Extra Problem: Three parts Show that the reciprocal lattice primitive vectors satisfy the relation

\[
b_1 \cdot (b_2 \times b_3) = \frac{(2\pi)^3}{a_1 \cdot (a_2 \times a_3)}
\]  

(1)

You might want to approach this problem by writing \( b \) in terms of \( a \) and then using the orthogonality relationship between them.

• Show that the volume of a Bravais lattice primitive cell is

\[
V_{\text{cell}} = |a_1 \cdot (a_2 \times a_3)|
\]  

(2)

• Show that the reciprocal lattice of the reciprocal lattice is the original real-space lattice—that is,

\[
2\pi \frac{b_2 \times b_3}{b_1 \cdot (b_2 \times b_3)} = a_1
\]  

(3)

In doing this, you will find the following vector identities useful:

\[
A \times (B \times C) = B(A \cdot C) - C(A \cdot B)
\]  

(4)

\[
(C \times A) \times (A \times B) = (C \cdot A \times B) \cdot A
\]  

(5)

\[
(A \times B) \cdot (C \times D) = (A \cdot C)(B \cdot D) - (A \cdot D)(B \cdot C)
\]  

(6)