Consider a uniform electron gas that interacts via a potential of the form $V(r) = V_0 e^{-r/a}/r$. a) Solve the Hartree-Fock equations for this system for the eigenfunctions and excitation spectrum, $\epsilon(\vec{p})$. Evaluate the Fermi energy $\epsilon_F = \mu$. b) At the Hartree-Fock level, show that the effective mass $m^*$ is determined solely by the exchange contribution. Compute explicitly $m^*$ in the limits $k_F a \ll 1$ and $k_F a \gg 1$. c) Show that the exchange interaction contribution to $\epsilon_F$ is negligible when $k_F a \gg 1$ and that the direct and exchange terms are comparable for a short-range interaction with $k_F a \ll 1$. 