for GaAs at RT ($\varepsilon_g = 0.66$, 1.1, and 1.4 eV, respectively).

$n$-type:

$n = n_D = \text{donor concentration}; \text{most donors are ionized at RT, and } n_i \text{ is negligible.}$

\[ p = \frac{n_i^2}{n_D} \ll n_i \quad \text{Carriers are essentially all electrons (majority carrier).} \]

$p$-type:  \( p = n_A = \text{acceptor concentration} \)

\[ n = \frac{n_i^2}{n_A} \ll n_i \quad \text{Carriers are essentially all holes (majority carrier).} \]

\[ \mu(T) = ? \quad \frac{n}{p} = \frac{N}{P} \exp \left( \frac{2\mu - \varepsilon_C - \varepsilon_V}{kT} \right) \]

Intrinsic case:

\[ \frac{n}{p} = 1 = \frac{N}{P} \exp \left( \frac{2\mu - \varepsilon_C - \varepsilon_V}{kT} \right) \quad \mu = \frac{\varepsilon_C + \varepsilon_V}{2} + \frac{kT}{2} \ln \left( \frac{P}{N} \right) \]
\[ \varepsilon_F = \mu(0) = \frac{\varepsilon_C + \varepsilon_V}{2} \] at \( T = 0 \); the Fermi level is at midgap.

\[ \ln\left( \frac{P}{N} \right) \] is very small; 0 if the effective masses are the same; \( \sim O(1) \) typically.

So, \( \mu = \frac{\varepsilon_C + \varepsilon_V}{2} + O(kT) \) is typically very close to midgap.

**n-type:**

\[ \frac{n}{p} = \frac{N}{P} \exp\left( \frac{2\mu - \varepsilon_C - \varepsilon_V}{kT} \right) = \frac{n_D}{n_i^2} = \frac{n_D^2}{n_i^2} \] (for "typical" doping)

With \( n_i = \sqrt{NP} \exp\left( \frac{-\varepsilon_g}{2kT} \right) \), \( \mu = \varepsilon_C + kT \ln\left( \frac{n_D}{N} \right) \).

For \( \frac{n_D}{N} = 10^{-2} \) to 1, \( \ln\left( \frac{n_D}{N} \right) \sim -5 \) to 0.

\( 5kT \sim 0.13 \text{ eV} \ll \varepsilon_g \) at RT. So, \( \mu \approx \varepsilon_C \) for typical n doping.

For \( \frac{n_D}{N} \sim 1 \) and higher, degenerate (n\(^+\)) doping; description needs revision (impurity levels are no longer independent).
**p-type:**

For typical doping levels, $\mu = \varepsilon_v$.

**Applications:** p-n junctions; npn, pnp transistors

**Degenerate doping:** high doping ($n^+$ or $p^+$), $\sim 10^{19}/\text{cm}^3$ and above ($\sim$above quantum concentration) $\Rightarrow$ impurity wave functions overlap $\Rightarrow$ band formation, overlapping with CB or VB $\Rightarrow$ metallic conduction (by e or h). Metal-insulator transition (Mott transition).

Radius of impurity ground state wave function $\sim 50$ Å.

Critical $n_D$ or $n_A \sim (50$ Å$)^{-3} \sim 10^{19}/\text{cm}^3$
Optical Properties and Excitons

Optical absorption (or emission) -- energy and momentum conservation

Direct gap: $E_g \sim 1-2 \text{ eV}$, $h\nu \sim 1-2 \text{ eV}$, $k$ (photon) $\sim 10^{-3} \text{ Å}^{-1}$, or $k \sim 10^{-3}$ BZ, $k$ negligible, **vertical transitions**

Indirect gap: phonon-assisted absorption (with the creation or annihilation of a phonon) is much weaker; phonons have $E_{\text{phonon}} << E_g$, but the momenta can be large.

Optical transmission $I = I_0 \exp(-\alpha z)$

$\alpha = \text{absorption coefficient}$
Excitons in direct-gap semiconductors: at low \( T \), direct optical absorption yields a correlated e-h pair. Coulomb attraction: \( V = -\frac{e^2}{\varepsilon r} \). Similar to the hydrogen atom or the impurity problem. Binding energy \( \sim 0.01 \text{ eV} \).

An exciton peak appears \( \sim 0.01 \text{ eV} \) below the absorption edge. Sometimes, a hydrogenic sequence can be observed.

The exciton peak can be the dominant feature in optical emission (because carriers relax quickly to the lowest energy level).

The spectra of real systems are often dominated by impurity emission at lower energies.